BINOMIAL THEOREM

1. $(1-x)^{101}(x^2+x+1)^{100}$ के प्रसार में x^{256} का गुणांक है :

(1) ${}^{100}C_{16}$ (2) ${}^{100}C_{15}$

(3) $-{}^{100}C_{16}$ (4) $-{}^{100}C_{15}$

2. $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ के द्विपद प्रसार में परिमेय पदों की संख्या है ________ ।

3. यदि धन पूर्णांकों m तथा n के लिए

$(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + ... + a_{m+n}y^{m+n}$

तथा $a_1 = a_2 = 10$ हैं, तो $(m + n)$ बराबर है:

(1) 88 (2) 64 (3) 100 (4) 80

4. माना $k \in N$ के लिए,

$$\frac{1}{\alpha(\alpha+1)(\alpha+2).......(\alpha+20)}=\sum_{K=0}^{20}\frac{A_k}{\alpha+k},\ \alpha>0$$

है। तो $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ बराबर है ________ ।

5. यदि $\left(2x^r+\frac{1}{x^2}\right)^{10}$ के द्विपद प्रसार में अचर पद 180 है तो r बराबर है _______ ।

6. समुच्चय $\{n \in \{1, 2, 3, ..., 100\} \mid (11)^n > (10)^n + (9)^n\}$ में अवयवों की संख्या है _______ ।

7. यदि b,a से बहुत छोटा है, जिनके लिए निम्न सर्वसमिका

$$\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+....+\frac{1}{a-nb}=\alpha n+\beta n^2+\gamma n^3$$

में, $\frac{b}{a}$ की क्यूब और ऊँची घातों की उपेक्षा की जा सकती है, तो γ बराबर है :

(1) $\frac{a^2+b}{3a^3}$ (2) $\frac{a+b}{3a^2}$

(3) $\frac{b^2}{3a^3}$ (4) $\frac{a+b^2}{3a^3}$

8. $(1 + x)^{20}$ के प्रसार में मध्य पद का गुणांक तथा $(1 + x)^{19}$ के प्रसार में दो मध्य पदों के गुणांकों के योग का अनुपात है ________ ।

9. $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, $x \neq 0, 1$ के प्रसार में 'x' से स्वतंत्र पद बराबर है ________ ।

10. $(2^{1/3}+3^{1/4})^{12}$ के प्रसार में, उन सभी पदों, जो परिमेय संख्याएँ हैं, का योगफल है :

(1) 89 (2) 27 (3) 35 (4) 43

11. यदि $\left(x\sin\alpha+a\frac{\cos\alpha}{x}\right)^{10}$ के प्रसार में 'x' से स्वतंत्र पद का अधिकतम मान $\frac{10!}{(5!)^2}$ है, तो 'a' बराबर है :

(1) –1 (2) 1 (3) –2 (4) 2

12. न्यूनतम पूर्णांक, जो कि $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ से बड़ा है, है :

(1) 3 (2) 4 (3) 2 (4) 1

13. माना $n \in \mathbf{N}$ तथा [x] महत्तम पूर्णांक $\le x$ है। यदि (n + 1) पदों ${}^nC_0, 3\cdot{}^nC_1, 5\cdot{}^nC_2, 7\cdot{}^nC_3, ...$ का योग $2^{100}\cdot 101$ है, तो $2\left[\frac{n-1}{2}\right]$ बराबर है ________ ।

14. यदि $\left(2+\frac{x}{3}\right)^n$ के प्रसार में x^7 तथा x^8 के गुणांक बराबर हैं, तो n बराबर है _______ ।

15. यदि $\left(x^2+\frac{1}{bx}\right)^{11}$, $b \neq 0$, में x^7 का गुणांक तथा $\left(x-\frac{1}{bx^2}\right)^{11}$, में x^{-7} का गुणांक बराबर हैं, तो b का मान बराबर है ?

