

DIFFERENTIAL EQUATION

- माना अवकल समीकरण $x \tan\left(\frac{y}{x}\right) dy = \left(y \tan\left(\frac{y}{x}\right) - x\right) dx$, $-1 \leq x \leq 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}$ का हल $y = y(x)$ है। तो वक्रों $x = 0, x = \frac{1}{\sqrt{2}}$ तथा $y = y(x)$ द्वारा ऊपरी आधे निर्देशांक तल में घिरे क्षेत्र का क्षेत्रफल है :
 (1) $\frac{1}{8}(\pi - 1)$ (2) $\frac{1}{12}(\pi - 3)$
 (3) $\frac{1}{4}(\pi - 2)$ (4) $\frac{1}{6}(\pi - 1)$
- माना अवकल समीकरण $e^x \sqrt{1-y^2} dx + \left(\frac{y}{x}\right) dy = 0, y(1) = -1$ का हल $y = y(x)$ है। तो $(y(3))^2$ का मान बराबर है :
 (1) $1 - 4e^3$ (2) $1 - 4e^6$
 (3) $1 + 4e^3$ (4) $1 + 4e^6$
- यदि $[x]$ महत्तम पूर्णांक $\leq x$ है, तो समाकलन $\int_{-\pi/2}^{\pi/2} [[x] - \sin x] dx$ का मान बराबर है:
 (1) $-\pi$ (2) π (3) 0 (4) 1
- यदि $f : \mathbf{R} \rightarrow \mathbf{R}, f(x) = x + 1$ द्वारा परिभाषित है, तो $\lim_{n \rightarrow \infty} \frac{1}{n} \left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \dots + f\left(\frac{5(n-1)}{n}\right) \right]$ का मान है:
 (1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{1}{2}$ (4) $\frac{7}{2}$
- माना वक्र $y = y(x)$, अवकल समीकरण $\cos\left(\frac{1}{2} \cos^{-1}(e^{-x})\right) dx = \sqrt{e^{2x} - 1} dy$ का हल है। यदि यह y -अक्ष को $y = -1$ तथा x -अक्ष को $(\alpha, 0)$ पर काटता है, तो e^α बराबर है _____ है।
- माना अवकल समीकरण $\operatorname{cosec}^2 x dy + 2 dx = (1 + y \cos 2x) \operatorname{cosec}^2 x dx$ जबकि $y\left(\frac{\pi}{4}\right) = 0$ है, का हल $y = y(x)$ है। तो $(y(0) + 1)^2$ का मान बराबर है -
 (1) $e^{1/2}$ (2) $e^{-1/2}$ (3) e^{-1} (4) e

- माना अवकल समीकरण $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1) \right) dx = (x+2) dy, y(1) = 1$ का हल $y = y(x)$ है। यदि $y = y(x)$ का प्रान्त विवक्त अन्तराल (α, β) है, तो $|\alpha + \beta|$ बराबर है _____।
- माना अवकल समीकरण, $\frac{dy}{dx} = 1 + x e^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$ का हल $y = y(x)$ है। तो $x \in (-\sqrt{2}, \sqrt{2})$ में $y(x)$ का न्यूनतम मान बराबर है :
 (1) $(2 - \sqrt{3}) - \log_e 2$
 (2) $(2 + \sqrt{3}) + \log_e 2$
 (3) $(1 + \sqrt{3}) - \log_e(\sqrt{3} - 1)$
 (4) $(1 - \sqrt{3}) - \log_e(\sqrt{3} - 1)$
- माना $y = y(x)$, अवकल समीकरण $e^y \frac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0, y\left(\frac{\pi}{2}\right) = 0$ का हल है। यदि $y(0) = \log_e(\alpha + \beta e^{-2})$ है, तो $4(\alpha + \beta)$ बराबर है _____।
- माना अवकल समीकरण $x dy = (y + x^3 \cos x) dx, y(\pi) = 0$ का हल $y = y(x)$ है। तो $y\left(\frac{\pi}{2}\right)$ बराबर है :
 (1) $\frac{\pi^2}{4} + \frac{\pi}{2}$ (2) $\frac{\pi^2}{2} + \frac{\pi}{4}$
 (3) $\frac{\pi^2}{2} - \frac{\pi}{4}$ (4) $\frac{\pi^2}{4} - \frac{\pi}{2}$
- माना एक वक्र $y = f(x)$, बिंदु $(2, (\log_e 2)^2)$ से होकर जाता है तथा x के सभी धनात्मक वास्तविक मानों के लिए इसकी प्रवणता $\frac{2y}{x \log_e x}$ है, तो $f(e)$ का मान है _____।
- माना अवकल समीकरण $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y, y(0) = 0$ का हल $y = y(x)$ है। यदि $y\left(-\frac{2}{3} \log_e 2\right) = \alpha \log_e 2$ है, तो α का मान बराबर है :
 (1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) 2 (4) $-\frac{1}{2}$

24. यदि $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, $y(0) = 1$, हैं, तो $y(1)$ बराबर है:

- (1) $\log_2(2 + e)$ (2) $\log_2(1 + e)$
 (3) $\log_2(2e)$ (4) $\log_2(1 + e^2)$

25. यदि $\frac{dy}{dx} = \frac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y} \log_e 2}$, $y(0) = 0$, हैं, तो $y = 1$

के लिए x का मान निम्न में से किस अंतराल में है?

- (1) (1, 2) (2) $\left(\frac{1}{2}, 1\right]$
 (3) (2, 3) (4) $\left(0, \frac{1}{2}\right]$

26. यदि $y \frac{dy}{dx} = x \left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)} \right]$, $x > 0$, $\phi > 0$, तथा

$y(1) = -1$ हैं, तो $\phi\left(\frac{y^2}{4}\right)$ बराबर है :

- (1) $4\phi(2)$ (2) $4\phi(1)$
 (3) $2\phi(1)$ (4) $\phi(1)$

27. यदि $y = y(x)$ अवकल समीकरण

$$x^2 dy + \left(y - \frac{1}{x}\right) dx = 0; \quad x > 0 \text{ का हल वक्र है}$$

तथा $y(1) = 1$, तब $y\left(\frac{1}{2}\right)$ बराबर है :

- (1) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (2) $3 + \frac{1}{\sqrt{e}}$
 (3) $3 + e$ (4) $3 - e$

28. यदि एक वक्र $y = f(x)$ बिन्दु (1, 2) से होकर जाता है तथा $x \frac{dy}{dx} + y = bx^4$ को संतुष्ट करता है, तो b

के किस मान के लिए $\int_1^2 f(x) dx = \frac{62}{5}$ है ?

