

DEFINITE INTEGRATION

- माना a एक धनात्मक वास्तविक संख्या है जिसके लिए $\int_0^a e^{x-[x]} dx = 10e - 9$ है, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है, तो a बराबर है :
 - $10 - \log_e(1 + e)$
 - $10 + \log_e 2$
 - $10 + \log_e 3$
 - $10 + \log_e(1 + e)$
- समाकलन $\int_{-1}^1 \log_e(\sqrt{1-x} + \sqrt{1+x}) dx$ का मान बराबर है :
 - $\frac{1}{2} \log_e 2 + \frac{\pi}{4} - \frac{3}{2}$
 - $2 \log_e 2 + \frac{\pi}{4} - 1$
 - $\log_e 2 + \frac{\pi}{2} - 1$
 - $2 \log_e 2 + \frac{\pi}{2} - \frac{1}{2}$
- माना $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx$, जबकि $f(x) = \log_e(x + \sqrt{x^2 + 1})$, $x \in \mathbf{R}$ है। तो निम्न में से कौन सा सही है ?
 - $g(1) = g(0)$
 - $\sqrt{2}g(1) = g(0)$
 - $g(1) = \sqrt{2}g(0)$
 - $g(1) + g(0) = 0$
- यदि $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1 + 4\pi^2}$, $\alpha \in \mathbf{R}$ है, जबकि $[x]$ महत्तम पूर्णांक $\leq x$ है, तो α बराबर है -
 - $200(1 - e^{-1})$
 - $100(1 - e)$
 - $50(e - 1)$
 - $150(e^{-1} - 1)$
- निश्चित समाकलन $\int_{\pi/24}^{5\pi/24} \frac{dx}{1 + \sqrt[3]{\tan 2x}}$ बराबर है :
 - $\frac{\pi}{3}$
 - $\frac{\pi}{6}$
 - $\frac{\pi}{12}$
 - $\frac{\pi}{18}$

- माना $f : [0, \infty) \rightarrow [0, \infty)$
 $f(x) = \int_0^x [y] dy$
 द्वारा परिभाषित है, जबकि $[x]$ महत्तम पूर्णांक $\leq x$ है। तो निम्न में से कौन-सा कथन सत्य है ?
 - $[0, \infty)$ के प्रत्येक बिन्दु पर f संतत है और पूर्णांक बिन्दुओं के अतिरिक्त सभी बिन्दुओं पर अवकलनीय है।
 - पूर्णांक बिन्दुओं के अतिरिक्त, $[0, \infty)$ के सभी बिन्दुओं पर f संतत तथा अवकलनीय दोनों है।
 - पूर्णांक बिन्दुओं के अतिरिक्त, $[0, \infty)$ के सभी बिन्दुओं पर f संतत है।
 - $[0, \infty)$ के प्रत्येक बिन्दु पर f अवकलनीय है।
- यदि $f(x) = \begin{cases} \int_0^x (5 + |1-t|) dt, & x > 2 \\ 5x + 1, & x \leq 2 \end{cases}$ है, तो
 - $x = 2$ पर $f(x)$ संतत नहीं है
 - $f(x)$ प्रत्येक x के लिए अवकलनीय है
 - $x = 2$ पर $f(x)$ संतत है परन्तु अवकलनीय नहीं है
 - $x = 1$ पर $f(x)$ अवकलनीय नहीं है
- समाकलन $\int_{-1}^1 \log(x + \sqrt{x^2 + 1}) dx$ का मान है :
 - 2
 - 0
 - 1
 - 1
- निश्चित समाकलन $\int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{x \cos x})(\sin^4 x + \cos^4 x)}$ का मान बराबर :
 - $-\frac{\pi}{2}$
 - $\frac{\pi}{2\sqrt{2}}$
 - $-\frac{\pi}{4}$
 - $\frac{\pi}{\sqrt{2}}$

10. माना फलन

$$f(x) = \log_4 \left(\log_5 \left(\log_3 (18x - x^2 - 77) \right) \right) \text{ का प्रांत}$$

(a, b) है। तो समाकलन

$$\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a+b-x))} dx \text{ का मान बराबर है}$$

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11. माना फलन $f : (a, b) \rightarrow \mathbf{R}$ दो बार अवकलनीय है,

जबकि एक अवकलनीय फलन $g(x)$ के लिए

$$f(x) = \int_a^x g(t) dt \text{ है। यदि } (a, b) \text{ में } f(x) = 0 \text{ के}$$

ठीक पाँच भिन्न मूल हैं, तो $g(x)g'(x) = 0$ के (a, b) में

कम से कम :

- (1) बारह मूल हैं (2) पाँच मूल
(3) सात मूल है (4) तीन मूल है

12. यदि $\int_0^\pi (\sin^3 x) e^{-\sin^2 x} dx = \alpha - \frac{\beta}{e} \int_0^1 \sqrt{t} e^t dt$ है, तो

$\alpha + \beta$ बराबर है _____.

13. $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right)^{1/2} dx$ का मान है :

- (1) $\log_e 4$ (2) $\log_e 16$
(3) $2\log_e 16$ (4) $4\log_e (3+2\sqrt{2})$

14. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{n^2}{n^2 + 4r^2}$ का मान है :

- (1) $\frac{1}{2} \tan^{-1}(2)$ (2) $\frac{1}{2} \tan^{-1}(4)$
(3) $\tan^{-1}(4)$ (4) $\frac{1}{4} \tan^{-1}(4)$

15. यदि समाकलन $\int_0^5 \frac{x+[x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta$ है, जहाँ

$\alpha, \beta \in \mathbf{R}, 5\alpha + 6\beta = 0$ तथा $[x]$ महत्तम पूर्णांक $\leq x$

है, तो $(\alpha + \beta)^2$ का मान बराबर है -

- (1) 100 (2) 25 (3) 16 (4) 36

16. $\int_{-\pi/2}^{\pi/2} \left(\frac{1 + \sin^2 x}{1 + \pi^{\sin x}} \right) dx$ का मान है -

- (1) $\frac{\pi}{2}$ (2) $\frac{5\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

17. यदि $U_n = \left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2^2}{n^2}\right)^2 \dots \left(1 + \frac{n^2}{n^2}\right)^n$ है, तो

$\lim_{n \rightarrow \infty} (U_n)^{\frac{-4}{n^2}}$ बराबर है :

- (1) $\frac{e^2}{16}$ (2) $\frac{4}{e}$ (3) $\frac{16}{e^2}$ (4) $\frac{4}{e^2}$

18. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$ बराबर है:

