QUADRATIC EQUATION

1. यदि α तथा β, समीकरण $x^2 + (3)^{1/4}x + 3^{1/2} = 0$ के दो भिन्न मूल हैं, तो $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ का मान बराबर है :

(1) 56×3^{25} (2) 56×3^{24}
(3) 52×3^{24} (4) 28×3^{25}

2. समीकरण, $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ के वास्तविक मूलों की संख्या है :

(1) 2 (2) 4 (3) 6 (4) 1

3. यदि समीकरण, $x^2 + 5(\sqrt{2})x + 10 = 0$, के α तथा β, $\alpha > \beta$ दो मूल है तथा $P_n = \alpha^n - \beta^n$,(प्रत्येक धन पूर्णांक n के लिए) है, तो $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ का मान है ________।

4. समीकरण, $x^2 - |x| - 12 = 0$ के वास्तविक हलों की संख्या है :

(1) 2 (2) 3 (3) 1 (4) 4

5. यदि $a + b + c = 1$, $ab + bc + ca = 2$ तथा $abc = 3$ हैं, तो $a^4 + b^4 + c^4$ बराबर है __________।

6. मान α, β समीकरण $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$ के दो मूल हैं। तो $\alpha^8 + \beta^8$ बराबर है :

(1) 10 (2) 100 (3) 50 (4) 160

7. समीकरण $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ के वास्तविक मूलों की संख्या है____।

8. k ($k \neq 0$) के सभी पूर्णांक मानों, जिनके लिए x में समीकरण $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ का कोई वास्तविक मूल नहीं है, का योग है_______।

9. माना $\lambda \neq 0$, **R** में है। यदि समीकरण $x^2 - x + 2\lambda = 0$ के मूल α तथा β है और समीकरण $3x^2 - 10x + 27\lambda = 0$ के मूल α तथा γ है, तो $\frac{\beta\gamma}{\lambda}$ बराबर है ________.

10. यदि $x^2 + 9y^2 - 4x + 3 = 0$, $x, y \in \mathbb{R}$ हैं, तो x तथा y क्रमशः निम्न में से किस अंतराल में है?

(1) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ तथा $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(2) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ तथा $[1, 3]$

(3) $[1, 3]$ तथा $[1, 3]$

(4) $[1, 3]$ तथा $\left[-\frac{1}{3}, \frac{1}{3}\right]$

11. समीकरण $3x^4 + 4x^3 - 12x^2 + 4 = 0$ के भिन्न वास्तविक मूलों की संख्या है _________।

12. $k > -1$ के सभी मानों, जिनके लिए समीकरण $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ के वास्तविक मूल है, का समुच्चय है:

(1) $\left(1, \frac{5}{2}\right]$ (2) $[2, 3)$
(3) $\left[-\frac{1}{2}, 1\right)$ (4) $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

13. cosec18° निम्न में से किस समीकरण का एक मूल है?

(1) $x^2 + 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$
(3) $x^2 - 2x + 4 = 0$ (4) $x^2 - 2x - 4 = 0$

14. जब–जब α समीकरण $x^2 + ax + b = 0$, का एक मूल है, $\alpha^2 - 2$ भी इस समीकरण का एक मूल है। इसके लिए वास्ताविक संख्याओं के युग्मों (a, b) की संख्या है :

(1) 6 (2) 2 (3) 4 (4) 8

15. समीकरण $(x + 1)^2 + |x - 5| = \frac{27}{4}$ के वास्तविक मूलों की संख्या है ________।

16. यदि p तथा q दो धनात्मक संख्याएँ हैं, जिनके लिए $p + q = 2$ तथा $p^4 + q^4 = 272$ हैं, तो p तथा q जिस समीकरण के मूल हैं, वह है:

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$
(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

17. पूर्णांक 'k', जिसके लिए असमिका

$x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$, R में प्रत्येक x के लिए, मान्य है, है :

(1) 3 (2) 2 (3) 0 (4) 4

18. यदि $\alpha, \beta \in R$ है, जिसके लिए $z^2 + \alpha z + \beta = 0$, का एक मूल $1 - 2i$ (यहाँ $i^2 = -1$) है, तो $(\alpha - \beta)$ बराबर है :