(1) 2 (2) –1 (3) 1 (4) –2

16. 'x' का एक संभव मान, जिसके लिए व्यंजक $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ के $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ की बढ़ती घातों में प्रसार में नौवाँ पद 180 के बराबर है, है:

(1) 0 (2) –1 (3) 2 (4) 1

17. यदि $(1 + x)^{20}$ के प्रसार में x^r का गुणांक ${}^{20}C_r$ है, तो $\sum_{r=0}^{20} r^2\,{}^{20}C_r$ का मान बराबर है...... ।

(1) 420×2^{19} (2) 380×2^{19}

(3) 380×2^{18} (4) 420×2^{18}

18. यदि ${}^1P_1 + 2\cdot{}^2P_2 + 3\cdot{}^3P_3 + \ldots + 15\cdot{}^{15}P_{15} = {}^qP_r - s$, $0 \le s \le 1$, है, तो ${}^{q+s}C_{r-s}$ बराबर है ________ ।

19. माना $\binom{n}{k}$, ${}^nC_k$ को दर्शाता है तथा

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k}, & \text{यदि } 0 \le k \le n \\ 0, & \text{अन्यथा} \end{cases} \text{ है।}$$

यदि $A_k = \sum_{i=0}^{9} \binom{9}{i}\begin{bmatrix} 12 \\ 12-k+i \end{bmatrix} + \sum_{i=0}^{8} \binom{8}{i}\begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$

तथा $A_4 - A_3 = 190\,p$, है, तो p बराबर है

20. यदि $0 < x < 1$ है, तो $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots$ बराबर है:

(1) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(2) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

(3) $\frac{1-x}{1+x} + \log_e(1-x)$

(4) $\frac{1+x}{1-x} + \log_e(1-x)$

21. $\sum_{k=0}^{20} \left({}^{20}C_k\right)^2$ बराबर है :

(1) ${}^{40}C_{21}$ (2) ${}^{40}C_{19}$ (3) ${}^{40}C_{20}$ (4) ${}^{41}C_{20}$

22. $3 \times 7^{22} + 2 \times 10^{22} - 44$ को 18 से भाग देने पर शेषफल __________ है।

23. यदि $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$ के द्विपद प्रसार में x से स्वतंत्र पद $\left(\frac{3^6}{4^4}\right)k$ हो, तो k बराबर होगा _________

24. यदि $(a + 2b + 4ab)^{10}$ के प्रसार में a^7b^8 का गुणांक $K{:}2^{16}$ है, तो K बराबर है_____ ।

25. यदि $(x + y)^n$ के प्रसार में गुणांकों का योगफल 4096 है, तब प्रसार में महत्तम गुणांक है _______ ।

26. यदि $n \ge 2$ एक धनात्मक पूर्णांक है, तो श्रेणी ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2)$ का योग है:

(1) $\frac{n(n-1)(2n+1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$

(3) $\frac{n(2n+1)(3n+1)}{6}$ (4) $\frac{n(n+1)^2(n+2)}{12}$

27. पूर्णांकों n तथा r के लिए,

माना $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{यदि } n \ge r \ge 0 \\ 0, & \text{अन्यथा} \end{cases}$

तो k का वह अधिकतम मान, जिसके लिए, योगफल

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ का अस्तित्व है, ________ है।

28. $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots\ldots - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$ का मान है:

(1) $2^{16} - 1$ (2) $2^{13} - 14$

(3) 2^{14} (4) $2^{13} - 13$

29. यदि x को 4 से विभाजित करने पर शेषफल 3 है, तो $(2020 + x)^{2022}$ को 8 से विभाजित करने पर शेषफल है _______.

30. माना $m,n \in \mathbb{N}$ तथा $\gcd(2,n) = 1$ हैं। यदि

$30\binom{30}{0} + 29\binom{30}{1} + \ldots + 2\binom{30}{28} + 1\binom{30}{29} = n.2^m$ है,

तो $n + m$ बराबर है _____ । (यहाँ) $\binom{n}{k} = {}^nC_k$ है।

31. $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$, जहाँ $x \in (0,1)$ है, के प्रसार में, 't' से स्वतंत्र पद का अधिकतम मान है :

(1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(3) $\frac{2.10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

32. माना n एक धनात्मक पूर्णांक है तथा

$$A=\sum_{k=0}^{n}(-1)^k\,{}^nC_k\left[\left(\frac{1}{2}\right)^k+\left(\frac{3}{4}\right)^k+\left(\frac{7}{8}\right)^k+\left(\frac{15}{16}\right)^k+\left(\frac{31}{32}\right)^k\right]$$

है। यदि $63A=1-\frac{1}{2^{30}}$ है, तो n बराबर है ______ ।

33. यदि $(3^{1/4}+5^{1/8})^{60}$ के प्रसार में अपरिमेय पदों की संख्या n है, (n – 1) निम्न में से किस से विभाज्य है ?