- (1) 5 (2) 10 (3) $\frac{62}{5}$ (4) $\frac{31}{5}$

29. किसी प्रजाति की समय 't' पर जनसंख्या, $P = P(t)$, अवकल समीकरण, $\frac{dP}{dt} = 0.5P - 450$ को संतुष्ट

करती है। यदि $P(0) = 850$ है, तो वह समय, जब प्रजाति की जनसंख्या शून्य हो जाती है, है:

- (1) $\log_e 18$ (2) $\log_e 9$
 (3) $\frac{1}{2} \log_e 18$ (4) $2 \log_e 18$

30. यदि एक वक्र मूलबिंदु से होकर जाता है तथा इसके किसी बिंदु (x, y) पर स्पर्श रेखा की प्रवणता $\frac{x^2 - 4x + y + 8}{x - 2}$ है, तो यह वक्र निम्न में से किस

बिंदु से भी होकर जाता है?

- (1) (5, 4) (2) (4, 5) (3) (4, 4) (4) (5, 5)

31. यदि अवकल समीकरण $(2xy^2 - y)dx + xdy = 0$ का हल $y = y(x)$, रेखाओं $2x - 3y = 1$ तथा $3x + 2y = 8$ के प्रतिच्छेदन बिन्दु से होकर जाता है, तो $|y(1)|$ बराबर है _____

32. माना एक वक्र के किसी बिंदु $P(x, y)$ पर स्पर्श रेखा की प्रवणता $\frac{xy^2 + y}{x}$ द्वारा दी गई है। यदि यह वक्र,

रेखा $x + 2y = 4$ को $x = -2$ पर काटता है, तो y का वह मान, जिसके लिए बिंदु (3, y) वक्र पर है, है :

- (1) $\frac{18}{35}$ (2) $-\frac{4}{3}$ (3) $-\frac{18}{19}$ (4) $-\frac{18}{11}$

33. एक कलचर में जीवाणुओं की वृद्धि की दर जीवाणुओं की संख्या के समानुपातिक है तथा प्रारम्भ में समय $t = 0$ पर जीवाणुओं की संख्या 1000 है। 2 घंटों में जीवाणुओं की संख्या में 20% की वृद्धि

होती है। यदि $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ घंटों के पश्चात

जीवाणुओं की संख्या 2000 है, तो $\left(\frac{k}{\log_e 2}\right)^2$

बराबर है :

- (1) 4 (2) 8 (3) 2 (4) 16

34. यदि समीकरण

$$e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x, y(0) = 0 \text{ का}$$

हल $y = y(x)$ है, तो $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$

बराबर है ____।

35. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ द्वारा दिए गए वक्र-कुल

के अवकल समीकरण की घात तथा कोटि का अंतर है ____।

36. यदि अवकल समीकरण $\frac{dy}{dx} + (\tan x)y = \sin x,$

$0 \leq x \leq \frac{\pi}{3}$, का हल $y = y(x)$ है जबकि $y(0) = 0$ है,

तो $y\left(\frac{\pi}{4}\right)$ बराबर है :

(1) $\frac{1}{4} \log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right) \log_e 2$

(3) $\log_e 2$ (4) $\frac{1}{2} \log_e 2$

37. माना अवकल समीकरण $2xy \frac{dy}{dx} = y^2 - x^2,$

$x > 0$ का हल वक्र C_1 है तथा $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$ का

हल वक्र C_2 है। यदि दोनों वक्र (1,1) से होकर जाते हैं, तो वक्रों C_1 तथा C_2 द्वारा परिबद्ध क्षेत्र का क्षेत्रफल बराबर है :

(1) $\pi - 1$ (2) $\frac{\pi}{2} - 1$ (3) $\pi + 1$ (4) $\frac{\pi}{4} + 1$

38. यदि अवकल समीकरण

$$\frac{dy}{dx} + 2y \tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0 \text{ का हल } y = y(x)$$

है, तो R पर $y(x)$ का अधिकतम मान है :

(1) 8 (2) $\frac{1}{2}$ (3) $-\frac{15}{4}$ (4) $\frac{1}{8}$

39. माना अवकल समीकरण

$$\cos x (3 \sin x + \cos x + 3) dy =$$

$$(1 + y \sin x (3 \sin x + \cos x + 3)) dx, 0 \leq x \leq \frac{\pi}{2},$$

$y(0) = 0$ का हल $y = y(x)$ है। तो $y\left(\frac{\pi}{3}\right)$ बराबर है :

(1) $2 \log_e \left(\frac{2\sqrt{3}+9}{6}\right)$ (2) $2 \log_e \left(\frac{2\sqrt{3}+10}{11}\right)$

(3) $2 \log_e \left(\frac{\sqrt{3}+7}{2}\right)$ (4) $2 \log_e \left(\frac{3\sqrt{3}-8}{4}\right)$

40. यदि अवकल समीकरण

$$2(x^2 + x^{5/4}) dy - y(x + x^{1/4}) dx = 2x^{9/4} dx, x > 0$$

का हल वक्र $y = y(x)$ है, जो बिन्दु $\left(1, 1 - \frac{4}{3} \log_e 2\right)$

से होकर जाता है, तो $y(16)$ का मान बराबर है :

(1) $4\left(\frac{31}{3} + \frac{8}{3} \log_e 3\right)$ (2) $\left(\frac{31}{3} + \frac{8}{3} \log_e 3\right)$

(3) $4\left(\frac{31}{3} - \frac{8}{3} \log_e 3\right)$ (4) $\left(\frac{31}{3} - \frac{8}{3} \log_e 3\right)$

41. $y(x)$, जो अवकल समीकरण $\frac{dy}{dx} = xy - 1 + x - y;$

$y(0) = 0$ को सन्तुष्ट करता है, के लिए निम्न में कौन सा सत्य है?

(1) $y(1) = e^{\frac{1}{2}} - 1$ (2) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

(3) $y(1) = 1$ (4) $y(1) = e^{\frac{1}{2}} - 1$

42. यदि $[\cdot]$ महत्तम पूर्णांक फलन को दर्शाता है, तो

$$\left| \int_0^{\sqrt{\pi}} \left[[x^2] - \cos x \right] dx \right| \text{ का मान है } \underline{\hspace{2cm}}$$

43. निम्न में से कौन सा अवकल समीकरण परवलयों के निकाय $y^2 = 4a(x + a)$ द्वारा संतुष्ट होता है?

