

LOGARITHM

1. समीकरण

$$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0,$$

 $x > 0$ के हलों की संख्या है _____।

2. समीकरण

$$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0 \quad \text{के}$$

मूलों का योग है :

(1) $\log_2 14$

(2) $\log_2 11$

(3) $\log_2 12$

(4) $\log_2 13$

SOLUTION

$$1. \quad \log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$$

$$\log_{(x+1)}(2x+5)(x+1) + 2\log_{(2x+5)}(x+1) = 4$$

$$\log_{(x+1)}(2x+5) + 1 + 2\log_{(2x+5)}(x+1) = 4$$

$$\text{Put } \log_{(x+1)}(2x+5) = t$$

$$t + \frac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$$

$$t = 1, 2$$

$$\log_{(x+1)}(2x+5) = 1 \quad \& \quad \log_{(x+1)}(2x+5) = 2$$

$$x+1 = 2x+5 \quad \& \quad 2x+5 = (x+1)^2$$

$$x = -4 \quad (\text{rejected})$$

$$x^2 = 4 \Rightarrow x = 2, -2 \quad (\text{rejected})$$

$$\text{So, } x = 2$$

$$\text{No. of solution} = 1$$

$$2. \quad x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$$

$$\log_2(2^{x+1}) - \log_2(3 + 2^x)^2 + \log_2(10 - 2^{-x}) = 0$$

$$\log_2 \left(\frac{2^{x+1} \cdot (10 - 2^{-x})}{(3 + 2^x)^2} \right) = 0$$

$$\frac{2(10 \cdot 2^x - 1)}{(3 + 2^x)^2} = 1$$

$$\Rightarrow 20 \cdot 2^x - 2 = 9 + 2^{2x} + 6 \cdot 2^x$$

$$\therefore (2^x)^2 - 14(2^x) + 11 = 0$$

$$\text{Roots are } 2^{x_1} \text{ \& } 2^{x_2}$$

$$\therefore 2^{x_1} \cdot 2^{x_2} = 11$$

$$x_1 + x_2 = \log_2(11)$$