(1) 26 (2) 30 (3) 8 (4) 7

34. मान [x] महत्तम पूर्णांक $\le x$ है। यदि $n\in\mathbb{N}$ के लिए

$(1-x+x^3)^n=\sum_{j=0}^{3n}a_j x^j$ है, तो $\sum_{j=0}^{\left[\frac{3n}{2}\right]}a_{2j}+4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]}a_{2j}+1$

बराबर है :

(1) 2 (2) 2^{n-1} (3) 1 (4) n

35. $\sum_{r=0}^{6}\left({}^6C_r\cdot{}^6C_{6-r}\right)$ का मान बराबर है :

(1) 1124 (2) 1324 (3) 1024 (4) 924

36. माना $\left(x+\frac{a}{x^2}\right)^n$, $x \neq 0$, के प्रसार में तीसरे, चौथे तथा पाँचवें पदों के गुणांक 12 : 8 : 3 के अनुपात में है। तो इस प्रसार में x से स्वतंत्र पद है_______ ।

37. दो पासे फेंकें जाते हैं। यदि दोनों पासों के छः तलों (faces) पर अंकित संख्याएँ 1, 2, 3, 5, 7 तथा 11 हैं तो ऊपर के तलों पर प्रकट होने वाली संख्याओं का योगफल ≤ 8 होने की प्रायिकता है :

(1) $\frac{4}{9}$ (2) $\frac{17}{36}$ (3) $\frac{5}{12}$ (4) $\frac{1}{2}$

38. यदि $(x+x^{\log_2 x})^7$ के प्रसार में चौथा पद 4480 है, तो x ($x \in N$) का मान है :

(1) 2 (2) 4 (3) 3 (4) 1

39. $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+......\infty}}}}$ का मान है :

(1) $2+\frac{2}{5}\sqrt{30}$ (2) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(3) $4+\frac{4}{\sqrt{5}}\sqrt{30}$ (4) $5+\frac{2}{5}\sqrt{30}$

40. $(2021)^{3762}$ को 17 से विभाजित करने पर शेषफल है _______ ।

41. माना $(1+x+2x^2)^{20}=a_0+a_1x+a_2x^2+...+a_{40}x^{40}$ है। तो $a_1+a_3+a_5+...+a_{37}$ बराबर है :

(1) $2^{20}(2^{20}-21)$ (2) $2^{19}(2^{20}-21)$

(3) $2^{19}(2^{20}+21)$ (4) $2^{20}(2^{20}+21)$

42. यदि $\sum_{r=1}^{10}r!\left(r^3+6r^2+2r+5\right)=\alpha(11!)$ है, तो α का मान बराबर है_______ ।

43. माना घात 3 का एक वास्तविक बहुपद P(x) है, जो x = –3 पर शून्य हो जाता है। माना P(x) का स्थानीय निम्नतम x = 1 पर, स्थानीय अधिकतम x = –1 पर तथा $\int_{-1}^{1}P(x)dx=18$ हैं। तो बहुपद P(x) के सभी गुणांकों का योगफल बराबर है_______.

44. माना $(1+x)^n$ के प्रसार में x^r का द्विपद गुणांक ${}^nC_r$ है। यदि $\sum_{k=0}^{10}(2^2+3k)\,{}^nC_k=\alpha.3^{10}+\beta.2^{10}$, $\alpha, \beta \in R$ हैं, $\alpha+\beta$ बराबर है______ ।

SOLUTION

1. Official Ans. by NTA (2)

Sol. $(1-x)^{100} \cdot (x^2+x+1)^{100} \cdot (1-x)$

$=\left((1-x)(x^2+x+1)\right)^{100}(1-x)$

$=(1^3-x^3)^{100}(1-x)$

$=(1-x^3)^{100}(1-x)$

$=\underbrace{(1-x^3)^{100}}_{\text{No term of } x^{256}}-\underbrace{x(1-x^3)^{100}}_{\text{We find cofficient of } x^{255}}$

Required coefficient $(-1)\times(-{}^{100}C_{85})$

$={}^{100}C_{85}={}^{100}C_{15}$

2. Official Ans. by NTA (21)

Sol. $\left(4^{1/4}+5^{1/6}\right)^{120}$

$T_{r+1}={}^{120}C_r(2^{1/2})^{120-r}(5)^{r/6}$

for rational terms $r=6\lambda \quad 0\le r\le 120$

so total no of forms are 21.