(1) $y \left(\frac{dy}{dx} \right)^2 - 2x \left(\frac{dy}{dx} \right) - y = 0$

(2) $y \left(\frac{dy}{dx} \right)^2 - 2x \left(\frac{dy}{dx} \right) + y = 0$

(3) $y \left(\frac{dy}{dx} \right)^2 + 2x \left(\frac{dy}{dx} \right) - y = 0$

(4) $y \left(\frac{dy}{dx} \right) + 2x \left(\frac{dy}{dx} \right) - y = 0$

44. माना अवकल समीकरण

$$\frac{dy}{dx} = (y+1)((y+1)e^{x^{2/2}} - x),$$

$0 < x < 2.1$, $y(2) = 0$ का हल $y = y(x)$ है। तो

$x = 1$ पर $\frac{dy}{dx}$ का मान बराबर है :

(1) $\frac{-e^{3/2}}{(e^2 + 1)^2}$ (2) $-\frac{2e^2}{(1 + e^2)^2}$

(3) $\frac{e^{5/2}}{(1 + e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2 + 1)^2}$

45. माना अवकल समीकरण $xdy - ydx = \sqrt{(x^2 - y^2)}$

dx , $x \geq 1$ का हल $y = y(x)$ है तथा $y(1) = 0$ है।

यदि रेखाओं $x = 1$, $x = e^\pi$, $y = 0$ तथा $y = y(x)$ द्वारा

घिरे क्षेत्र का क्षेत्रफल $\alpha e^{2\pi} + \beta$ है, तो $10(\alpha + \beta)$

का मान बराबर है _____.

SOLUTION

1. Official Ans. by NTA (1)

Sol. We have

$$\frac{dy}{dx} = \frac{x \left(\frac{y}{x} \cdot \tan \frac{y}{x} - 1 \right)}{x \tan \frac{y}{x}}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} - \cot \left(\frac{y}{x} \right)$$

$$\text{Put } \frac{y}{x} = v$$

$$\Rightarrow y = vx$$

$$\therefore \frac{dy}{dx} = v + \frac{ndv}{dx}$$

Now, we get

$$v + n \frac{dv}{dx} = v - \cot(v)$$

$$\Rightarrow \int (\tan) dv = - \int \frac{dx}{x}$$

$$\therefore \ell n \left| \sec \left(\frac{y}{x} \right) \right| = -\ell n |x| + c$$

$$\text{As } \left(\frac{1}{2} \right) = \left(\frac{y}{x} \right) \Rightarrow \boxed{C=0}$$

$$\therefore \sec \left(\frac{y}{x} \right) = \frac{1}{x}$$

$$\Rightarrow \cos \left(\frac{y}{x} \right) = x$$

$$\therefore \boxed{y = x \cos^{-1}(x)}$$

So, required bounded area

$$= \int_0^{1/\sqrt{2}} x \underset{(I)}{\cos^{-1}(x)} dx = \left(\frac{\pi-1}{8} \right)$$

(I.B.P.)

\therefore option (1) is correct.

2. Official Ans. by NTA (2)

$$\text{Sol. } e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$$

$$\Rightarrow e^x \sqrt{1-y^2} dx + \frac{-y}{x} dy$$

$$\Rightarrow \int \frac{-y}{\sqrt{1-y^2}} dy = \int \frac{e^x}{x} dx$$

$$\Rightarrow \sqrt{1-y^2} = e^x(x-1) + c$$

$$\text{Given : At } x=1, y=-1$$

$$\Rightarrow 0 = 0 + c \Rightarrow c = 0$$

$$\therefore \sqrt{1-y^2} = e^x(x-1)$$

$$\text{At } x=3 \quad 1-y^2 = (e^3 \cdot 2)^2 \Rightarrow y^2 = 1 - 4e^6$$

3. Official Ans. by NTA (1)

$$\text{Sol. } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([x] + [-\sin x]) dx \quad \dots(1)$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([-x] + [\sin x]) dx \quad \dots(2)$$

(King property)

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\underbrace{[x] + [-x]}_{-1} \right) + \left(\underbrace{[\sin x] + [-\sin x]}_{-1} \right) dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-2) dx = -2(\pi)$$

$$I = -\pi$$

4. Official Ans. by NTA (4)

$$\text{Sol. } I = \sum_{r=0}^{n-1} f \left(\frac{5r}{n} \right) \frac{1}{n}$$

$$I = \int_0^1 f(5x) dx$$

$$I = \int_0^1 (5x+1) dx$$

$$I = \left[\frac{5x^2}{2} + x \right]_0^1$$

$$I = \frac{5}{2} + 1 = \frac{7}{2}$$

5. Official Ans. by NTA (2)

Sol. $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1} dy$

Put $\cos^{-1}(e^{-x}) = \theta, \theta \in [0, \pi]$

$\cos\theta = e^{-x} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 = e^{-x}$

$\cos\frac{\theta}{2} = \sqrt{\frac{e^{-x}+1}{2}} = \sqrt{\frac{e^x+1}{2e^x}}$

$\sqrt{\frac{e^x+1}{2e^x}}dx = \sqrt{e^{2x}-1} dy$

$\frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{e^x}\sqrt{e^x-1}} = \int dy$

Put $e^x = t, \frac{dt}{dx} = e^x$

$\frac{1}{\sqrt{2}} \int \frac{dt}{e^x \sqrt{e^x}\sqrt{e^x-1}} = \int dy$

$\int \frac{dt}{t\sqrt{t^2-t}} = \sqrt{2}y$

Put $t = \frac{1}{z}, \frac{dt}{dz} = -\frac{1}{z^2}$

$\int \frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}-\frac{1}{z}}} = \sqrt{2}y$

$-\int \frac{dz}{\sqrt{1-z}} = \sqrt{2}y$

$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$

$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}y + c$

$2(1-e^{-x})^{1/2} = \sqrt{2}y + c \xrightarrow{(0,-1)} \Rightarrow c = \sqrt{2}$

$2(1-e^{-x})^{1/2} = \sqrt{2}(y+1),$ passes through $(\alpha, 0)$

$2(1-e^{-\alpha})^{1/2} = \sqrt{2}$

$\sqrt{1-e^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1-e^{-\alpha} = \frac{1}{2}$

$e^{-\alpha} = \frac{1}{2} \Rightarrow e^{\alpha} = 2$

6. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} + 2\sin^2 x = 1 + y\cos 2x$

$\Rightarrow \frac{dy}{dx} + (-\cos 2x)y = \cos 2x$

I.F. = $e^{\int -\cos 2x dx} = e^{-\frac{\sin 2x}{2}}$

Solution of D.E.