(1) –3 (2) –7 (3) 7 (4) 3

19. माना $x^2 - 6x - 2 = 0$ के मूल α तथा β हैं। यदि $n \geq 1$, के लिए $a_n = \alpha^n - \beta^n$ है, तो $\frac{a_{10} - 2a_8}{3a_9}$ का मान है :

(1) 2 (2) 1 (3) 4 (4) 3

20. माना α तथा β दो वास्तविक संख्याऐं हैं जिनके लिए $\alpha + \beta = 1$ तथा $\alpha\beta = -1$ हैं। माना किसी पूर्णांक $n \geq 1$के लिए $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ तथा $p_{n+1} = 29$ हैं। तो p_n^2 का मान है ______

21. समीकरण $\log_4(x - 1) = \log_2(x - 3)$ के हलों की संख्या है _________।

22. माना $f : [-1, 1] \to R$, $f(x) = ax^2 + bx + c$ $\forall x \in [-1, 1]$, a, b, $c \in R$ द्वारा परिभाषित है, जबकि $f(-1) = 2$, $f'(-1) = 1$ हैं तथा $x \in (-1, 1)$ के लिए $f''(x)$ का अधिकतम मान $\frac{1}{2}$ है। यदि $f(x) \leq \alpha$, $x \in [-1, 1]$ है, तो α का निम्नतम मान है______।

23. $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$ का मान बराबर है :

(1) $1.5+\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $3+2\sqrt{3}$ (4) $4+\sqrt{3}$

SOLUTION

1. As, $\left(\alpha^2+\sqrt{3}\right)=-(3)^{1/4}\cdot\alpha$

$\Rightarrow \left(\alpha^4+2\sqrt{3}\alpha^2+3\right)=\sqrt{3}\alpha^2$ (On squaring)

$\therefore\ (\alpha^4+3)=(-)\sqrt{3}\alpha^2$

$\Rightarrow \alpha^8+6\alpha^4+9=3\alpha^4$ (Again squaring)

$\therefore\ \alpha^8+3\alpha^4+9=0$

$\Rightarrow \boxed{\alpha^8=-9-3\alpha^4}$

(Multiply by α^4)

So, $\alpha^{12}=-9\alpha^4-3\alpha^8$

$\therefore\ \alpha^{12}=-9\alpha^4-3(-9-3\alpha^4)$

$\Rightarrow \alpha^{12}=\cancel{-9\alpha^4}+27+\cancel{9\alpha^4}$

Hence, $\boxed{\alpha^{12}=(27)^2}$

$\Rightarrow (\alpha^{12})^8=(27)^8$

$\Rightarrow \alpha^{96}=(3)^{24}$

Similarly $\beta^{96}=(3)^{24}$

$\therefore\ \alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)=(3)^{24}\times 52$

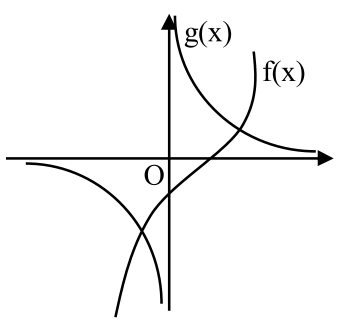
$\Rightarrow$ Option (3) is correct.

2. $e^{6x}-e^{4x}-2e^{3x}-12e^{2x}+e^x+1=0$

$\Rightarrow \left(e^{3x}-1\right)^2-e^x\left(e^{3x}-1\right)=12e^{2x}$

$\left(e^{3x}-1\right)^2\left(e^x-e^{-x}-e^{-2x}\right)=12$

$\Rightarrow \underbrace{e^x-e^{-x}-e^{-2x}}_{\text{increasing (let } f(x))}=\dfrac{12}{\underbrace{e^{3x}-1}_{\text{decreasing (let } g(x))}}$



$\Rightarrow$ No. of real roots = 2

3. $x^2+5\sqrt{2}\,x+10=0$

& $p_n=\alpha^n-\beta^n$ (Given)

Now $\dfrac{P_{17}P_{20}+5\sqrt{2}\,P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}=\dfrac{P_{17}(P_{20}+5\sqrt{2}\,P_{19})}{P_{18}(P_{19}+5\sqrt{2}P_{18})}$