- (1) 6 (2) 8 (3) 5 (4) 10

19. समाकलन $\int_0^1 \frac{\sqrt{x} dx}{(1+x)(1+3x)(3+x)}$ का मान है :

- (1) $\frac{\pi}{8} \left(1 - \frac{\sqrt{3}}{2}\right)$ (2) $\frac{\pi}{4} \left(1 - \frac{\sqrt{3}}{6}\right)$
(3) $\frac{\pi}{8} \left(1 - \frac{\sqrt{3}}{6}\right)$ (4) $\frac{\pi}{4} \left(1 - \frac{\sqrt{3}}{2}\right)$

20. माना $[t]$ महत्तम पूर्णांक $\leq t$ है। तो

$$8 \cdot \int_{-\frac{1}{2}}^1 ([2x] + |x|) dx \text{ का मान है _____.$$

21. यदि $x \phi(x) = \int_5^x (3t^2 - 2\phi'(t)) dt$, $x > -2$, तथा

$\phi(0) = 4$, है, तो $\phi(2)$ बराबर है _____

22. यदि $[x]$ महत्तम पूर्णांक $\leq x$ है, तो

$$\pi^2 \int_0^2 \left(\sin \frac{\pi x}{2} \right) (x - [x])^{[x]} dx \text{ बराबर है :}$$

- (1) $2(\pi - 1)$ (2) $4(\pi - 1)$
(3) $4(\pi + 1)$ (4) $2(\pi + 1)$

23. फलन $f(x)$, जो $f(x) = x + \int_0^{\pi/2} \sin x \cdot \cos y f(y) dy$

को संतुष्ट करता है, है :

- (1) $x + \frac{2}{3}(\pi - 2)\sin x$ (2) $x + (\pi + 2)\sin x$
 (3) $x + \frac{\pi}{2}\sin x$ (4) $x + (\pi - 2)\sin x$

24. समाकलन $\int_1^3 [x^2 - 2x - 2] dx$ का मान, जबकि $[x]$,

महत्तम पूर्णांक $\leq x$ है, है :

- (1) $-\sqrt{2} - \sqrt{3} + 1$ (2) $-\sqrt{2} - \sqrt{3} - 1$
 (3) -5 (4) -4

25. माना $[0, 2]$ में $f(x)$ एक अवकलनीय फलन है, जिसके लिए $f'(x) = f'(2-x) \forall x \in (0, 2)$, $f(0) = 1$

तथा $f(2) = e^2$ है, तो $\int_0^2 f(x) dx$ का मान है :

- (1) $1 - e^2$ (2) $1 + e^2$
 (3) $2(1 - e^2)$ (4) $2(1 + e^2)$

26. यदि $\int_{-a}^a (|x| + |x - 2|) dx = 22$, ($a > 2$) है तथा

$[x]$, महत्तम पूर्णांक $\leq x$ को दर्शाता है, तो

$\int_a^{-a} (x + [x]) dx$ बराबर है _____ ।

27. $\int_{-1}^1 x^2 e^{[x^3]} dx$, जहाँ $[t]$ महत्तम पूर्णांक $\leq t$ है, का मान है:

- (1) $\frac{e-1}{3e}$ (2) $\frac{e+1}{3}$ (3) $\frac{e+1}{3e}$ (4) $\frac{1}{3e}$

28. $\lim_{x \rightarrow \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right]$

बराबर है :

- (1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

29. $\int_{-2}^2 |3x^2 - 3x - 6| dx$ का मान है _____.

30. $x > 0$ के लिए यदि $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$ है, तो

$f(e) + f\left(\frac{1}{e}\right)$ बराबर है

- (1) 1 (2) -1 (3) $\frac{1}{2}$ (4) 0

31. माना $f(x) = \int_0^x e^t f(t) dt + e^x$ सभी $x \in \mathbb{R}$ के लिए

एक अवकलनीय फलन है। तो $f(x)$ बराबर है:

- (1) $2e^{(e^x-1)} - 1$ (2) $e^x - 1$
 (3) $2e^{e^x} - 1$ (4) $e^{(e^x-1)}$

32. यदि $I_{m,n} = \int_0^1 x^{m-1} (1-x)^{n-1} dx$, $m, n \geq 1$ तथा

$\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \alpha I_{m,n}$, $\alpha \in \mathbb{R}$, है, तो α बराबर है _____

33. समाकलन $\int_0^{\pi} |\sin 2x| dx$ का मान बराबर है _____ ।

34. $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ का मान है :

- (1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

35. यदि $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx$, है, तो :

(1) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ G.P. में है

(2) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ A.P. में है

(3) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ G.P. में है

(4) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ A.P. में है

36. $\sum_{n=1}^{100} \int_{n-1}^n e^{x-[x]} dx$, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है, का

मान है :

- (1) $100(e-1)$ (2) $100(1-e)$
 (3) $100e$ (4) $100(1+e)$

37. समाकलन

$$I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}} dx,$$

का विचार कीजिए, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है।

तो I का मान बराबर है :

- (1) $9(e-1)$ (2) $45(e+1)$
 (3) $45(e-1)$ (4) $9(e+1)$

38. माना वास्तविक गुणांकों का एक द्विघातीय बहुपद

$$P(x) = x^2 + bx + c \text{ इस प्रकार है कि } \int_0^1 P(x) dx = 1 \text{ है}$$

तथा $P(x)$ को $(x-2)$ से विभाजित करने पर शेषफल 5 आता है। तो $9(b+c)$ का मान बराबर है :

- (1) 9 (2) 15 (3) 7 (4) 11

39. मान $f: \mathbb{R} \rightarrow \mathbb{R}$ एक संतत फलन है जिसके लिए

$$f(x) + f(x+1) = 2, \forall x \in \mathbb{R} \text{ है। यदि}$$

$$I_1 = \int_0^8 f(x) dx \text{ तथा } I_2 = \int_{-1}^3 f(x) dx \text{ है, तो } I_1 + 2I_2$$

का मान बराबर है _____।

40. माना $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^{-x} \sin x$ द्वारा परिभाषित है। यदि $F: [0, 1] \rightarrow \mathbb{R}$ एक अवकलनीय फलन

$$\text{है जिसके लिए } F(x) = \int_0^x f(t) dt \text{ है, तो}$$

$$\int_0^1 (F'(x) + f(x))e^x dx \text{ का मान निम्न में से किस}$$

अंतराल में है ?

