

**INDEFINITE INTEGRATION**

1. यदि  $\int \frac{dx}{(x^2+x+1)^2} = a \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b$

$\left(\frac{2x+1}{x^2+x+1}\right) + C, x > 0$  हैं, जहाँ C समाकलन अचर

है, तो  $9(\sqrt{3}a + b)$  का मान बराबर है \_\_\_\_\_ ।

2. यदि  $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14} (ux + v \log_e(4e^x + 7e^{-x})) + C$

है, जहाँ C एक समाकलन अचर है, तो  $u + v$  बराबर है \_\_\_\_\_.

3. समाकलन  $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}} dx$  का बराबर है :

(जहाँ C एक समाकलन अचर है)

(1)  $\frac{3}{4} \left(\frac{x+2}{x-1}\right)^{\frac{1}{4}} + C$       (2)  $\frac{3}{4} \left(\frac{x+2}{x-1}\right)^{\frac{5}{4}} + C$

(3)  $\frac{4}{3} \left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$       (4)  $\frac{4}{3} \left(\frac{x-1}{x+2}\right)^{\frac{5}{4}} + C$

4. यदि  $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx =$

$\alpha \log_e |1 + \tan x| + \beta \log_e |1 - \tan x + \tan^2 x| + \gamma \tan^{-1}\left(\frac{2 \tan x - 1}{\sqrt{3}}\right) + C,$

जहाँ C एक समाकलन अचर है, तो  $18(\alpha + \beta + \gamma^2)$  का मान बराबर है \_\_\_\_\_ ।

5. यदि  $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$

है, जबकि c एक समाकलन अचर है, तो क्रमित युग्म (a, b) बराबर है:

- (1) (-1, 3)                      (2) (3, 1)  
 (3) (1, 3)                      (4) (1, -3)

6. समाकलन

$\int \frac{\sin \theta \cdot \sin 2\theta (\sin^6 \theta + \sin^4 \theta + \sin^2 \theta) \sqrt{2 \sin^4 \theta + 3 \sin^2 \theta + 6}}{1 - \cos 2\theta} d\theta$

बराबर है :

(जहाँ c एक समाकलन अचर है)

(1)  $\frac{1}{18} [11 - 18 \sin^2 \theta + 9 \sin^4 \theta - 2 \sin^6 \theta]^{\frac{3}{2}} + c$

(2)  $\frac{1}{18} [9 - 2 \cos^6 \theta - 3 \cos^4 \theta - 6 \cos^2 \theta]^{\frac{3}{2}} + c$

(3)  $\frac{1}{18} [9 - 2 \sin^6 \theta - 3 \sin^4 \theta - 6 \sin^2 \theta]^{\frac{3}{2}} + c$

(4)  $\frac{1}{18} [11 - 18 \cos^2 \theta + 9 \cos^4 \theta - 2 \cos^6 \theta]^{\frac{3}{2}} + c$

7. समाकलन  $\int \frac{e^{3 \log_e 2x} + 5e^{2 \log_e 2x}}{e^{4 \log_e x} + 5e^{3 \log_e 2x} - 7e^{2 \log_e x}} dx, x > 0,$

बराबर है : (जहाँ c एक समाकलन अचर है)

(1)  $\log_e |x^2 + 5x - 7| + c$

(2)  $4 \log_e |x^2 + 5x - 7| + c$

(3)  $\frac{1}{4} \log_e |x^2 + 5x - 7| + c$

(4)  $\log_e \sqrt{x^2 + 5x - 7} + c$

8. वास्तविक संख्याओं  $\alpha, \beta, \gamma$  तथा  $\delta$  के लिए, यदि

$\int \frac{(x^2 - 1) + \tan^{-1}\left(\frac{x^2 + 1}{x}\right)}{(x^4 + 3x^2 + 1) \tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx$

$= \alpha \log_e \left( \tan^{-1}\left(\frac{x^2 + 1}{x}\right) \right) + \beta \tan^{-1}\left(\frac{\gamma(x^2 - 1)}{x}\right)$

$+ \delta \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + C$  है, जहाँ C एक स्वेच्छ अचर है,