$\dfrac{P_{17}(\alpha^{20}-\beta^{20}+5\sqrt{2}(\alpha^{19}-\beta^{19}))}{P_{18}(\alpha^{19}-\beta^{19}+5\sqrt{2}(\alpha^{18}-\beta^{18}))}$

$\dfrac{P_{17}(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2}))}{P_{18}(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2}))}$

Since $\alpha+5\sqrt{2}=-10/\alpha$ ....(1)

& $\beta+5\sqrt{2}=-10/\beta$ ....(2)

Now put there values in above expression

$=-\dfrac{10P_{17}P_{18}}{-10P_{18}P_{17}}=1$

4. $|x|^2-|x|-12=0$

$(|x|+3)(|x|-4)=0$

$|x|=4\Rightarrow x=\pm 2$

5. $a^2+b^2+c^2=(a+b+c)^2-2\Sigma ab=-3$

$(ab+bc+ca)^2=\Sigma(ab)^2+2abc\Sigma a$

$\Rightarrow \Sigma(ab)^2=-2$

$a^4+b^4+c^4=(a^2+b^2+c^2)^2-2\Sigma(ab)^2$

$=9-2(-2)=13$

6. $\left(x^2+\sqrt{5}\right)^2=\sqrt{20}\,x^2$

$x^4=-5\Rightarrow x^8=25$

$\alpha^8+\beta^8=50$

7. $t^4-t^3-4t^2-t+1=0,\ e^x=t>0$

$\Rightarrow t^2-t-4-\dfrac{1}{t}+\dfrac{1}{t^2}=0$

$\Rightarrow \alpha^2-\alpha-6=0,\ \alpha=t+\dfrac{1}{t}\geq 2$

$\Rightarrow \alpha=3,-2$ (reject)

$\Rightarrow t+\dfrac{1}{t}=3$

$\Rightarrow$ The number of real roots = 2

8. $\frac{2}{x-1}-\frac{1}{x-2}=\frac{2}{k}$

$x \in R - \{1, 2\}$

$\Rightarrow k(2x-4-x+1) = 2(x^2-3x+2)$

$\Rightarrow k(x-3) = 2(x^2-3x+2)$

for $x \neq 3$, $k = 2\left(x-3+\frac{2}{x-3}+3\right)$

$x-3+\frac{2}{x-3} \geq 2\sqrt{2},\ \forall x > 3$

& $x-3+\frac{2}{x-3} \leq -2\sqrt{2},\ \forall x < -3$

$\Rightarrow 2\left(x-3+\frac{2}{x-3}+3\right) \in \left(-\infty, 6-4\sqrt{2}\right] \cup \left[6+4\sqrt{2}, \infty\right)$

for no real roots

$k \in (6-4\sqrt{2}, 6+4\sqrt{2}) - \{0\}$

Integral $k \in \{1, 2\ldots..\ 11\}$

Sum of k = 66

9. $3\alpha^2 - 10\alpha + 27\lambda = 0$ ______(1)

$\alpha^2 - \alpha + 2\lambda = 0$ ______(2)

(1) – 3(2) gives

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (1) we get

$9\lambda^2 - 3\lambda + 2\lambda - 0$

$9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \neq 0$

Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$

$\alpha + \beta = 1 \Rightarrow \beta = 2/3$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}\times 3}{\frac{1}{9}} = 18$

10. $x^2 + 9y^2 - 4x + 3 = 0$

$(x^2 - 4x) + (9y^2) + 3 = 0$

$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$

$(x-2)^2 + (3y)^2 = 1$

$\frac{(x-2)^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$ (equation of an ellipse).

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x-2 \in [-1,1]$ and $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

$x \in [1,3]$ $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

11. $3x^4 + 4x^3 - 12x^2 + 4 = 0$

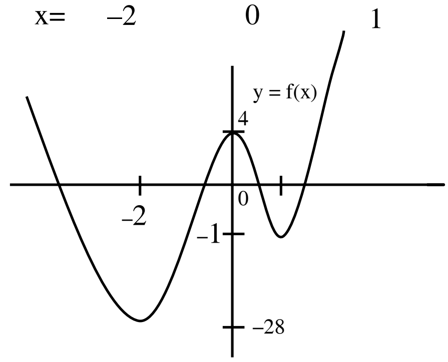
So, Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$\therefore f'(x) = 12x(x^2 + x - 2)$