$y\left(e^{-\frac{\sin 2x}{2}}\right) = \int (\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right) dx + c$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + c$

Given

$y\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 0 = -e^{-1/2} + c \Rightarrow \boxed{c = e^{-1/2}}$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + e^{-1/2}$

at $x = 0$

$y = -1 + e^{-1/2}$

$\Rightarrow y(0) = -1 + e^{-1/2} \Rightarrow (y(0) + 1)^2 = e^{-1}$

7. Official Ans. by NTA (4)

Sol. $y + 1 = Y \Rightarrow dy = dY$

$$x + 2 = X \Rightarrow dx = dX$$

$$\Rightarrow \left(Xe^{\frac{Y}{X}} + Y \right) dX = X dY$$

$$\Rightarrow X dY - Y dX = X e^{\frac{Y}{X}} dX$$

$$\Rightarrow d\left(\frac{Y}{X}\right) e^{\frac{Y}{X}} = \frac{dX}{X}$$

$$-e^{-\frac{Y}{X}} = \ln|X| + c$$

$$(3, 2) \rightarrow -e^{-\frac{2}{3}} = \ln|3| + c$$

$$-e^{-\frac{Y}{X}} = \ln|X| - e^{-\frac{2}{3}} - \ln 3$$

$$e^{\frac{Y}{X}} = e^{2/3} + \ln 3 - \ln|X| > 0$$

$$\ln|X| < (e^{2/3} + \ln 3)$$

$$\text{Let } \lambda = (e^{2/3} + \ln 3)$$

$$|x + 2| < e^\lambda$$

$$-e^\lambda < x + 2 < e^\lambda$$

$$-e^\lambda - 2 < x < e^\lambda - 2$$

$$\alpha \quad \beta$$

$$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$$

Although $x = -2$ should be excluded from domain but according to the given problem it will the most appropriate solution.

8. Official Ans. by NTA (4)

Sol. $\frac{dy - dx}{e^{y-x}} = x dx$

$$\Rightarrow \frac{dy - dx}{e^{y-x}} = x dx$$

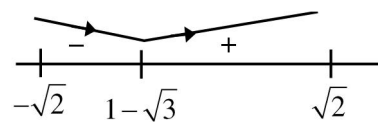
$$\Rightarrow -e^{x-y} = \frac{x^2}{2} + c$$

$$\text{At } x = 0, y = 0 \Rightarrow c = -1$$

$$\Rightarrow e^{x-y} = \frac{2-x^2}{2}$$

$$\Rightarrow y = x - \ln\left(\frac{2-x^2}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{2x}{2-x^2} = \frac{2+2x-x^2}{2-x^2}$$



So minimum value occurs at $x = 1 - \sqrt{3}$

$$y(1-\sqrt{3}) = (1-\sqrt{3}) - \ln\left(\frac{2-(4-2\sqrt{3})}{2}\right)$$

$$= (1-\sqrt{3}) - \ln(\sqrt{3}-1)$$

9. Official Ans. by NTA (4)

Sol. Let $e^y = t$

$$\Rightarrow \frac{dt}{dx} - (2 \sin x)t = -\sin x \cos^2 x$$

$$\text{I.F.} = e^{2 \cos x}$$

$$\Rightarrow t \cdot e^{2 \cos x} = \int e^{2 \cos x} \cdot (-\sin x \cos^2 x) dx$$

$$\Rightarrow e^y \cdot e^{2 \cos x} = \int e^{2z} \cdot z^2 dz, z = e^{\cos x}$$

$$\Rightarrow e^y \cdot e^{2 \cos x} = \frac{1}{2} \cdot \cos^2 x \cdot e^{2 \cos x} - \frac{1}{2} \cos x \cdot e^{2 \cos x} + \frac{e^{2 \cos x}}{4} + C$$

$$\text{at } x = \frac{\pi}{2}, y = 0 \Rightarrow C = \frac{3}{4}$$

$$\Rightarrow e^y = \frac{1}{2} \cos^2 x - \frac{1}{2} \cos x + \frac{1}{4} + \frac{3}{4} \cdot e^{-2 \cos x}$$

$$\Rightarrow y = \log \left[\frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + \frac{3}{4} e^{-2 \cos x} \right]$$

Put $x = 0$

$$\Rightarrow y = \log \left[\frac{1}{4} + \frac{3}{4} e^{-2} \right] \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$$

10. Official Ans. by NTA (1)

Sol. $x dy = (y + x^3 \cos x) dx$

$$x dy = y dx + x^3 \cos x dx$$

$$\frac{x dy - y dx}{x^2} = \frac{x^3 \cos x dx}{x^2}$$

$$\frac{d}{dx} \left(\frac{y}{x} \right) = \int x \cos x dx$$

$$\Rightarrow \frac{y}{x} = x \sin x - \int 1 \cdot \sin x dx$$

$$\frac{y}{x} = x \sin x + \cos x + C$$

$$\Rightarrow 0 = -1 + C \Rightarrow C = 1, x = \pi, y = 0$$

$$\text{so } \frac{y}{x} = x \sin x + \cos x + 1$$

$$y = x^2 \sin x + x \cos x + x \quad x = \frac{\pi}{2}$$

$$y \left(\frac{\pi}{2} \right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

11. Official Ans. by NTA (1)

Sol. $y' = \frac{2y}{x \ln x}$

$$\Rightarrow \frac{dy}{y} = \frac{2 dx}{x \ln x}$$

$$\Rightarrow \ln |y| = 2 \ln |\ln x| + C$$

$$\text{put } x = 2, y = (\ln 2)^2$$

$$\Rightarrow c = 0$$

$$\Rightarrow y = (\ln x)^2$$

$$\Rightarrow f(e) = 1$$

12. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = e^{3x} \cdot e^{4y} \Rightarrow \int e^{-4y} dy = \int e^{3x} dx$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C \Rightarrow -\frac{1}{4} - \frac{1}{3} = C \Rightarrow C = -\frac{7}{12}$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow e^{-4y} = \frac{4e^{3x} - 7}{-3}$$

$$e^{4y} = \frac{3}{7 - 4e^{3x}} \Rightarrow 4y = \ln \left(\frac{3}{7 - 4e^{3x}} \right)$$

$$4y = \ln \left(\frac{3}{6} \right) \text{ when } x = -\frac{2}{3} \ln 2$$

$$y = \frac{1}{4} \ln \left(\frac{1}{2} \right) = -\frac{1}{4} \ln 2$$

13. Official Ans. by NTA (16)**Sol.** $F(3) = 0$

$$e^x F(x) = \int_3^x (3t^2 + 2t + 4F'(t)) dt$$

$$e^x F(x) + e^x F'(x) = 3x^2 + 2x + 4F'(x)$$

$$(e^x - 4) \frac{dy}{dx} + e^x y = (3x^2 + 2x)$$

$$\frac{dy}{dx} + \frac{e^x}{(e^x - 4)} y = \frac{(3x^2 + 2x)}{(e^x - 4)}$$

$$y e^{\int \frac{e^x}{(e^x - 4)} dx} = \int \frac{(3x^2 + 2x)}{(e^x - 4)} e^{\int \frac{e^x}{e^x - 4} dx} dx$$