तो  $10(\alpha + \beta\gamma + \delta)$  का मान बराबर है \_\_\_\_\_ ।

9. समाकलन  $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}} dx$  बराबर

है : (जहाँ  $c$  एक समाकलन अचर है)

(1)  $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

(2)  $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(3)  $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

(4)  $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

10. यदि  $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx, (x \geq 0), f(0) = 0$

तथा  $f(1) = \frac{1}{K}$  हैं, तो  $K$  का मान है \_\_\_\_\_।

SOLUTION

1. Official Ans. by NTA (15)

Sol. 
$$I = \int \frac{dx}{\left[ \left( x + \frac{1}{2} \right)^2 + \frac{3}{4} \right]^2}$$

$$\int \frac{dt}{\left( t^2 + \frac{3}{4} \right)^2} \quad \left( \text{Put } x + \frac{1}{2} = t \right)$$

$$= \frac{\sqrt{3}}{2} \int \frac{\sec^2 \theta d\theta}{\frac{9}{16} \sec^4 \theta} \quad \left( \text{Put } t = \frac{\sqrt{3}}{2} \tan \theta \right)$$

$$= \frac{4\sqrt{3}}{9} \int (1 + \cos 2\theta) d\theta$$

$$= \frac{4\sqrt{3}}{9} \left[ \theta + \frac{\sin 2\theta}{2} \right] + c$$

$$= \frac{4\sqrt{3}}{9} \left[ \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + \frac{\sqrt{3}(2x+1)}{3+(2x+1)^2} \right] + c$$

$$= \frac{4\sqrt{3}}{9} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + \frac{1}{3} \left( \frac{2x+1}{x^2+x+1} \right) + c$$

Hence,  $9(\sqrt{3}a + b) = 15$

2. Official Ans. by NTA (7)

Sol. 
$$\int \frac{2e^x}{4e^x + 7e^{-x}} dx + 3 \int \frac{e^{-x}}{4e^x + 7e^{-x}} dx$$

$$= \int \frac{2e^{2x}}{4e^{2x} + 7} dx + 3 \int \frac{e^{-2x}}{4 + 7e^{-2x}} dx$$

Let  $4e^{2x} + 7 = T$       Let  $4 + 7e^{-2x} = t$

$8e^{2x} dx = dT$        $-14e^{-2x} dx = dt$

$2e^{2x} dx = \frac{dT}{4}$        $e^{-2x} dx = -\frac{dt}{14}$

$$\int \frac{dT}{4T} - \frac{3}{14} \int \frac{dt}{t}$$

$$= \frac{1}{4} \log T - \frac{3}{14} \log t + C$$

$$= \frac{1}{4} \log(4e^{2x} + 7) - \frac{3}{14} \log(4 + 7e^{-2x}) + C$$

$$= \frac{1}{14} \left[ \frac{1}{2} \log(4e^x + 7e^{-x}) + \frac{13}{2} x \right] + C$$

$$u = \frac{13}{2}, v = \frac{1}{2} \Rightarrow u + v = 7$$

Aliter :

$$2e^x + 3e^{-x} = A(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x}) + \lambda$$

$$2 = 4A + 4B \quad ; \quad 3 = 7A - 7B \quad ; \quad \lambda = 0$$

$$A + B = \frac{1}{2}$$

$$A - B = \frac{3}{7}$$

$$A = \frac{1}{2} \left( \frac{1}{2} + \frac{3}{7} \right) = \frac{7+6}{28} = \frac{13}{28}$$

$$B = A - \frac{3}{7} = \frac{13}{28} - \frac{3}{7} = \frac{13-12}{28} = \frac{1}{28}$$

$$\int \frac{13}{28} dx + \frac{1}{28} \int \frac{4e^x - 7e^{-x}}{4e^x + 7e^{-x}} dx$$