- (1) $\left[\frac{327}{360}, \frac{329}{360}\right]$ (2) $\left[\frac{330}{360}, \frac{331}{360}\right]$
 (3) $\left[\frac{331}{360}, \frac{334}{360}\right]$ (4) $\left[\frac{335}{360}, \frac{336}{360}\right]$

41. यदि समाकलन $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$ है,

जहाँ α, β, γ पूर्णांक है तथा $[x]$ महत्तम पूर्णांक $\leq x$ है, तो $\alpha + \beta + \gamma$ का मान बराबर है :

- (1) 0 (2) 20 (3) 25 (4) 10

42. माना $I_n = \int_1^e x^{19} (\log|x|)^n dx$ है, जहाँ $n \in \mathbb{N}$ है।

यदि धन पूर्णाकों α तथा β के लिए $(20)I_{10} = \alpha I_9 + \beta I_8$ है, तो $\alpha - \beta$ बराबर है _____।

43. फलन $g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^\alpha x}{\cos^\alpha x + \sin^\alpha x} dx$, $\alpha \in \mathbb{R}$ के

लिए निम्न में से कौन सा कथन असत्य है?

- (1) $g(\alpha)$ एक निरंतर वर्धमान फलन है
 (2) $g(\alpha)$ का $\alpha = -\frac{1}{2}$ पर एक नति परिवर्तन बिन्दु है
 (3) $g(\alpha)$ एक निरंतर ह्रसमान फलन है
 (4) $g(\alpha)$ एक सम फलन है

44. माना $f(x)$ तथा $g(x)$ दो फलन हैं, जो

$$f(x^2) + g(4-x) = 4x^3 \text{ तथा } g(4-x) + g(x) = 0 \text{ को}$$

संतुष्ट करते हैं। तो $\int_{-4}^4 f(x)^2 dx$ का मान है _____।

45. माना $g(x) = \int_0^x f(t) dt$, जहाँ $[0, 3]$ में f एक संतत

$$\text{फलन है, } \frac{1}{3} \leq f(t) \leq 1 \forall t \in [0, 1] \text{ है तथा}$$

$$0 \leq f(t) \leq \frac{1}{2} \forall t \in (1, 3] \text{ है। अधिकतम संभव}$$

अंतराल, जिसमें $g(3)$ स्थित है, है।

- (1) $\left[-1, -\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2}, -1\right]$
 (3) $\left[\frac{1}{3}, 2\right]$ (4) $[1, 3]$

SOLUTION

1. Official Ans. by NTA (2)

Sol. $a > 0$

Let $n \leq a < n + 1, n \in W$

$$\therefore a = [a] + \{a\}$$

$$\Downarrow \quad \Downarrow$$

G.I.F Fractional part

Here $[a] = n$

$$\text{Now, } \int_0^a e^{x-[x]} dx = 10e - 9$$

$$\Rightarrow \int_0^n e^{\{x\}} dx + \int_n^a e^{x-[x]} dx = 10e - 9$$

$$\therefore n \int_0^1 e^x dx + \int_n^a e^{x-n} dx = 10e - 9$$

$$\Rightarrow n(e - 1) + (e^{a-n} - 1) = 10e - 9$$

$$\therefore \boxed{n=0} \text{ and } \{a\} = \log_e 2$$

$$\text{So, } a = [a] + \{a\} = (0 + \log_e 2)$$

\Rightarrow Option (2) is correct.

2. Official Ans. by NTA (2)

ALLEN Ans. (3)

Sol. Let $I = 2 \int_0^1 \underbrace{\ln(\sqrt{1-x} + \sqrt{1+x})}_{(I)} \cdot \underbrace{1}_{(II)} dx$
(I.B.P.)

$$\therefore I = 2 \left[(x \cdot \ln(\sqrt{1-x} + \sqrt{1+x})) \Big|_0^1 \right.$$

$$\left. - \int_0^1 x \cdot \left(\frac{1}{\sqrt{1-x} + \sqrt{1+x}} \right) \cdot \left(\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}} \right) dx \right]$$

$$= 2(\ln \sqrt{2} - 0) - \frac{2}{2} \int_0^1 \frac{x\sqrt{1-x} - \sqrt{1+x} dx}{(\sqrt{1-x} + \sqrt{1+x})\sqrt{1-x^2}}$$

$$= (\log_e 2) - \int_0^1 \frac{x \cdot (2 - 2\sqrt{1-x^2})}{-2x\sqrt{1-x^2}} dx \quad (\text{After}$$

rationalisation)

$$= (\log_e 2) + \int_0^1 \left(\frac{1 - \sqrt{1-x^2}}{\sqrt{1-x^2}} \right) dx$$

$$= (\log_e 2) + (\sin^{-1} x) \Big|_0^1 - 1$$

$$= \log_e 2 + \left(\frac{\pi}{2} - 0 \right) - 1$$

$$\therefore I = (\log_e 2) + \frac{\pi}{2} - 1$$

\Rightarrow Option (3) is correct.

3. Official Ans. by NTA (2)

Sol. $g(t) = \int_{-\pi/2}^{\pi/2} \left(\cos \frac{\pi}{4} t + f(x) \right) dx$

$$g(t) = \pi \cos \frac{\pi}{4} t + \int_{-\pi/2}^{\pi/2} f(x) dx$$

$$g(t) = \pi \cos \frac{\pi}{4} t$$

$$g(1) = \frac{\pi}{\sqrt{2}}, g(0) = \pi$$

4. Official Ans. by NTA (1)

$$\text{Sol. } I = \int_0^{100\pi} \frac{\sin^2 x}{e^{\{x/\pi\}}} dx = 100 \int_0^\pi \frac{\sin^2 x}{e^{x/\pi}} dx$$

$$100 \int_0^\pi e^{-x/\pi} \frac{(1 - \cos 2x)}{2} dx$$

$$= 50 \left\{ \int_0^\pi e^{-x/\pi} dx - \int_0^\pi e^{-x/\pi} \cos 2x dx \right\}$$

$$I_1 = \int_0^\pi e^{-x/\pi} dx = \left[-\pi e^{-x/\pi} \right]_0^\pi = \pi(1 - e^{-1})$$

$$I_2 = \int_0^\pi e^{-x/\pi} \cos 2x dx$$

$$= -\pi e^{-x/\pi} \cos 2x \Big|_0^\pi - \int -\pi e^{-x/\pi} (-2 \sin 2x) dx$$

$$= \pi(1 - e^{-1}) - 2\pi \int_0^\pi e^{-x/\pi} \sin 2x dx$$

$$= \pi(1 - e^{-1}) - 2\pi \left\{ -\pi e^{-x/\pi} \sin 2x \Big|_0^\pi - \int_0^\pi -\pi e^{-x/\pi} 2 \cos 2x dx \right\}$$