$$y \cdot (e^x - 4) = \int (3x^2 + 2x) dx + c$$

$$y(e^x - 4) = x^3 + x^2 + c$$

$$\text{Put } x = 3 \Rightarrow c = -36$$

$$F(x) = \frac{(x^3 + x^2 - 36)}{(e^x - 4)}$$

$$F'(x) = \frac{(3x^2 + 2x)(e^x - 4) - (x^3 + x^2 - 36)e^x}{(e^x - 4)^2}$$

Now put value of $x = 4$ we will get $\alpha = 12$ & $\beta = 4$

14. Official Ans. by NTA (2)**Sol.** $\sec y \frac{dy}{dx} = 2 \sin x \cos y$

$$\sec^2 y dy = 2 \sin x dx$$

$$\tan y = -2 \cos x + c$$

$$c = 2$$

$$\tan y = -2 \cos x + 2 \Rightarrow \text{at } x = \frac{\pi}{2}$$

$$\tan y = 2$$

$$\sec^2 y \frac{dy}{dx} = 2 \sin x$$

$$5 \frac{dy}{dx} = 2$$

15. Official Ans. by NTA (2)**Sol.** $(x - x^3)dy = (y + yx^2 - 3x^4)dx$

$$\Rightarrow xdy - ydx = (yx^2 - 3x^4)dx + x^3dy$$

$$\Rightarrow \frac{xdy - ydx}{x^2} = (ydx + xdy) - 3x^2dx$$

$$\Rightarrow d\left(\frac{y}{x}\right) = d(xy) - d(x^3)$$

Integrate

$$\Rightarrow \frac{y}{x} = xy - x^3 + c$$

$$\text{given } f(3) = 3$$

$$\Rightarrow \frac{3}{3} = 3 \times 3 - 3^3 + c$$

$$\Rightarrow c = 19$$

$$\therefore \frac{y}{x} = xy - x^3 + 19$$

$$\text{at } x = 4, \frac{y}{4} = 4y - 64 + 19$$

$$15y = 4 \times 45$$

$$\Rightarrow y = 12$$

16. Official Ans. by NTA (2)**Sol.** $\int e^{-y} dy = \int e^{\alpha x} dx$

$$\Rightarrow e^{-y} = \frac{e^{\alpha x}}{\alpha} + c \quad \dots(i)$$

$$\text{Put } (x, y) = (\ln 2, \ln 2)$$

$$\frac{-1}{2} = \frac{2^\alpha}{\alpha} + C \quad \dots(ii)$$

$$\text{Put } (x, y) \equiv (0, -\ln 2) \text{ in (i)}$$

$$-2 = \frac{1}{\alpha} + C \quad \dots(iii)$$

$$(ii) - (iii)$$

$$\frac{2^\alpha - 1}{\alpha} = \frac{3}{2}$$

$$\Rightarrow \alpha = 2 \text{ (as } \alpha \in \mathbb{N})$$

17. Official Ans. by NTA (4)

Sol. $(y + 1)\tan^2 x \, dx + \tan x \, dy + y \, dx = 0$

or $\frac{dy}{dx} + \frac{\sec^2 x}{\tan x} \cdot y = -\tan x$

IF = $e^{\int \frac{\sec^2 x}{\tan x} dx} = e^{\ln \tan x} = \tan x$

$\therefore y \tan x = - \int \tan^2 x \, dx$

or $y \tan x = -\tan x + x + C$

or $y = -1 + \frac{x}{\tan x} + \frac{C}{\tan x}$

or $\lim_{x \rightarrow 0} xy = -x + \frac{x^2}{\tan x} + \frac{Cx}{\tan x} = 1$

or $C = 1$

$y(x) = \cot x + x \cot x - 1$

$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

18. Official Ans. by NTA (3)

Sol. $2x^2 dy + (e^y - 2x) dx = 0$

$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$

$e^{-y} \frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow$ Put $e^{-y} = z$

$-\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$

$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2} \log_e x + c$

$xe^{-y} = \frac{1}{2} \log_e x + c$, passes through $(e, 1)$

$\Rightarrow C = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

19. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} - 2xy = 2(2 \sin x - 5)x - 2 \cos x$

IF = e^{-x^2}

so,

$y \cdot e^{-x^2} = \int e^{-x^2} (2x(2 \sin x - 5) - 2 \cos x) dx$

$\Rightarrow y \cdot e^{-x^2} = e^{-x^2} (5 - 2 \sin x) + c$

$\Rightarrow y = 5 - 2 \sin x + c \cdot e^{x^2}$

Given at $x = 0, y = 7$

$\Rightarrow 7 = 5 + c \Rightarrow c = 2$

So, $y = 5 - 2 \sin x + 2e^{x^2}$

Now at $x = \pi$,

$y = 5 + 2e^{\pi^2}$

20. Official Ans. by NTA (3)

Sol. $y + \frac{xy}{dx} = x^2$ (given)

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$

If = $e^{\int \frac{1}{x} dx} = x$

Solution of DE

$\Rightarrow y \cdot x = \int x \cdot x \, dx$

$\Rightarrow xy = \frac{x^3}{3} + \frac{c}{3}$

Passes through $(-2, 2)$, so

$-12 = -8 + c \Rightarrow c = -4$

$\therefore 3xy = x^3 - 4$

ie. $3x \cdot f(x) = x^3 - 4$

21. Official Ans. by NTA (4)

$$\text{Sol. } \alpha. R = \frac{|3(2) + 4(-3) - 5|}{5} = \frac{11}{5}$$

$$(x-h)^2 = \frac{11}{5}(y-k)$$

differentiate w.r.t 'x' :-

$$2(x-h) = \frac{11}{5} \frac{dy}{dx}$$

again differentiate

$$2 = \frac{11}{5} \frac{d^2y}{dx^2}$$

$$\frac{11d^2y}{dx^2} = 10.$$

22. Official Ans. by NTA (4)

$$\text{Sol. } (2x - 10y^3) dy + y dx = 0$$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{2}{y}\right)x = 10y^2$$

$$\text{I. F.} = e^{\int \frac{2}{y} dy} = e^{2 \ln(y)} = y^2$$

Solution of D.E. is

$$\therefore x \cdot y = \int (10y^2) y^2 \cdot dy$$

$$xy^2 = \frac{10y^5}{5} + C \Rightarrow xy^2 = 2y^5 + C$$

$$\text{It passes through } (0, 1) \rightarrow 0 = 2 + C \Rightarrow C = -2$$

$$\therefore \text{Curve is } \boxed{xy^2 = 2y^5 - 2}$$

Now, it passes through (2, β)

$$2\beta^2 = 2\beta^5 - 2 \Rightarrow \beta^5 - \beta^2 - 1 = 0$$

$$\therefore \beta \text{ is root of an equation } \boxed{y^5 - y^2 - 1 = 0} \text{ Ans.}$$