$$\frac{13}{28} x + \frac{1}{28} \ln |4e^x + 7e^{-x}| + C$$

$$u = \frac{13}{2}; v = \frac{1}{2}$$

$$\Rightarrow u + v = 7$$

3. Official Ans. by NTA (3)

Sol. 
$$\int \frac{dx}{(x-1)^{3/4} (x+2)^{5/4}}$$

$$= \int \frac{dx}{\left( \frac{x+2}{x-1} \right)^{5/4} \cdot (x-1)^2}$$

put  $\frac{x+2}{x-1} = t$

$$= -\frac{1}{3} \int \frac{dt}{t^{5/4}}$$

$$= \frac{4}{3} \cdot \frac{1}{t^{1/4}} + C$$

$$= \frac{4}{3} \left( \frac{x-1}{x+2} \right)^{1/4} + C$$

## 4. Official Ans. by NTA (3)

$$\text{Sol. } = \int \frac{\sin x}{1 + \tan^3 x} dx = \int \frac{\tan x \cdot \sec^2 x}{(\tan x + 1)(1 + \tan^2 x - \tan x)} dx$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x \cdot dx = dt$$

$$= \int \frac{t}{(t+1)(t^2-t+1)} dt$$

$$= \int \left( \frac{A}{t+1} + \frac{B(2t-1)}{t^2-t+1} + \frac{C}{t^2-t+1} \right) dx$$

$$\Rightarrow A(t^2-t+1) + B(2t-1)(t^2-t+1) + C(t+1)$$

$$= t$$

$$\Rightarrow t^2(A+2B) + t(-A+B+C) + A-B+C = 1$$

$$\therefore A+2B=0 \quad \dots(1)$$

$$-A+B+C=1 \quad \dots(2)$$

$$A-B+C=0 \quad \dots(3)$$

$$\Rightarrow C = \frac{1}{2} \Rightarrow A-B = -\frac{1}{2} \quad \dots(4)$$

$$A+2B=0$$

$$A-B = -\frac{1}{2}$$

$$\Rightarrow 3B = \frac{1}{2} \Rightarrow B = \frac{1}{6}$$

$$A = -\frac{1}{3}$$

$$I = -\frac{1}{3} \int \frac{dt}{1+t} + \frac{1}{6} \int \frac{2t-1}{t^2-t+1} dt + \frac{1}{2} \int \frac{dt}{t^2-t+1}$$

$$= -\frac{1}{3} \ln|(1+\tan x)| + \frac{1}{6} \ln|\tan^2 x - \tan x + 1| +$$

$$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{\tan x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right)$$

$$= -\frac{1}{3} \ln|(1+\tan x)| + \frac{1}{6} \ln|\tan^2 x - \tan x + 1| +$$

$$\frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{2 \tan x - 1}{\sqrt{3}} \right) + C$$

$$\alpha = -\frac{1}{3}, \beta = \frac{1}{6}, \gamma = \frac{1}{\sqrt{3}}$$

$$18(\alpha + \beta + \gamma^2) = 18 \left( -\frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right) = 3$$

## 5. Official Ans. by NTA (3)

$$\text{Sol. } \int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx$$

$$= \int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}} dx$$

$$\text{Let } \sin x + \cos x = t$$

$$\int \frac{dt}{\sqrt{9-t^2}} = \sin^{-1} \frac{t}{3} + c$$

$$= \sin^{-1} \left( \frac{\sin x + \cos x}{3} \right) + c$$

$$\text{So } a = 1, b = 3.$$

## 6. Official Ans. by NTA (4)

$$\text{Sol. } I = \int \frac{\sin \theta \cdot \sin 2\theta (\sin^6 \theta + \sin^4 \theta + \sin^2 \theta) \sqrt{2 \sin^4 \theta + 3 \sin^2 \theta + 6}}{1 - \cos 2\theta} d\theta$$

$$\Rightarrow I = \int \frac{\sin \theta \cdot 2 \sin \theta \cos \theta \cdot \sin^2 \theta (\sin^4 \theta + \sin^2 \theta + 1) (2 \sin^4 \theta + 3 \sin^2 \theta + 6)^{1/2}}{2 \sin^2 \theta} d\theta$$

$$= \int \sin^2 \theta \cdot \cos \theta (\sin^4 \theta + \sin^2 \theta + 1) (2 \sin^4 \theta + 3 \sin^2 \theta + 6)^{1/2} d\theta$$