$$= \pi(1 - e^{-1}) - 4\pi^2 I_2$$

$$\Rightarrow I_2 = \frac{\pi(1 - e^{-1})}{1 + 4\pi^2}$$

$$\therefore I = 50 \left\{ \pi(1 - e^{-1}) - \frac{\pi(1 - e^{-1})}{1 + 4\pi^2} \right\}$$

$$= \frac{200(1 - e^{-1})\pi^3}{1 + 4\pi^2}$$

5. Official Ans. by NTA (3)

$$\text{Sol. Let } I = \int_{\pi/24}^{5\pi/24} \frac{(\cos 2x)^{1/3}}{(\cos 2x)^{1/3} + (\sin 2x)^{1/3}} dx \quad \dots(i)$$

$$\Rightarrow I = \int_{\pi/24}^{5\pi/24} \frac{\left(\cos \left\{ 2 \left(\frac{\pi}{4} - x \right) \right\} \right)^{1/3}}{\left(\cos \left\{ 2 \left(\frac{\pi}{4} - x \right) \right\} \right)^{1/3} + \left(\sin \left\{ 2 \left(\frac{\pi}{4} - x \right) \right\} \right)^{1/3}} dx$$

$$\left\{ \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right\}$$

$$\text{So } I = \int_{\pi/24}^{5\pi/24} \frac{(\sin 2x)^{1/3}}{(\sin 2x)^{1/3} + (\cos 2x)^{1/3}} dx \quad \dots(ii)$$

$$\text{Hence } 2I = \int_{\pi/24}^{5\pi/24} dx \quad [(i) + (ii)]$$

$$\Rightarrow 2I = \frac{4\pi}{24} \Rightarrow \boxed{I = \frac{\pi}{12}}$$

6. Official Ans. by NTA (1)

$$\text{Sol. } f : [0, \infty) \rightarrow [0, \infty), f(x) = \int_0^x [y] dy$$

$$\text{Let } x = n + f, f \in (0, 1)$$

$$\text{So } f(x) = 0 + 1 + 2 + \dots + (n-1) + \int_n^{n+f} n dy$$

$$f(x) = \frac{n(n-1)}{2} + nf$$

$$= \frac{[x]([x]-1)}{2} + [x]\{x\}$$

$$\text{Note } \lim_{x \rightarrow n^+} f(x) = \frac{n(n-1)}{2},$$

$$\lim_{x \rightarrow n^-} f(x) = \frac{(n-1)(n-2)}{2} + (n-1) = \frac{n(n-1)}{2}$$

$$f(x) = \frac{n(n-1)}{2} \quad (n \in \mathbb{N}_0)$$

so $f(x)$ is cont. $\forall x \geq 0$ and diff. except at integer points

7. Official Ans. by NTA (3)

Sol. $f(x) = \int_0^1 (5 + (1-t)) dt + \int_1^x (5 + (t-1)) dt$

$$= 6 - \frac{1}{2} + \left(4t + \frac{t^2}{2}\right) \Big|_1^x$$

$$= \frac{11}{2} + 4x + \frac{x^2}{2} - 4 - \frac{1}{2}$$

$$= \frac{x^2}{2} + 4x + 1$$

$f(2^+) = 2 + 8 + 1 = 11$

$f(2) = f(2^-) = 5 \times 2 + 1 = 11$

\Rightarrow continuous at $x = 2$

Clearly differentiable at $x = 1$

$Lf'(2) = 5$

$Rf'(2) = 6$

\Rightarrow not differentiable at $x = 2$

8. Official Ans. by NTA (2)

Sol. Let $I = \int_{-1}^1 \log(x + \sqrt{x^2 + 1}) dx$

$\because \log(x + \sqrt{x^2 + 1})$ is an odd function

$\therefore I = 0$

9. Official Ans. by NTA (2)

Sol. $I = \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{x \cos x})(\sin^4 x + \cos^4 x)} \dots(1)$

Using $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$

$I = \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{-x \cos x})(\sin^4 x + \cos^4 x)}$

Add (1) and (2)

$$2I = \int_{-\pi/4}^{\pi/4} \frac{dx}{\sin^4 x + \cos^4 x}$$

$$2I = 2 \int_0^{\pi/4} \frac{dx}{\sin^4 x + \cos^4 x}$$

$$I = \int_0^{\pi/4} \frac{(1 + \tan^2 x) \sec^2 x}{\tan^4 x + 1} dx$$

$$I = \int_0^{\pi/4} \frac{\left(1 + \frac{1}{\tan^2 x}\right) \sec^2 x}{\left(\tan x - \frac{1}{\tan x}\right)^2 + 2} dx$$

$\tan x - \frac{1}{\tan x} = t$

$\left(1 + \frac{1}{\tan^2 x}\right) \sec^2 x dx = dt$

$$I = \int_{-\infty}^0 \frac{dt}{t^2 + 2} = \left[\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) \right]_{-\infty}^0$$

$$I = 0 - \frac{1}{\sqrt{2}} \left(-\frac{\pi}{2} \right) = \frac{\pi}{2\sqrt{2}}$$

10. Official Ans. by NTA (1)

Sol. For domain

$\log_3(\log_3(18x - x^2 - 77)) > 0$

$\log_3(18x - x^2 - 77) > 1$

$18x - x^2 - 77 > 3$

$x^2 - 18x + 80 < 0$

$x \in (8, 10)$

$\Rightarrow a = 8$ and $b = 10$

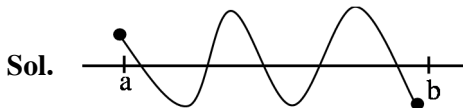
$$I = \int_a^b \frac{\sin^3 x}{\sin^3 x + \sin^3(a + b - x)} dx$$

$$I = \int_a^b \frac{\sin^3(a + b - x)}{\sin^3 x + \sin^3(a + b - x)}$$

$2I = (b - a) \Rightarrow I = \frac{b - a}{2} \quad (\because a = 8 \text{ and } b = 10)$

$$I = \frac{10 - 8}{2} = 1$$

11. Official Ans. by NTA (3)



$$f(x) = \int_a^x g(t) dt$$

$$f(x) \rightarrow 5$$

$$f'(x) \rightarrow 4$$

$$g(x) \rightarrow 4$$

$$g'(x) \rightarrow 3$$

12. Official Ans. by NTA (5)

Sol. $I = 2 \int_0^{\pi/2} \sin^3 x e^{-\sin^2 x} dx$

$$= 2 \int_0^{\pi/2} \sin x e^{-\sin^2 x} dx +$$

$$\int_0^{\pi/2} \underbrace{\cos x}_I \underbrace{e^{-\sin^2 x} (-\sin 2x)}_{II} dx$$