23. Official Ans. by NTA (4)

$$\text{Sol. } \int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt \quad 0 \leq x \leq 1$$

differentiating both the sides

$$\sqrt{1 - (f'(x))^2} = f(x)$$

$$\Rightarrow 1 - (f'(x))^2 = f^2(x)$$

$$\frac{f'(x)}{\sqrt{1 - f^2(x)}} = 1$$

$$\sin^{-1} f(x) = x + C$$

$$\therefore f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$$

$$\text{Now } \lim_{x \rightarrow 0} \frac{\int_0^x \sin t dt}{x^2} \left(\frac{0}{0} \right) = \frac{1}{2}$$

24. Official Ans. by NTA (2)

$$\text{Sol. } \frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2^y}$$

$$2^y \frac{dy}{dx} = 2^x (2^y - 1)$$

$$\int \frac{2^y}{2^y - 1} dy = \int 2^x dx$$

$$\frac{\ln(2^y - 1)}{\ln 2} = \frac{2^x}{\ln 2} + C$$

$$\Rightarrow \log_2(2^y - 1) = 2^x \log_2 e + C$$

$$\therefore y(0) = 1 \Rightarrow 0 = \log_2 e + C$$

$$C = -\log_2 e$$

$$\Rightarrow \log_2(2^y - 1) = (2^x - 1) \log_2 e$$

$$\text{put } x = 1, \log_2(2^y - 1) = \log_2 e$$

$$2^y = e + 1$$

$$y = \log_2(e + 1) \text{ Ans.}$$

25. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = \frac{2^x(y+2^y)}{2^x(1+2^y \ln 2)}$

$$\Rightarrow \int \frac{(1+2^y)\ln 2}{(y+2^y)} dy = \int dx$$

$$\Rightarrow \ln|y+2^y| = x + c$$

$$x=0; y=0 \Rightarrow c=0$$

$$\Rightarrow x = \ln|y+2^y|$$

$$\Rightarrow \text{at } y=1, x = \ln 3$$

$$\therefore 3 \in (e, e^2) \Rightarrow x \in (1, 2)$$

26. Official Ans. by NTA (2)

Sol. Let, $y = tx$

$$\frac{dy}{dx} = t + x \frac{dt}{dx}$$

$$\therefore tx \left(t + x \frac{dt}{dx} \right) = x \left(t^2 + \frac{\phi(t^2)}{\phi'(t^2)} \right)$$

$$t^2 + xt \frac{dt}{dx} = t^2 + \frac{\phi(t^2)}{\phi'(t^2)}$$

$$\int \frac{t\phi'(t^2)}{\phi(t^2)} dt = \int \frac{dx}{x}$$

Let $\phi(t^2) = p$

$$\therefore \phi'(t^2)2tdt = dp$$

$$\Rightarrow \int \frac{dy}{2p} = \int \frac{dx}{x}$$

$$\frac{1}{2} \ln \phi(t^2) = \ln x + \ln c$$

$$\phi(t^2) = x^2 k$$

$$\phi\left(\frac{y^2}{x^2}\right) = kx^2, \phi(1) = k$$

$$\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$$

27. Official Ans. by NTA (4)

Sol. $x^2 dy + \left(y - \frac{1}{x}\right) dx = 0 : x > 0, y(1) = 1$

$$x^2 dy + \frac{(xy-1)}{x} dx = 0$$

$$x^2 dy = \frac{(xy-1)}{x} dx$$

$$\frac{dy}{dx} = \frac{1-xy}{x^3}$$

$$\frac{dy}{dx} = \frac{1}{x^3} - \frac{y}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2} \cdot y = \frac{1}{x^3}$$

$$\text{If } e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$$

$$ye^{-\frac{1}{x}} = \int \frac{1}{x^3} \cdot e^{-\frac{1}{x}}$$

$$ye^{-\frac{1}{x}} = e^{-x} \left(1 + \frac{1}{x}\right) + C$$

$$1 \cdot e^{-1} = e^{-1}(2) + C$$

$$C = -e^{-1} = -\frac{1}{e}$$

$$ye^{-\frac{1}{x}} = e^{-\frac{1}{x}} \left(1 + \frac{1}{x}\right) - \frac{1}{e}$$

$$y\left(\frac{1}{2}\right) = 3 - \frac{1}{e} \times e^2$$

$$y\left(\frac{1}{2}\right) = 3 - e$$

28. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = x$$

So, solution of D.E. is given by

$$y \cdot x = \int b \cdot x^3 \cdot x dx + c$$

$$y = \frac{c}{x} + \frac{bx^4}{5}$$

Passes through (1, 2)

$$2 = c + \frac{b}{5} \quad \dots(1)$$

$$\int_1^2 f(x) dx = \frac{62}{5}$$

$$\left[c \ln x + \frac{bx^5}{25} \right]_1^2 = \frac{62}{5}$$

$$c \ln 2 + \frac{31b}{25} = \frac{62}{5} \quad \dots(2)$$

By equation (1) & (2)

$$c = 0 \text{ and } b = 10$$

29. Official Ans. by NTA (4)

Sol. $\frac{dP}{dt} = 0.5P - 450$

$$\Rightarrow \int_0^t \frac{dp}{P-900} = \int_0^t \frac{dt}{2}$$

$$\Rightarrow \left[\ln |P(t) - 900| \right]_0^t = \left[\frac{t}{2} \right]_0^t$$

$$\Rightarrow \ln |P(t) - 900| - \ln |P(0) - 900| = \frac{t}{2}$$

$$\Rightarrow \ln |P(t) - 900| - \ln |50| = \frac{t}{2}$$

for $P(t) = 0$

$$\Rightarrow \ln \left| \frac{900}{50} \right| = \frac{t}{2} \Rightarrow t = 2 \ln 18$$

30. Official Ans. by NTA (4)

Sol. Given

$$y(0) = 0$$

$$\& \frac{dy}{dx} = \frac{(x-2)^2 + y + 4}{x-2}$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x-2} = (x-2) + \frac{4}{x-2}$$

$$\Rightarrow \text{I.F.} = e^{-\int \frac{1}{x-2} dx} = \frac{1}{x-2}$$

Solution of L.D.E.

$$\Rightarrow y \frac{1}{x-2} = \int \frac{1}{x-2} \left((x-2) + \frac{4}{x-2} \right) dx$$

$$\Rightarrow \frac{y}{x-2} = x - \frac{4}{x-2} + C$$

$$\text{Now, at } x = 0, y = 0 \Rightarrow C = -2$$

$$y = x(x-2) - 4 - 2(x-2)$$

$$\Rightarrow y = x^2 - 4x$$

This curve passes through (5, 5)