3. Official Ans. by NTA (4)

Sol. $(1-y)^m(1+y)^n$

Coefficient of y $(a_1) = 1.{}^nC_1 + {}^mC_1(-1)$

$= n - m = 10$ .....(1)

Coefficient of y^2 (a_2)

$= 1.{}^nC_2 - {}^mC_1.{}^nC_1. + 1.{}^mC_2 = 10$

$=\frac{n(n-1)}{2}-m.n+\frac{m(m-1)}{2}=10$

$m^2+n^2-2mn-(n+m)=20$

$(n-m)^2-(n+m)=20$

$n+m=80$ ..... (2)

By equation (1) & (2)

$m = 35, n = 45$

4. Official Ans. by NTA (9)

Sol. $\frac{1}{\alpha(\alpha+1).....(\alpha+20)}=\sum_{k=0}^{20}\frac{A_k}{\alpha+k}$

$A_{14}=\frac{1}{(-14)(-13).....(-1)(1).....(6)}=\frac{1}{14!\cdot 6!}$

$A_{15}=\frac{1}{(-15)(-14).....(-1)(1).....(5)}=\frac{1}{15!\cdot 5!}$

$A_{13}=\frac{1}{(-13).....(-1)(1).....(7)}=\frac{-1}{13!\cdot 7!}$

$\frac{A_{14}}{A_{13}}=\frac{1}{14!\cdot 6!}\times -13!\times 7!=\frac{-7}{14}=-\frac{1}{2}$

$\frac{A_{15}}{A_{13}}=-\frac{1}{15!\times 5!}\times -13!\times 7!=\frac{42}{15\times 14}=\frac{1}{5}$

$100\left(\frac{A_{14}}{A_{13}}+\frac{A_{15}}{A_{13}}\right)^2=100\left(-\frac{1}{2}+\frac{1}{5}\right)^2=9$

5. Official Ans. by NTA (8)

Sol. $\left(2x^r+\frac{1}{x^2}\right)^{10}$

General term $= {}^{10}C_R(2x^2)^{10-R}x^{-2R}$

$\Rightarrow 2^{10-R}\,{}^{10}C_R=180$ .... (1)

& $(10-R)r-2R=0$

$r=\frac{2R}{10-R}$

$r=\frac{2(R-10)}{10-R}+\frac{20}{10-R}$

$\Rightarrow r=-2+\frac{20}{10-R}$ .... (2)

R = 8 or 5 reject equation (1) not satisfied

At R = 8

$2^{10-R}\,{}^{10}C_R=180\Rightarrow \boxed{r=8}$

6. Official Ans. by NTA (96)

Sol. $11^n > 10^n + 9^n$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10 + 1)^n - (10 - 1)^n > 10^n$

$\Rightarrow \{ {}^nC_1 \cdot 10^{n-1} + {}^nC_3 10^{n-0} + {}^nC_5 10^{n-5} + \cdots\} > 10^n$

$\Rightarrow 2n \cdot 10^{n-1} + 2\{ {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\} > 10^n$ .... (1)

For n = 5

$10^5 + 2\{ {}^5C_3 10^2 + {}^5C_5\} > 10^5$ (True)

For n = 6, 7, 8, ….. 100

$2n10^{n-1} > 10^n$

$\Rightarrow 2n10^{n-1} + 2\ \{ {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\} > 10^n$

$\Rightarrow 11^n - 9^n > 10^n$ For n = 5, 6, 7, ….. 100

For n = 4, Inequality (1) is not satisfied

$\Rightarrow$ Inequality does not hold good for

N = 1, 2, 3, 4

So, required number of elements

= 96

7. Official Ans. by NTA (3)

Sol. $(a - b)^{-1} + (a - 2b)^{-1} + \ldots + (a - nb)^{-1}$

$$= \frac{1}{a}\sum_{r=1}^{n}\left(1 - \frac{rb}{a}\right)^{-1}$$

$$= \frac{1}{a}\sum_{r=1}^{n}\left\{\left(1 + \frac{rb}{a} + \frac{r^2b^2}{a^2}\right) + (\text{terms to be neglected})\right\}$$