$$= 2 \int_0^{\pi/2} \sin x e^{-\sin^2 x} dx + \left[\cos x e^{-\sin^2 x} \right]_0^{\pi/2}$$

$$+ \int_0^{\pi/2} \sin x e^{-\sin^2 x} dx$$

$$= 3 \int_0^{\pi/2} \sin x e^{-\sin^2 x} dx - 1$$

$$= \frac{3}{2} \int_{-1}^0 \frac{e^\alpha d\alpha}{\sqrt{1+\alpha}} - 1 \quad (\text{Put } -\sin^2 x = t)$$

$$= \frac{3}{2e} \int_0^1 \frac{e^x}{\sqrt{x}} dx - 1 \quad (\text{put } 1 + \alpha = x)$$

$$= \frac{3}{2e} \int_0^1 \frac{e^x}{\sqrt{x}} dx - 1$$

$$= 2 - \frac{3}{e} \int_0^1 e^x \sqrt{x} dx$$

Hence, $\alpha + \beta = \boxed{5}$

13. Official Ans. by NTA (2)

Sol. $I = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1} - \frac{x-1}{x+1} \right)^2 \right)^{1/2} dx$

$$I = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left| \frac{4x}{x^2-1} \right| dx \Rightarrow I = 2.4 \int_0^{1/\sqrt{2}} \left| \frac{x}{x^2-1} \right| dx$$

$$\Rightarrow I = -4 \int_0^{1/\sqrt{2}} \frac{2x}{x^2-1} dx \Rightarrow I = -4 \ln |x^2-1|_0^{1/\sqrt{2}}$$

$$\Rightarrow I = 4 \ln 2 \Rightarrow I = \ln 16$$

14. Official Ans. by NTA (2)

Sol. $L = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{1}{1 + 4 \left(\frac{r}{n} \right)^2}$

$$\Rightarrow L = \int_0^2 \frac{1}{1 + 4x^2} dx$$

$$\Rightarrow L = \frac{1}{2} \tan^{-1}(2x) \Big|_0^2 \Rightarrow L = \frac{1}{2} \tan^{-1} 4$$

15. Official Ans. by NTA (2)

Sol. $I = \int_0^5 \frac{x + [x]}{e^{x-[x]}} dx$

$$\int_0^1 \frac{x}{e^x} dx + \int_1^2 \frac{x+1}{e^{x-1}} dx + \int_2^3 \frac{x+2}{e^{x-2}} dx + \dots + \int_4^5 \frac{x+4}{e^{x-4}} dx$$

$$\Downarrow \qquad \qquad \qquad \Downarrow \qquad \qquad \qquad \Downarrow$$

$$x = t + 1 \qquad x = z + 2 \qquad x = y + 4$$

$$\int_0^1 \frac{t+2}{e^t} dt + \int_0^1 \frac{z+4}{e^z} dz + \dots + \int_0^1 \frac{y+8}{e^y} dy$$

$$\Rightarrow \int_0^5 \frac{5x+20}{e^x} dx = 5 \int_0^1 \frac{x+4}{e^x} dx$$

$$\Rightarrow 5 \int_0^1 (x+4)e^{-x} dx$$

$$\Rightarrow 5e^{-x}(-x-5) \Big|_0^1 \Rightarrow -\frac{30}{e} + 25$$

$$\alpha = -30$$

$$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$$

$$(\alpha + \beta)^2 = 5^2 = 25$$

16. Official Ans. by NTA (3)

Sol. $I = \int_0^{\pi/2} \frac{(1 + \sin^2 x)}{(1 + \pi^{\sin x})} + \frac{\pi^{\sin x} (1 + \sin^2 x)}{(1 + \pi^{\sin x})} dx$

$$I = \int_0^{\pi/2} (1 + \sin^2 x) dx$$

$$I = \frac{\pi}{2} + \frac{\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$$

17. Official Ans. by NTA (1)

Sol. $U_n = \prod_{r=1}^n \left(1 + \frac{r^2}{n^2}\right)^r$

$$L = \lim_{n \rightarrow \infty} (U_n)^{-4/n^2}$$

$$\log L = \lim_{n \rightarrow \infty} \frac{-4}{n^2} \sum_{r=1}^n \log \left(1 + \frac{r^2}{n^2}\right)^r$$

$$\Rightarrow \log L = \lim_{n \rightarrow \infty} \sum_{r=1}^n -\frac{4r}{n} \cdot \frac{1}{n} \log \left(1 + \frac{r^2}{n^2}\right)$$

$$\Rightarrow \log L \Rightarrow -4 \int_0^1 x \log(1 + x^2) dx$$

put $1 + x^2 = t$

Now, $2x dx = dt$

$$= -2 \int_1^2 \log(t) dt = -2 [t \log t - t]_1^2$$

$$\Rightarrow \log L = -2(2 \log 2 - 1)$$

$$\therefore L = e^{-2(2 \log 2 - 1)}$$

$$= e^{-2 \left(\log \left(\frac{4}{e}\right)\right)}$$

$$= e^{\log \left(\frac{4}{e}\right)^{-2}}$$

$$= \left(\frac{e}{4}\right)^2 = \frac{e^2}{16}$$

18. Official Ans. by NTA (3)

Sol. Let $I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$

$$I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x - 22)^2} dx \quad \dots(1)$$

We know

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx \quad (\text{king})$$

So

$$I = \int_6^{16} \frac{\log_e (22 - x)^2}{\log_e (22 - x)^2 + \log_e (22 - (22 - x))^2} dx$$

$$I = \int_0^{16} \frac{\log_e (22 - x)^2}{\log_e x^2 + \log_e (22 - x)^2} dx \quad \dots(2)$$

$$(1) + (2)$$

$$2I = \int_6^{16} 1 dx = 10$$

$$I = 5$$

19. Official Ans. by NTA (1)

$$\text{Sol. } I = \int_0^1 \frac{\sqrt{x}}{(1+x)(1+3x)(3+x)} dx$$

$$\text{Let } x = t^2 \Rightarrow dx = 2t \cdot dt$$

$$I = \int_0^1 \frac{t(2t)}{(t^2+1)(1+3t^2)(3+t^2)} dt$$

$$I = \int_0^1 \frac{(3t^2+1) - (t^2+1)}{(3t^2+1)(t^2+1)(3+t^2)} dt$$

$$I = \int_0^1 \frac{dt}{(t^2+1)(3+t^2)} - \int_0^1 \frac{dt}{(1+3t^2)(3+t^2)}$$

$$= \frac{1}{2} \int_0^1 \frac{(3+t^2) - (t^2+1)}{(t^2+1)(3+t^2)} dt + \frac{1}{8} \int_0^1 \frac{(1+3t^2) - 3(3+t^2)}{(1+3t^2)(3+t^2)} dt$$