31. Official Ans. by NTA (1)

Sol. $(2xy^2 - y)dx + xdy = 0$

$$2xy^2 dx - y dx + x dy = 0$$

$$2x dx = \frac{y dx - x dy}{y^2} = d\left(\frac{x}{y}\right)$$

Now integrate

$$x^2 = \frac{x}{y} + c$$

Now point of intersection of lines are (2, 1)

$$4 = \frac{2}{1} + c \quad \Rightarrow c = 2$$

$$x^2 = \frac{x}{y} + 2$$

$$\text{Now } y(1) = -1$$

$$\Rightarrow |y(1)| = 1$$

32. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} = \frac{xy^2 + y}{x}$
 $\frac{xdy - ydx}{y^2} = xdx$
 $-d\left(\frac{x}{y}\right) = xdx$
 $-\frac{x}{y} = \frac{x^2}{2} + c$

∴ curve intersects the line $x + 2y = 4$ at $x = -2$

⇒ point of intersection is $(-2, 3)$

∴ curve passes through $(-2, 3)$

⇒ $\frac{2}{3} = 2 + c \Rightarrow c = -\frac{4}{3}$

⇒ $\frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$

Now put $(3, y)$

⇒ $\frac{-3}{y} = \frac{19}{6}$

⇒ $y = \frac{-18}{19}$

33. Official Ans. by NTA (1)

Sol. $\frac{dB}{dt} = \lambda B \Rightarrow \int_{1000}^{1200} \frac{dB}{B} = \lambda \int_0^2 dt \Rightarrow \lambda = \frac{1}{2} \ln\left(\frac{6}{5}\right)$

$\int_{1000}^{2000} \frac{dB}{B} = \frac{1}{2} \ln\left(\frac{6}{5}\right) \int_0^T dt \Rightarrow T = \frac{2 \ln 2}{\ln\left(\frac{6}{5}\right)}$

⇒ $k = 2 \ln 2$

34. Official Ans. by NTA (1)

Sol. Put $e^{\sin y} = t$

⇒ $e^{\sin y} \cos y \frac{dy}{dx} = \frac{dt}{dx}$

⇒ D.E is $\frac{dt}{dx} + t \cos x = \cos x$

I.F. = $e^{\int \cos x dx} = e^{\sin x}$

⇒ solution is $t \cdot e^{\sin x} = \int \cos x e^{\sin x}$

⇒ $e^{\sin y} e^{\sin x} = e^{\sin x} + c$

∴ $x = 0, y = 0 \Rightarrow c = 0$

⇒ $e^{\sin y} = 1$

⇒ $y = 0$

⇒ $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right) =$

35. Official Ans. by NTA (2)

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) = ax + \frac{a^{3/2}}{2} \dots(1)$

⇒ $2yy' = a$

put in equation (1)

$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$

$(y^2 - 2xyy') = \frac{(2yy')^{3/2}}{2}$

squaring

$(y^2 - 2xyy')^2 = \frac{y^3(y')^3}{2}$

∴ order = 1

degree = 3

Degree - order = 3 - 1 = 2

36. Official Ans by NTA (2)

Sol. $\frac{dy}{dx} + (\tan x)y = \sin x$; $0 \leq x \leq \frac{\pi}{3}$

$$\text{I.F.} = e^{\int \tan x \, dx} = e^{\ln \sec x} = \sec x$$

$$y \sec x = \int \tan x \, dx$$

$$y \sec x = \int \tan x \, dx$$

$$y \sec x = \ln |\sec x| + C$$

$$x = 0, y = 0 \Rightarrow \therefore c = 0$$

$$y \sec x = \ln |\sec x|$$

$$y = \cos x \cdot \ln |\sec x|$$

$$y|_{x=\frac{\pi}{4}} = \left(\frac{1}{\sqrt{2}}\right) \cdot \ln \sqrt{2}$$

$$y|_{x=\frac{\pi}{4}} = \frac{1}{2\sqrt{2}} \log_e 2$$

37. Official Ans by NTA (2)

Sol. $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$, $x \in (0, \infty)$

put $y = vx$

$$x \frac{dv}{dx} + v = \frac{v^2 - 1}{2v}$$

$$\frac{2v}{v^2 + 1} dv = -\frac{dx}{x}$$

Integrate,

$$\ln(v^2 + 1) = -\ln x + C$$

$$\ln\left(\frac{y^2}{x^2} + 1\right) = -\ln x + C$$

put $x = 1, y = 1, C = \ln 2$

$$\ln\left(\frac{y^2}{x^2} + 1\right) = -\ln x + \ln 2$$

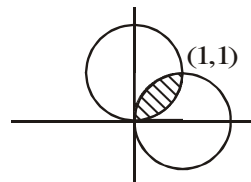
$$\Rightarrow x^2 + y^2 - 2x = 0 \quad (\text{Curve } C_1)$$

Similarly,

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

Put $y = vx$

$$x^2 + y^2 - 2y = 0$$



$$\text{required area} = 2 \int_0^1 (\sqrt{2x - x^2} - x) dx = \frac{\pi}{2} - 1$$

38. Official Ans. by NTA (4)

Sol. $\frac{dy}{dx} + 2y \tan x = \sin x$

$$\text{I.F.} = e^{\int 2 \tan x \, dx} = e^{2 \ln \sec x}$$

$$\text{I.F.} = \sec^2 x$$

$$y \cdot (\sec^2 x) = \int \sin x \cdot \sec^2 x \, dx$$

$$y \cdot (\sec^2 x) = \int \sec x \tan x \, dx$$

$$y \cdot (\sec^2 x) = \sec x + C$$

$$x = \frac{\pi}{3}; y = 0$$

$$\Rightarrow C = -2$$

$$\Rightarrow y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2 \cos^2 x$$

$$y = t - 2t^2 \Rightarrow \frac{dy}{dt} = 1 - 4t = 0 \Rightarrow t = \frac{1}{4}$$

$$\therefore \max = \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$$

39. Official Ans. by NTA (2)

Sol. $\cos x (3 \sin x + \cos x + 3) dy = (1 + y \sin x (3 \sin x + \cos x + 3)) dx$

$$\frac{dy}{dx} - (\tan x)y = \frac{1}{(3 \sin x + \cos x + 3) \cos x}$$

I.F. = $e^{\int -\tan x dx} = e^{\ln|\cos x|} = |\cos x|$

$$= \cos x \quad \forall x \in \left[0, \frac{\pi}{2}\right)$$

Solution of D.E.

$$y(\cos x) = \int (\cos x) \cdot \frac{1}{\cos x (3 \sin x + \cos x + 3)} dx + C$$

$$y(\cos x) = \int \frac{dx}{3 \sin x + \cos x + 3} dx + C$$

$$y(\cos x) = \int \frac{\left(\sec^2 \frac{x}{2}\right)}{2 \tan^2 \frac{x}{2} + 6 \tan \frac{x}{2} + 4} dx + C$$