$$\text{Let } \sin \theta = t \Rightarrow \cos \theta d\theta = dt$$

$$\therefore I = \int t^2 (t^4 + t^2 + 1) (2t^4 + 3t^2 + 6)^{1/2} dt$$

$$= \int (t^5 + t^3 + t) t (2t^4 + 3t^2 + 6)^{1/2} dt$$

$$= \int (t^5 + t^3 + t) (t^2)^{1/2} (2t^4 + 3t^2 + 6)^{1/2} dt$$

$$= \int (t^5 + t^3 + t) (2t^6 + 3t^4 + 6t^2)^{1/2} dt$$

$$\text{Let } 2t^6 + 3t^4 + 6t^2 = u^2$$

$$\Rightarrow 12(t^5 + t^3 + t) dt = 2u du$$

$$\therefore I = \int (u^2)^{1/2} \cdot \frac{2u du}{12}$$

$$= \int \frac{u^2}{6} du = \frac{u^3}{18} + C$$

$$= \frac{(2t^6 + 3t^4 + 6t^2)^{3/2}}{18} + C$$

$$\text{when } t = \sin \theta$$

$$\text{and } t^2 = 1 - \cos^2 \theta \text{ will give option (4)}$$

7. Official Ans. by NTA (2)

$$\begin{aligned} \text{Sol. } \int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx, x > 0 \\ = \int \frac{(2x)^3 + 5(2x)^2}{x^4 + 5x^3 - 7x^2} dx = \int \frac{4x^2(2x+5)}{x^2(x^2+5x-7)} dx \\ = 4 \int \frac{d(x^2+5x-7)}{(x^2+5x-7)} = 4 \log_e |x^2+5x-7| + c \end{aligned}$$

option (2)

8. Official Ans by NTA (6)

$$\begin{aligned} \text{Sol. } \int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)} + \int \frac{dx}{x^4+3x^2+1} \\ \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)} + \frac{1}{2} \int \frac{(x^2+1)-(x^2-1)dx}{x^4+3x^2+1} \\ \text{Put } \tan^{-1}\left(x+\frac{1}{x}\right) = t \\ \int \frac{dt}{t} + \frac{1}{2} \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5} - \frac{1}{2} \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1} \\ \text{Put } x-\frac{1}{x} = y, x+\frac{1}{x} = z \\ \log_e t + \frac{1}{2} \int \frac{dy}{y^2+5} - \frac{1}{2} \int \frac{dz}{z^2+1} \\ = \log_e \tan^{-1}\left(x+\frac{1}{x}\right) + \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right) \\ - \frac{1}{2} \tan^{-1}\left(\frac{x^2+1}{x}\right) + C \end{aligned}$$

$$\alpha = 1, \beta = \frac{1}{2\sqrt{5}}, \gamma = \frac{1}{\sqrt{5}}, \delta = \frac{-1}{2}$$

or

$$\alpha = 1, \beta = \frac{-1}{2\sqrt{5}}, \gamma = \frac{-1}{\sqrt{5}}, \delta = \frac{-1}{2}$$

$$10(\alpha + \beta\gamma + \delta) = 10\left(1 + \frac{1}{10} - \frac{1}{2}\right) = 6$$

9. Official Ans. by NTA (1)

$$\begin{aligned} \text{Sol. } \int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}} dx \\ (2x-1)^2+5 = t^2 \\ 2(2x-1)2dx = 2t dt \\ 2\sqrt{t^2-5}dx = t dt \\ \text{So } \int \frac{\sqrt{t^2-5}\cos t}{2\sqrt{t^2-5}} dt = \frac{1}{2}\sin t + c \\ = \frac{1}{2}\sin\sqrt{(2x-1)^2+5} + c \end{aligned}$$

10. Official Ans. by NTA (4)

$$\begin{aligned} \text{Sol. } f(x) = \int \frac{(5x^8+7x^6)dx}{x^{14}(x^{-5}+x^{-7}+2)^2} \\ \text{Let } x^{-5}+x^{-7}+2 = t \\ (-5x^{-6}-7x^{-8})dx = dt \\ \Rightarrow f(x) = \int -\frac{dt}{t^2} = \frac{1}{t} + c \\ f(x) = \frac{x^7}{x^2+1+2x^7} \\ f(1) = \frac{1}{4} \end{aligned}$$