$$= \frac{1}{2} \int_0^1 \frac{dt}{1+t^2} - \frac{1}{2} \int_0^1 \frac{dt}{t^2+3}$$

$$+ \frac{1}{8} \int_0^1 \frac{dt}{t^2+3} - \frac{3}{8} \int_0^1 \frac{dt}{(1+3t^2)}$$

$$= \frac{1}{2} \int_0^1 \frac{dt}{t^2+1} - \frac{3}{8} \int_0^1 \frac{dt}{t^2+3} - \frac{3}{8} \int_0^1 \frac{dt}{1+3t^2}$$

$$= \frac{1}{2} (\tan^{-1}(t))_0^1 - \frac{3}{8\sqrt{3}} \left(\tan^{-1} \left(\frac{t}{\sqrt{3}} \right) \right)_0^1$$

$$- \frac{3}{8\sqrt{3}} (\tan^{-1}(\sqrt{3}t))_0^1$$

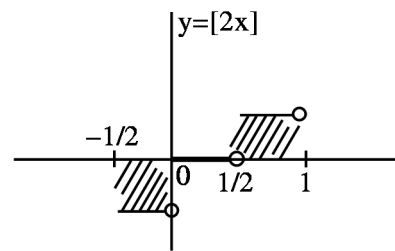
$$= \frac{1}{2} \left(\frac{\pi}{4} \right) - \frac{\sqrt{3}}{8} \left(\frac{\pi}{6} \right) - \frac{\sqrt{3}}{8} \left(\frac{\pi}{3} \right)$$

$$= \frac{\pi}{8} - \frac{\sqrt{3}}{16} \pi$$

$$= \frac{\pi}{8} \left(1 - \frac{\sqrt{3}}{2} \right)$$

20. Official Ans. by NTA (5)

$$\text{Sol. } I = \int_{-1/2}^1 ([2x] + |x|) dx$$



$$= \int_{-1/2}^0 [2x] dx + \int_{-1/2}^0 |x| dx$$

$$= 0 + \int_{-1/2}^0 (-x) dx + \int_0^1 x dx$$

$$= \left(-\frac{x^2}{2} \right)_{-1/2}^0 + \left(\frac{x^2}{2} \right)_0^1$$

$$= \left(0 + \frac{1}{8} \right) + \frac{1}{2}$$

$$= \frac{5}{8}$$

$$8I = 5$$

21. Official Ans. by NTA (4)

$$\text{Sol. } x\phi(x) = \int_5^x 3t^2 - 2\phi'(t) dt$$

$$x\phi(x) = x^3 - 125 - 2[\phi(x) - \phi(5)]$$

$$x\phi(x) = x^3 - 125 - 2\phi(x) + 2\phi(5)$$

$$\phi(0) = 4 \Rightarrow \phi(5) = -\frac{133}{2}$$

$$\phi(x) = \frac{x^3 + 8}{x + 2}$$

$$\phi(2) = 4$$

22. Official Ans. by NTA (2)

$$\text{Sol. } \pi^2 \left[\int_0^1 \sin \frac{\pi x}{2} dx + \int_1^2 \sin \frac{\pi x}{2} (x-1) dx \right]$$

$$= \pi^2 \left[-\frac{2}{\pi} \left(\cos \frac{\pi x}{2} \right) + \left((x-1) \left(-\frac{2}{\pi} \cos \frac{\pi x}{2} \right) \right)_1^2 - \int_1^2 \frac{2}{\pi} \cos \frac{\pi x}{2} dx \right]$$

$$= \pi^2 \left[0 + \frac{2}{\pi} + \frac{2}{\pi} + \frac{2}{\pi} \cdot \frac{2}{\pi} \left(\sin \frac{\pi x}{2} \right)_1^2 \right]$$

$$= 4\pi - 4 = 4(\pi - 1)$$

23. Official Ans. by NTA (4)

Sol. $f(x) = x + \int_0^{\pi/2} \sin x \cos y f(y) dy$

$$f(x) = x + \sin x \underbrace{\int_0^{\pi/2} \cos y f(y) dy}_K$$

$$\Rightarrow f(x) = x + K \sin x$$

$$\Rightarrow f(y) = y + K \sin y$$

Now $K = \int_0^{\pi/2} \cos y (y + K \sin y) dy$

$$K = \int_0^{\pi/2} y \cos y dy + \int_0^{\pi/2} \cos y \sin y dy$$

Apply IBP Put $\sin y = t$

$$K = (y \sin y)_0^{\pi/2} - \int_0^{\pi/2} \sin y dy + K \int_0^1 t dt$$

$$\Rightarrow K = \frac{\pi}{2} - 1 + K \left(\frac{1}{2} \right)$$

$$\Rightarrow K = \pi - 2$$

So $f(x) = x + (\pi - 2) \sin x$

Option (4)

24. Official Ans. by NTA (2)

Sol. $\int_1^3 \left(\left[(x-1)^2 \right] - 3 \right) dx$

$$= \int_1^2 [x^2] - 3 \int_1^3 dx$$

$$= \int_1^3 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 dx + \int_{\sqrt{3}}^2 3 dx - 6$$

$$= \sqrt{2} - 1 + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) - 6$$

$$= -\sqrt{2} - \sqrt{3} - 1$$

25. Official Ans. by NTA (2)

Sol. $f'(x) = f'(2-x)$

$$f(x) = -f(2-x) + c$$

put $x = 0$

$$f'(0) = -f'(2) + c$$

$$c = f(0) + f(2) = 1 + e^2$$

so, $f(x) + f(2-x) = 1 + e^2$

$$I = \int_0^2 f(x) dx$$

$$I = \int_0^2 f(2-x) dx$$

$$2I = \int_0^2 (f(x) + f(2-x)) dx$$

$$2I = (1 + e^2) \int_0^2 dx$$

$$I = 1 + e^2$$

26. Official Ans. by NTA (3)

Sol. $\int_{-a}^0 (-2x+2) dx + \int_0^2 (x+2-x) dx + \int_2^a (2x-2) dx = 22$

$$x^2 - 2x \Big|_0^{-a} + 2x \Big|_0^2 + x^2 - 2x \Big|_0^a = 22$$

$$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$$

$$2a^2 = 18 \Rightarrow a = 3$$

$$\int_3^{-3} (x + [x]) dx = -(-3 - 2 - 1 + 1 + 2) = 3$$

27. Official Ans. by NTA (3)

Sol. $I = \int_{-1}^1 x^2 e^{[x^3]} dx$

$$= \int_{-1}^0 x^2 e^{[x^3]} dx + \int_0^1 x^2 e^{[x^3]} dx$$

$$= \int_{-1}^0 x^2 e^{-1} dx + \int_0^1 x^2 e^0 dx$$

$$= \frac{1}{e} \times \frac{x^3}{3} \Big|_{-1}^0 + \frac{x^3}{3} \Big|_0^1$$

$$= \frac{1}{e} \times \left(0 - \left(\frac{-1}{3} \right) \right) + \frac{1}{3}$$