$= 12x(x+2)(x-1)$

f '(x) = – + – +

x= –2 0 1



12. $(3x^2+4x+3)^2-(k+1)(3x^2+4x+3)(3x^2+4x+2)$
$+k(3x^2+4x+2)^2=0$

Let $3x^2+4x+3=a$

and $3x^2+4x+2=b \Rightarrow b=a-1$

Given equation becomes

$\Rightarrow a^2-(k+1)ab+kb^2=0$

$\Rightarrow a(a-kb)-b(a-kb)=0$

$\Rightarrow (a-kb)(a-b)=0 \Rightarrow a=kb$ or $a=b$ (reject)

$\because a=kb$

$\Rightarrow 3x^2+4x+3=k(3x^2+4x+2)$

$\Rightarrow 3(k-1)x^2+4(k-1)x+(2k-3)=0$

for real roots $D \geq 0$

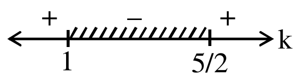
$\Rightarrow 16(k-1)^2-4(3(k-1))(2k-3)\geq 0$

$\Rightarrow 4(k-1)\{4(k-1)-3(2k-3)\}\geq 0$

$\Rightarrow 4(k-1)\{-2k+5\}\geq 0$

$\Rightarrow -4(k-1)\{2k-5\}\geq 0$

$\Rightarrow (k-1)(2k-5)\leq 0$



$\therefore k \in \left[1, \frac{5}{2}\right]$

$\because k \neq 1$

$\therefore k \in \left(1, \frac{5}{2}\right]$ Ans.

13. $\text{cosec}18° = \frac{1}{\sin 18°} = \frac{4}{\sqrt{5}-1} = \sqrt{5}+1$

Let $\text{cosec}18° = x = \sqrt{5}+1$

$\Rightarrow x-1=\sqrt{5}$

Squaring both sides, we get

$x^2-2x+1=5$

$\Rightarrow x^2-2x-4=0$

14. Consider the equation $x^2+ax+b=0$

If has two roots (not necessarily real α & β)

Either $\alpha=\beta$ or $\alpha\neq\beta$

Case (1) If $\alpha=\beta$, then it is repeated root. Given that α^2-2 is also a root

So, $\alpha=\alpha^2-2 \Rightarrow (\alpha+1)(\alpha-2)=0$

$\Rightarrow \alpha=-1$ or $\alpha=2$

When $\alpha=-1$ then $(a, b)=(2, 1)$

$\alpha=2$ then $(a, b)=(-4, 4)$

Case (2) If $\alpha\neq\beta$ Then

(I) $\alpha=\alpha^2-2$ and $\beta=\beta^2-2$

Here $(\alpha, \beta)=(2, -1)$ or $(-1, 2)$

Hence $(a, b)=(-(\alpha+\beta), \alpha\beta)$

$=(-1, -2)$

(II) $\alpha=\beta^2-2$ and $\beta=\alpha^2-2$

Then $\alpha-\beta=\beta^2-\alpha^2=(\beta-\alpha)(\beta+\alpha)$

Since $\alpha\neq\beta$ we get $\alpha+\beta=\beta^2+\alpha^2-4$

$\alpha+\beta=(\alpha+\beta)^2-2\alpha\beta-4$

Thus $-1=1-2\alpha\beta-4$ which implies

$\alpha\beta=-1$ Therefore $(a, b)=(-(\alpha+\beta), \alpha\beta)$

$=(1, -1)$

(III) $\alpha=\alpha^2-2=\beta^2-2$ and $\alpha\neq\beta$

$\Rightarrow \alpha=-\beta$

Thus $\alpha=2, \beta=-2$

$\alpha=-1, \beta=1$

Therefore $(a, b)=(0, -4)$ & $(0, -1)$

(IV) $\beta=\alpha^2-2=\beta^2-2$ and $\alpha\neq\beta$ is same as (III)

Therefore we get 6 pairs of (a, b)

Which are (2, 1), (–4, 4), (–1, –2), (1, –1) (0, –4)

Option (1)

15. **Case-I**

$x \le 5$

$(x+1)^2 - (x-5) = \frac{27}{4}$

$(x+1)^2 - (x+1) - \frac{3}{4} = 0$

$x + 1 = \frac{3}{2}, -\frac{1}{2}$

$x = \frac{1}{2}, -\frac{3}{2}$

Case-II

$x > 5$

$(x+1) + (x-5) = \frac{27}{4}$

$(x+1)^2 + (x+1) - \frac{51}{4} = 0$

$x = \frac{-1 \pm \sqrt{52}}{2}$ (rejected as $x > 5$)

So, the equation have two real root.

