

METHOD OF DIFFERENTIATION

- फलन  $f(x) = \frac{P(x)}{\sin(x-2)}$ ,  $x \neq 2$ ,  
 $= 7$ ,  $x = 2$   
 का विचार कीजिए, जहाँ  $P(x)$  एक बहुपद है, जिसके लिए  $P''(x)$  हमेशा एक अचर है तथा  $P(3) = 9$  है। यदि  $x = 2$  पर  $f(x)$  संतत है, तो  $P(5)$  बराबर है \_\_\_\_\_।
- माना  $f(x) = \cos\left(2 \tan^{-1} \sin\left(\cot^{-1} \sqrt{\frac{1-x}{x}}\right)\right)$ ,  
 $0 < x < 1$ . है। तो :  
 (1)  $(1-x)^2 f'(x) - 2(f(x))^2 = 0$   
 (2)  $(1+x)^2 f'(x) + 2(f(x))^2 = 0$   
 (3)  $(1-x)^2 f'(x) + 2(f(x))^2 = 0$   
 (4)  $(1+x)^2 f'(x) - 2(f(x))^2 = 0$
- यदि  $y = y(x)$ ,  $x$  का एक अस्पष्ट फलन इस प्रकार है कि  $\log_e(x+y) = 4xy$  है, तो  $x = 0$  पर  $\frac{d^2y}{dx^2}$  बराबर है \_\_\_\_\_।
- यदि  $y^{14} + y^{-14} = 2x$ , तथा  $(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x \frac{dy}{dx} + \beta y = 0$  हैं, तो  $|\alpha - \beta|$  बराबर है \_\_\_\_\_।
- यदि  $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$ ,  
 $x \in \left(\frac{\pi}{2}, \pi\right)$  है, तो  $x = \frac{5\pi}{6}$  पर  $\frac{dy}{dx}$  का मान है :  
 (1)  $-\frac{1}{2}$  (2)  $-1$  (3)  $\frac{1}{2}$  (4)  $0$

- $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} (\sin \sqrt{t}) dt}{x^3}$  बराबर है:  
 (1)  $\frac{2}{3}$  (2)  $\frac{3}{2}$  (3)  $0$  (4)  $\frac{1}{15}$
- माना एक फलन  $f(x)$ ,  $x = a$  पर अवकलनीय है तथा  $f'(a) = 2$  और  $f(a) = 4$  हैं। तो  $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x-a}$  बराबर है :  
 (1)  $2a + 4$  (2)  $4 - 2a$  (3)  $2a - 4$  (4)  $a + 4$
- वक्र  $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$  की अधिकतम प्रवणता निम्न में से किस बिंदु पर है ?  
 (1)  $(2, 2)$  (2)  $(0, 0)$  (3)  $(2, 9)$  (4)  $\left(3, \frac{21}{2}\right)$
- माना  $R$  पर एक फलन  $f$  दो बार अवकलनीय है, जिसके लिए  $f(0) = 1, f'(0) = 2$  तथा  $f'(x) \neq 0 \forall x \in R$  हैं। यदि  $\left| \frac{f(x)}{f'(x)} - \frac{f'(x)}{f''(x)} \right| = 0, \forall x \in R$  है, तो  $f(1)$  का मान जिस अन्तराल में है, वह है :  
 (1)  $(9, 12)$  (2)  $(6, 9)$  (3)  $(0, 3)$  (4)  $(3, 6)$
- माना  $f : S \rightarrow S$ , जहाँ  $S = (0, \infty)$  है, दो बार अवकलनीय फलन है जिसके लिए  $f(x+1) = xf(x)$  है। यदि  $g : S \rightarrow R, g(x) = \log_e f(x)$  द्वारा परिभाषित है, तो  $|g''(5) - g''(1)|$  का मान बराबर है :  
 (1)  $\frac{205}{144}$  (2)  $\frac{197}{144}$  (3)  $\frac{187}{144}$  (4)  $1$
- यदि  $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$  है तथा इसका  $x$  के सापेक्ष प्रथम अवकलज  $-\frac{b}{a} \log_e 2$  है जब  $x = 1$  है, जहाँ  $a$  तथा  $b$  पूर्णांक है, तो  $|a^2 - b^2|$  का न्यूनतम मान है \_\_\_\_\_

## SOLUTION

## 1. Official Ans. by NTA (39)

$$\text{Sol. } f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)}, & x \neq 2 \\ 7, & x = 2 \end{cases}$$