$$= \frac{1}{3e} + \frac{1}{3} = \frac{1+e}{3e}$$

28. Official Ans. by NTA (1)

$$\begin{aligned} \text{Sol. } \lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right] \\ = \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \frac{n}{(n+r)^2} = \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \frac{n}{n^2 + 2nr + r^2} \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{n-1} \frac{1}{(r/n)^2 + 2(r/n) + 1} \\ = \int_0^1 \frac{dx}{(x+1)^2} = \left[\frac{-1}{(x+1)} \right]_0^1 = \frac{1}{2} \end{aligned}$$

29. Official Ans. by NTA (19)

$$\begin{aligned} \text{Sol. } \int_{-2}^2 3|x^2 - x - 2| dx \\ = 3 \int_{-2}^2 |x^2 - x - 2| dx \\ = 3 \left[\int_{-2}^{-1} (x^2 - x - 2) dx + \int_{-1}^2 -(x^2 - x - 2) dx \right] \\ = 3 \left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x \right) \Big|_{-2}^{-1} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x \right) \Big|_{-1}^2 \right] \\ = 3 \left[7 - \frac{2}{3} \right] \\ = 19 \end{aligned}$$

30. Official Ans. by NTA (3)

$$\begin{aligned} \text{Sol. } f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt \\ f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{\log_e t}{1+t} dt, \text{ let } t = \frac{1}{y} \\ = + \int_1^x \frac{\log_e y}{1+y} \cdot \frac{y}{y^2} dy \\ = \int_1^x \frac{\log_e y}{y(1+y)} dy \\ \text{hence} \\ f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{(1+t) \log_e t}{t(1+t)} dt = \int_1^x \frac{\log_e t}{t} dt \\ = \frac{1}{2} \log_e^2(x) \\ \text{so } f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2} \dots(3) \end{aligned}$$

31. Official Ans. by NTA (1)

$$\text{Sol. } f(x) = \int_0^x e^t f(t) dt + e^x \Rightarrow f(0) = 1$$

differentiating with respect to x

$$f'(x) = e^x f(x) + e^x$$

$$f'(x) = e^x (f(x) + 1)$$

$$\int_0^x \frac{f'(x)}{f(x)+1} dx = \int_0^x e^x dx$$

$$\ln(f(x)+1) \Big|_0^x = e^x \Big|_0^x$$

$$\ln(f(x)+1) - \ln(f(0)+1) = e^x - 1$$

$$\ln\left(\frac{f(x)+1}{2}\right) = e^x - 1 \quad \{\text{as } f(0) = 1\}$$

$$f(x) = 2e^{(e^x-1)} - 1$$

32. Official Ans. by NTA (1)

$$\text{Sol. } I_{m,n} = \int_0^1 x^{m-1} (1-x)^{n-1} dx = I_{n,m}$$

$$\text{Now Let } x = \frac{1}{y+1} \Rightarrow dx = -\frac{1}{(y+1)^2} dy$$

so

$$I_{m,n} = -\int_{\infty}^0 \frac{1}{(y+1)^{m-1}} \frac{y^{n-1}}{(y+1)^{n-1}} \frac{dy}{(y+1)^2} = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$$

$$\text{similarly } I_{m,n} = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}} dy$$

$$\text{Now } 2I_{m,n} = \int_0^{\infty} \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy$$

$$= \int_0^{\infty} \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy$$

$$= \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy + \underbrace{\int_1^{\infty} \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy}_{\text{substitute } y = \frac{1}{t}}$$

$$\Rightarrow 2I_{m,n} = \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy - \int_1^0 \frac{t^{n-1} + t^{m-1}}{t^{m+n-2}} \frac{t^{m+n}}{(1+t)^{m+n}} dt$$

$$\Rightarrow \text{Hence } 2I_{m,n} = 2 \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy \Rightarrow \alpha = 1$$

33. Official Ans. by NTA (2)

Sol. Put $2x = t \Rightarrow 2dx = dt$

$$\Rightarrow I = \frac{1}{2} \int_0^{2\pi} |\sin t| dt$$

$$= \int_0^{\pi} |\sin t| dt$$

$$= 2$$

34. Official Ans. by NTA (1)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ (using king)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+3^x) \cos^2 x}{1+3^x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \cos^2 x dx = 2 \int_0^{\pi/2} \cos^2 x dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$$

35. Official Ans. by NTA (4)

Sol. $I_n = \int_{\pi/4}^{\pi/2} \cot^n x dx = \int_{\pi/4}^{\pi/2} \cot^{n-2} x (\operatorname{cosec}^2 x - 1) dx$

$$= -\frac{\cot^{n-1} x}{n-1} \Big|_{\pi/4}^{\pi/2} - I_{n-2}$$

$$= \frac{1}{n-1} - I_{n-2}$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$$

$$\Rightarrow I_2 + I_4 = \frac{1}{3}$$

$$I_3 + I_5 = \frac{1}{4}$$

$$I_4 + I_6 = \frac{1}{5}$$

$\therefore \frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in A.P.