16. Consider $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p+q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 - 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$\therefore$ $pq = 16$

$\therefore$ Required equation : $x^2 - (2)x + 16 = 0$

17. $x^2 - 2(3K-1)x + 8K^2 - 7 > 0$

Now, $D < 0$

$\Rightarrow 4(3K-1)^2 - 4 \times 1 \times (8K^2 - 7) < 0$

$\Rightarrow 9K^2 - 6K + 1 - 8K^2 + 7 < 0$

$\Rightarrow K^2 - 6K + 8 < 0$

$\Rightarrow (K-4)(K-2) < 0$

$\Rightarrow \boxed{K \in (2,4)}$

18. $\because \alpha, \beta \in R \Rightarrow$ other root is $1 + 2i$

$\alpha = -(\text{sum of roots}) = -(1 - 2i + 1 + 2i) = -2$

$\beta = \text{product of roots} = (1-2i)(1+2i) = 5$

$\therefore \alpha - \beta = -7$

option (2)

19. $\alpha^2 - 6\alpha - 2 = 0$

$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$

Similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$

$(\alpha^{10} - \beta^{10}) - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$\Rightarrow a_{10} - 6a_9 - 2a_8 = 0$

$\Rightarrow \frac{a_{10} - 2a_8}{3a_9} = 2$

20. $x^2 - x - 1 = 0$ roots $= \alpha, \beta$

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^{n+1} = \alpha^n + \alpha^{n-1}$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^{n+1} = \beta^n + \beta^{n-1}$

+

$P_{n+1} = P_n + P_{n-1}$

$29 = P_n + 11$

$P_n = 18$

$P_n^2 = 324$

21. $\log_4(x-1) = \log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1)^{1/2} = \log_2(x-3)$

$\Rightarrow (x-1)^{1/2} = x - 3$

$\Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x-2)(x-5) = 0$

$\Rightarrow x = 2, 5$

But $x \ne 2$ because it is not satisfying the domain of given equation i.e $\log_2(x-3) \to$ its domain $x > 3$

finally x is 5

$\therefore$ No. of solutions = 1.

22. $f : [-1, 1] \to R$

$f(x) = ax^2 + bx + c$

$f(-1) = a - b + c = 2$ ...(1)

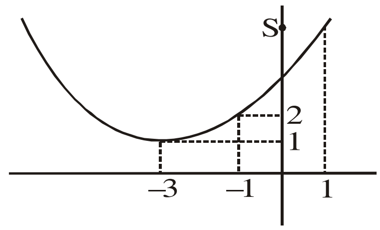
$f'(-1) = -2a + b = 1$ ...(2)

$f''(x) = 2a$

$\Rightarrow$ Max. value of $f''(x) = 2a = \frac{1}{2}$

$\Rightarrow a = \frac{1}{4};\ b = \frac{3}{2};\ c = \frac{13}{4}$

$\therefore f(x) = \frac{x^2}{4} + \frac{3}{2}x + \frac{13}{4}$



For, $x \in [-1, 1] \Rightarrow 2 \le f(x) \le 5$

$\therefore$ Least value of α is 5

24. Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + ...\infty}}}}$

So, $x = 3 + \cfrac{1}{4 + \cfrac{1}{x}} = 3 + \cfrac{1}{\cfrac{4x+1}{x}}$

$\Rightarrow (x-3) = \frac{x}{(4x+1)}$

$\Rightarrow (4x + 1)(x - 3) = x$

$\Rightarrow 4x^2 - 12x + x - 3 = x$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$x = \frac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \frac{12 \pm \sqrt{12(16)}}{8}$

$= \frac{12 \pm 4 \times 2\sqrt{3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$

$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$.

But only positive value is accepted

So, $x = 1.5 + \sqrt{3}$