$P''(x) = \text{const.} \Rightarrow P(x)$  is a 2 degree polynomial

$f(x)$  is cont. at  $x = 2$

$$f(2^+) = f(2^-)$$

$$\lim_{x \rightarrow 2^+} \frac{P(x)}{\sin(x-2)} = 7$$

$$\lim_{x \rightarrow 2^+} \frac{(x-2)(ax+b)}{\sin(x-2)} = 7 \Rightarrow \boxed{2a+b=7}$$

$$P(x) = (x-2)(ax+b)$$

$$P(3) = (3-2)(3a+b) = 9 \Rightarrow \boxed{3a+b=9}$$

$$\boxed{a=2, b=3}$$

$$P(5) = (5-2)(2.5+3) = 3.13 = 39$$

## 2. Official Ans. by NTA (3)

$$\text{Sol. } f(x) = \cos \left( 2 \tan^{-1} \sin \left( \cot^{-1} \sqrt{\frac{1-x}{x}} \right) \right)$$

$$\cot^{-1} \sqrt{\frac{1-x}{x}} = \sin^{-1} \sqrt{x}$$

$$\text{or } f(x) = \cos (2 \tan^{-1} \sqrt{x})$$

$$= \cos \tan^{-1} \left( \frac{2\sqrt{x}}{1-x} \right)$$

$$f(x) = \frac{1-x}{1+x}$$

$$\text{Now } f'(x) = \frac{-2}{(1+x)^2}$$

$$\text{or } f(x) (1-x)^2 = -2 \left( \frac{1-x}{1+x} \right)^2$$

$$\text{or } (1-x)^2 f'(x) + 2(f(x))^2 = 0.$$

## 3. Official Ans. by NTA (40)

$$\text{Sol. } \ln(x+y) = 4xy \quad (\text{At } x=0, y=1)$$

$$x+y = e^{4xy}$$

$$\Rightarrow 1 + \frac{dy}{dx} = e^{4xy} \left( 4x \frac{dy}{dx} + 4y \right)$$

$$\text{At } x=0 \quad \boxed{\frac{dy}{dx} = 3}$$

$$\frac{d^2y}{dx^2} = e^{4xy} \left( 4x \frac{dy}{dx} + 4y \right)^2 + e^{4xy} \left( 4x \frac{d^2y}{dx^2} + 4y \right)$$

$$\text{At } x=0, \frac{d^2y}{dx^2} = e^0(4)^2 + e^0(24)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 40$$

## 4. Official Ans. by NTA (17)

$$\text{Sol. } y^{\frac{1}{4}} + \frac{1}{y^{\frac{1}{4}}} = 2x$$

$$\Rightarrow \left( y^{\frac{1}{4}} \right)^2 - 2xy^{\left( \frac{1}{4} \right)} + 1 = 0$$

$$\Rightarrow y^{\frac{1}{4}} = x + \sqrt{x^2 - 1} \quad \text{or } x - \sqrt{x^2 - 1}$$

$$\text{So, } \frac{1}{4} \frac{1}{y^{\frac{3}{4}}} \frac{dy}{dx} = 1 + \frac{x}{\sqrt{x^2 - 1}}$$

$$\Rightarrow \frac{1}{4} \frac{1}{y^{\frac{3}{4}}} \frac{dy}{dx} = \frac{y^{\frac{1}{4}}}{\sqrt{x^2 - 1}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4y}{\sqrt{x^2 - 1}} \quad \dots (1)$$

$$\text{Hence, } \frac{d^2y}{dx^2} = 4 \frac{(\sqrt{x^2 - 1})y' - \frac{yx}{\sqrt{x^2 - 1}}}{x^2 - 1}$$

$$\Rightarrow (x^2 - 1)y'' = 4 \frac{(x^2 - 1)y' - xy}{\sqrt{x^2 - 1}}$$

$$\Rightarrow (x^2 - 1)y'' = 4 \left( \sqrt{x^2 - 1}y' - \frac{xy}{\sqrt{x^2 - 1}} \right)$$

$$\Rightarrow (x^2 - 1)y'' = 4 \left( 4y - \frac{xy'}{4} \right) \quad (\text{from I})$$

$$\Rightarrow (x^2 - 1)y'' + xy' - 16y = 0$$

$$\text{So, } |\alpha - \beta| = 17$$

5. Official Ans. by NTA (1)

$$\text{Sol. } y(x) = \cot^{-1} \left[ \frac{\cos \frac{x}{2} + \sin \frac{x}{2} + \sin \frac{x}{2} - \cos \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2} - \sin \frac{x}{2} + \cos \frac{x}{2}} \right]$$

$$y(x) = \cot^{-1} \left( \tan \frac{x}{2} \right) = \frac{\pi}{2} - \frac{x}{2}$$

$$y'(x) = \frac{-1}{2}$$

6. Official Ans. by NTA (1)

$$\text{Sol. } \lim_{x \rightarrow 0^+} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x^3} = \lim_{x \rightarrow 0^+} \frac{(\sin x)2x}{3x^2}$$

$$= \lim_{x \rightarrow 0^+} \left( \frac{\sin x}{x} \right) \times \frac{2}{3} = \frac{2}{3}$$