Now

Let $I_1 = \int \frac{\left(\sec^2 \frac{x}{2}\right)}{2 \left(\tan^2 \frac{x}{2} + 3 \tan \frac{x}{2} + 2\right)} dx + C$

Put $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

$$I_1 = \int \frac{dt}{t^2 + 3t + 2} = \int \frac{dt}{(t+2)(t+1)}$$

$$= \int \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt$$

$$= \ln \left| \frac{t+1}{t+2} \right| = \ln \left| \frac{\tan \frac{x}{2} + 1}{\tan \frac{x}{2} + 2} \right|$$

So solution of D.E.

$$y(\cos x) = \ln \left| \frac{1 + \tan \frac{x}{2}}{2 + \tan \frac{x}{2}} \right| + C$$

$$\Rightarrow y(\cos x) = \ln \left(\frac{1 + \tan \frac{x}{2}}{2 + \tan \frac{x}{2}} \right) + C \quad \text{for } 0 \leq x < \frac{\pi}{2}$$

Now, it is given $y(0) = 0$

$$\Rightarrow 0 = \ln \left(\frac{1}{2} \right) + C \Rightarrow \boxed{C = \ln 2}$$

$$\Rightarrow y(\cos x) = \ln \left(\frac{1 + \tan \frac{x}{2}}{2 + \tan \frac{x}{2}} \right) + \ln 2$$

For $x = \frac{\pi}{3}$

$$y\left(\frac{1}{2}\right) = \ln \left(\frac{1 + \frac{1}{\sqrt{3}}}{2 + \frac{1}{\sqrt{3}}} \right) + \ln 2$$

$$y = 2 \ln \left(\frac{2\sqrt{3} + 10}{11} \right)$$

Ans.(2)

40. Official Ans. by NTA (3)

$$\text{Sol. } \frac{dy}{dx} - \frac{y}{2x} = \frac{x^{9/4}}{x^{5/4}(x^{3/4} + 1)}$$

$$\text{IF} = e^{-\int \frac{dx}{2x}} = e^{-\frac{1}{2} \ln x} = \frac{1}{x^{1/2}}$$

$$y \cdot x^{-1/2} = \int \frac{x^{9/4} \cdot x^{-1/2}}{x^{5/4}(x^{3/4} + 1)} dx$$

$$\int \frac{x^{1/2}}{(x^{3/4} + 1)} dx$$

$$x = t^4 \Rightarrow dx = 4t^3 dt$$

$$\int \frac{t^2 \cdot 4t^3 dt}{(t^3 + 1)}$$

$$4 \int \frac{t^2(t^3 + 1 - 1)}{(t^3 + 1)} dt$$

$$4 \int t^2 dt - 4 \int \frac{t^2}{t^3 + 1} dt$$

$$\frac{4t^3}{3} - \frac{4}{3} \ln(t^3 + 1) + C$$

$$yx^{-1/2} = \frac{4x^{3/4}}{3} - \frac{4}{3} \ln(x^{3/4} + 1) + C$$

$$1 - \frac{4}{3} \log_e 2 = \frac{4}{3} - \frac{4}{3} \log_e 2 + C$$

$$\Rightarrow C = -\frac{1}{3}$$

$$y = \frac{4}{3} x^{5/4} - \frac{4}{3} \sqrt{x} \ln(x^{3/4} + 1) - \frac{\sqrt{x}}{3}$$

$$y(16) = \frac{4}{3} \times 32 - \frac{4}{3} \times 4 \ln 9 - \frac{4}{3}$$

$$= \frac{124}{3} - \frac{32}{3} \ln 3 = 4 \left(\frac{31}{3} - \frac{8}{3} \ln 3 \right)$$

41. Official Ans. by NTA (1)

$$\text{Sol. } \frac{dy}{dx} = (1+y)(x-1)$$

$$\frac{dy}{(y+1)} = (x-1) dx$$

$$\text{Integrate } \ln(y+1) = \frac{x^2}{2} - x + c$$

$$(0,0) \Rightarrow c = 0 \Rightarrow y = e^{\left(\frac{x^2}{2} - x\right)} - 1$$

42. Official Ans. by NTA (1)

$$\text{Sol. } I = \int_0^{\sqrt{\pi/2}} ([x^2] + [-\cos x]) dx$$

$$= \int_0^1 0 dx + \int_1^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}} (-1) dx$$

$$= \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} = -1$$

$$\Rightarrow |I| = 1$$

43. Official Ans. by NTA (3)

$$\text{Sol. } y^2 = 4ax + 4a^2$$

differentiate with respect to x

$$\Rightarrow 2y \frac{dy}{dx} = 4a$$

$$\Rightarrow a = \left(\frac{y}{2} \frac{dy}{dx} \right)$$

so, required differential equation is

$$y^2 = \left(4 \times \frac{y}{2} \frac{dy}{dx} \right) x + 4 \left(\frac{y}{2} \frac{dy}{dx} \right)^2$$

$$\Rightarrow y^2 \left(\frac{dy}{dx} \right)^2 + 2xy \left(\frac{dy}{dx} \right) - y^2 = 0$$

$$\Rightarrow y \left(\frac{dy}{dx} \right)^2 + 2x \left(\frac{dy}{dx} \right) - y = 0$$

44. Official Ans. by NTA (1)

Sol. Let $y + 1 = Y$

$$\therefore \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$$

Put $-\frac{1}{Y} = k$

$$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$$

I.F. = $e^{-\frac{x^2}{2}}$

$$\therefore k = (x + c)e^{x^2/2}$$

Put $k = -\frac{1}{y+1}$

$$\therefore y + 1 = -\frac{1}{(x + c)e^{x^2/2}} \quad \dots(i)$$

when $x = 2, y = 0$, then $c = -2 - \frac{1}{e^2}$

differentiate equation (i) & put $x = 1$

we get $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{(1 + e^2)^2}$

45. Official Ans. by NTA (4)

Sol. $x dy - y dx = \sqrt{x^2 - y^2} dx$

$$\Rightarrow \frac{x dy - y dx}{x^2} = \frac{1}{x} \sqrt{1 - \frac{y^2}{x^2}} dx$$

$$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln|x| + c$$

at $x = 1, y = 0 \Rightarrow c = 0$

$$y = x \sin(\ln x)$$

$$A = \int_1^{e^\pi} x \sin(\ln x) dx$$

$$x = e^t, dx = e^t dt \Rightarrow \int_0^\pi e^{2t} \sin(t) dt = A$$

$$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5} (2 \sin t - \cos t)\right)_0^\pi = \frac{1 + e^{2\pi}}{5}$$

$$\alpha = \frac{1}{5}, \beta = \frac{1}{5} \text{ so } 10(\alpha + \beta) = 4$$