36. Official Ans. by NTA (1)

Sol. $\sum_{n=1}^{100} \int_{n-1}^n e^{\{x\}} dx$, period of $\{x\} = 1$

$$\sum_{n=1}^{100} \int_0^1 e^{\{x\}} dx = \sum_{n=1}^{100} \int_0^1 e^x dx$$

$$\sum_{n=1}^{100} (e-1) = 100(e-1)$$

37. Official Ans by NTA (3)

Sol. $I = \int_0^{10} [x] \cdot e^{[x]-x+1} dx$

$$I = \int_0^1 0 dx + \int_1^2 1 \cdot e^{2-x} + \int_2^3 2 \cdot e^{3-x} + \dots + \int_9^{10} 9 \cdot e^{10-x} dx$$

$$\Rightarrow I = \sum_{n=0}^9 \int_n^{n+1} n \cdot e^{n+1-x} dx$$

$$= - \sum_{n=0}^9 n (e^{n+1-x})_n^{n+1}$$

$$= - \sum_{n=0}^9 n \cdot (e^0 - e^1)$$

$$= (e-1) \sum_{n=0}^9 n$$

$$= (e-1) \cdot \frac{9 \cdot 10}{2}$$

$$= 45(e-1)$$

38. Official Ans by NTA (3)

Sol. $\int_0^1 (x^2 + bx + c) dx = 1$

$$\frac{1}{3} + \frac{b}{2} + c = 1 \Rightarrow \frac{b}{2} + c = \frac{2}{3}$$

$$3b + 6c = 4 \quad \dots(1)$$

$$P(2) = 5$$

$$4 + 2b + c = 5$$

$$2b + c = 1 \quad \dots(2)$$

From (1) & (2)

$$b = \frac{2}{9} \quad \& \quad c = \frac{5}{9}$$

$$9(b+c) = 7$$

39. Official Ans. by NTA (16)

Sol. $f(x) + f(x+1) = 2$

$\Rightarrow f(x)$ is periodic with period = 2

$$I_1 = \int_0^8 f(x) dx = 4 \int_0^2 f(x) dx$$

$$= 4 \int_0^1 (f(x) + f(1+x)) dx = 8$$

Similarly $I_2 = 2 \times 2 = 4$

$$I_1 + 2I_2 = 16$$

40. Official Ans. by NTA (2)

Sol. $f(x) = e^{-x} \sin x$

Now, $F(x) = \int_0^x f(t) dt \Rightarrow F'(x) = f(x)$

$$I = \int_0^1 (F'(x) + f(x)) e^x dx = \int_0^1 (f(x) + f(x)) \cdot e^x dx$$

$$= 2 \int_0^1 f(x) \cdot e^x dx = 2 \int_0^1 e^{-x} \sin x \cdot e^x dx$$

$$= 2 \int_0^1 \sin x dx$$

$$= 2(1 - \cos 1)$$

$$I = 2 \left\{ 1 - \left(1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{6} + \frac{1}{8} - \dots \right) \right\}$$

$$I = 1 - \frac{2}{4} + \frac{2}{6} - \frac{2}{8} + \dots$$

$$1 - \frac{2}{4} < I < 1 - \frac{2}{4} + \frac{2}{6}$$

$$\frac{11}{12} < I < \frac{331}{360}$$

$$\Rightarrow I \in \left[\frac{11}{12}, \frac{331}{360} \right]$$

$$\Rightarrow I \in \left[\frac{330}{360}, \frac{331}{360} \right] \quad \text{Ans. (2)}$$

41. Official Ans. by NTA (1)

Sol. Let $I = \int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \int_0^{10} \frac{[\sin 2\pi x]}{e^{\{x\}}} dx$

Function $f(x) = \frac{[\sin 2\pi x]}{e^{\{x\}}}$ is periodic with period '1'

Therefore

$$I = 10 \int_0^1 \frac{[\sin 2\pi x]}{e^{\{x\}}} dx$$

$$= 10 \int_0^1 \frac{[\sin 2\pi x]}{e^x} dx$$

$$= 10 \left(\int_0^{1/2} \frac{[\sin 2\pi x]}{e^x} dx + \int_{1/2}^1 \frac{[\sin 2\pi x]}{e^x} dx \right)$$

$$= 10 \left(0 + \int_{1/2}^1 \frac{(-1)}{e^x} dx \right)$$

$$= -10 \int_{1/2}^1 e^{-x} dx$$

$$= 10(e^{-1} - e^{-1/2})$$

Now,

$$10 \cdot e^{-1} - 10 \cdot e^{-1/2} = \alpha e^{-1} + \beta e^{-1/2} + \gamma \text{ (given)}$$

$$\Rightarrow \alpha = 10, \beta = -10, \gamma = 0$$

$$\Rightarrow \alpha + \beta + \gamma = 0$$

Ans. (1)

42. Official Ans. by NTA (1)

Sol. $I_n = \int_1^e x^{19} (\log|x|)^n dx$

$$I_n = \left[(\log|x|)^{19} \frac{x^{20}}{20} \right]_1^e - \int_1^e n(\log|x|)^{n-1} \cdot \frac{1}{x} \cdot \frac{x^{20}}{20} dx$$

$$20I_n = e^{20} - nI_{n-1}$$

$$\therefore 20I_{10} = e^{20} - 10I_9$$

$$20I_9 = e^{20} - 9I_8$$

$$\Rightarrow 20I_{10} = 10I_9 + 9I_8$$

$$\alpha = 10, \beta = 9$$

43. Official Ans. by NTA (Bonus)

Sol. $g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3} \frac{\sin^\alpha x}{(\sin^\alpha x + \cos^\alpha x)} \dots(i)$

$g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3} \frac{\cos^\alpha x}{(\sin^\alpha x + \cos^\alpha x)} \dots(ii)$

(1) + (2)

$2g(\alpha) = \frac{\pi}{6}$

$g(\alpha) = \frac{\pi}{12}$

Constant and even function

Due to typing mistake it must be bonus.

44. Official Ans. by NTA (512)

Sol. $I = 2 \int_0^4 f(x^2) dx$ {Even function}

$= 2 \int_0^4 (4x^3 - g(4-x)) dx$

$= 2 \left(\frac{4x^4}{4} \Big|_0^4 - \int_0^4 g(4-x) dx \right)$

$= 2(256 - 0) = 512$

45. Official Ans. by NTA (3)

Sol. $\frac{1}{3} \leq f(t) \leq 1 \forall t \in [0, 1]$

$0 \leq f(t) \leq \frac{1}{2} \forall t \in (1, 3]$

Now, $g(3) = \int_0^3 f(t) dt = \int_0^1 f(t) dt + \int_1^3 f(t) dt$

$\therefore \int_0^1 \frac{1}{3} dt \leq \int_0^1 f(t) dt \leq \int_0^1 1 dt \dots(1)$

and $\int_1^3 0 dt \leq \int_1^3 f(t) dt \leq \int_1^3 \frac{1}{2} dt \dots(2)$

Adding, we get

$\frac{1}{3} + 0 \leq g(3) \leq 1 + \frac{1}{2}(3-1)$

$\frac{1}{3} \leq g(3) \leq 2$