7. Official Ans. by NTA (2)

$$\text{Sol. } f'(a) = 2, f(a) = 4$$

$$\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a}$$

$$\Rightarrow \lim_{x \rightarrow a} \frac{f(a) - af'(x)}{1} \quad (\text{Lopitals rule})$$

$$= f(a) - af'(a)$$

$$= 4 - 2a$$

8. Official Ans. by NTA (1)

$$\text{Sol. } \frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$$

Since, slope is maximum so,

$$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0 \quad \left| \quad \frac{d^3y}{dx^3} = 12x - 30 \right.$$

$$x = 2, 3 \quad \left| \quad \text{at } x = 2, \frac{d^3y}{dx^3} < 0 \right.$$

So, maxima

$$\text{at } x = 2$$

$$y = \frac{1}{2} \times 16 - 5 \times 8 + 18 \times 4 - 19 \times 2$$

$$= 8 - 40 + 72 - 38 = 80 - 78 = 2$$

point (2, 2)

9. Official Ans. by NTA (2)

$$\text{Sol. } f(x)f''(x) - (f'(x))^2 = 0$$

$$\frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

$$\ln(f'(x)) = \ln f(x) + \ln c$$

$$f'(x) = cf(x)$$

$$\frac{f'(x)}{f(x)} = c$$

$$\ln f(x) = cx + k_1$$

$$f(x) = ke^{cx}$$

$$f(0) = 1 = k$$

$$f'(0) = c = 2$$

$$f(x) = e^{2x}$$

$$f(1) = e^2 \in (6, 9)$$

10. Official Ans by NTA (1)

$$\text{Sol. } \ln f(x+1) = \ln(xf(x))$$

$$\ln f(x+1) = \ln x + \ln f(x)$$

$$\Rightarrow g(x+1) = \ln x + g(x)$$

$$\Rightarrow g(x+1) - g(x) = \ln x$$

$$\Rightarrow g''(x+1) - g''(x) = -\frac{1}{x^2}$$

Put  $x = 1, 2, 3, 4$

$$g''(2) - g''(1) = -\frac{1}{1^2} \quad \dots(1)$$

$$g''(3) - g''(2) = -\frac{1}{2^2} \quad \dots(2)$$

$$g''(4) - g''(3) = -\frac{1}{3^2} \quad \dots(3)$$

$$g''(5) - g''(4) = -\frac{1}{4^2} \quad \dots(4)$$

Add all the equation we get

$$g''(5) - g''(1) = -\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2}$$

$$|g''(5) - g''(1)| = \frac{205}{144}$$

## 11. Official Ans. by NTA (481)

Sol.  $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$  at  $x=1$ ;  $2^{2x} = 4$

for  $\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$ ;

Let  $\tan^{-1} x = \theta$ ;  $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \sin(\cos^{-1} \cos 2\theta) = \sin 2\theta$

$$\left\{ \begin{array}{l} \text{If } x > 1 \Rightarrow \frac{\pi}{2} > \theta > \frac{\pi}{4} \\ \therefore \pi > 2\theta > \frac{\pi}{2} \end{array} \right\}$$

$$= 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$= \frac{2x}{1+x^2}$$

Hence,  $f(x) = \frac{2 \cdot 2^x}{1+2^{2x}}$

$$\therefore f'(x) = \frac{(1+2^{2x})(2 \cdot 2^x \ln 2) - 2^{2x} \cdot 2 \cdot \ln 2 \cdot 2 \cdot 2^x}{(1+2^{2x})^2}$$

$$\therefore f'(1) = \frac{20 \ln 2 - 32 \ln 2}{25} = -\frac{12}{25} \ln 2$$

So,  $a = 25$ ,  $b = 12 \Rightarrow |a^2 - b^2| = 25^2 - 12^2$   
 $= 625 - 144$   
 $= 481$