$$= \frac{1}{a}\left[n + \frac{n(n+1)}{2}\cdot\frac{b}{a} + \frac{n(n+1)(2n+1)}{6}\cdot\frac{b^2}{a^2}\right]$$

$$= \frac{1}{a}\left[n^3\left(\frac{b^2}{3a^2}\right) + \ldots\right]$$

So $\gamma = \dfrac{b^2}{3a^3}$

8. Official Ans. by NTA (1)

Sol. Coeff. of middle term in $(1 + x)^{20} = {}^{20}C_{10}$

& Sum of Coeff. of two middle terms in

$(1 + x)^{19} = {}^{19}C_9 + {}^{19}C_{10}$

So required ratio = $\dfrac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \dfrac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

9. Official Ans. by NTA (210)

Sol. $\left(\left(x^{1/3} + 1\right) - \left(\dfrac{x^{1/2} + 1}{x^{1/2}}\right)\right)^{10}$

$$= \left(x^{1/3} - \frac{1}{x^{1/2}}\right)^{10}$$

Now General Term

$T_{r+1} = {}^{10}C_r\left(x^{1/3}\right)^{10-r}.\left(-\dfrac{1}{x^{1/2}}\right)^r$

For independent term

$\dfrac{10-r}{3} - \dfrac{r}{2} = 0 \Rightarrow r = 4$

$\Rightarrow T_5 = {}^{10}C_4 = 210$

10. Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^{12}C_r\left(2^{1/3}\right)^r.\left(3^{1/4}\right)^{12-r}$

T_{r+1} will be rational number

when r = 0, 3, 6, 9, 12

& r = 0, 4, 8, 12

$\Rightarrow$ r = 0, 12

$T_1 + T_{13} = 1 \times 3^3 + 1 \times 2^4 \times 1$

= 24 + 16 = 43

11. Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^{10}C_r(x\sin\alpha)^{10-r}\left(\dfrac{a\cos\alpha}{x}\right)^r$

r = 0, 1, 2, …, 10

T_{r+1} will be independent of x

when $10 - 2r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5(x\sin\alpha)^5 \times \left(\dfrac{a\cos\alpha}{x}\right)^5$

$= {}^{10}C_5 \times a^5 \times \dfrac{1}{2^5}(\sin 2\alpha)^5$

will be greatest when $\sin2\alpha = 1$

$\Rightarrow\ {}^{10}C_5\dfrac{a^5}{2^5} = {}^{10}C_5 \Rightarrow a = 2$

12. Official Ans. by NTA (1)

Sol. Let $P = \left(1+\frac{1}{10^{100}}\right)^{10^{100}}$,

Let $x = 10^{100}$

$\Rightarrow P = \left(1+\frac{1}{x}\right)^{x}$

$\Rightarrow P = 1+(x)\left(\frac{1}{x}\right)+\frac{(x)(x-1)}{\underline{|2}}\cdot\frac{1}{x^2}$

$+\frac{(x)(x-1)(x-2)}{\underline{|3}}\cdot\frac{1}{x^3}+\ldots$

(upto $10^{100}+1$ terms)

$\Rightarrow P = 1+1+\left(\frac{1}{\underline{|2}}-\frac{1}{\underline{|2}x^2}\right)+\left(\frac{1}{\underline{|3}}-\ldots\right)+\ldots$ so on

$\Rightarrow P = 2+\left(\text{Positive value less then } \frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}+\frac{1}{\underline{|4}}+\ldots\right)$

Also $e = 1+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}+\frac{1}{\underline{|4}}+\ldots$

$\Rightarrow \frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}+\frac{1}{\underline{|4}}+\ldots = e-2$

$\Rightarrow$ P = 2 + (positive value less then e – 2)

$\Rightarrow P \in (2, 3)$

$\Rightarrow$ least integer value of P is 3

13. Official Ans. by NTA (98)

Sol. $1.{}^nC_0 + 3.{}^nC_1 + 5.{}^nC_2 + \ldots + (2n+1).{}^nC_n$

$T_r = (2r+1){}^nC_r$

$S = \Sigma T_r$

$S = \Sigma(2r+1){}^nC_r = \Sigma 2r{}^nC_r + \Sigma{}^nC_r$

$S = 2(n.2^{n-1}) + 2^n = 2^n(n+1)$

$2^n(n+1) = 2^{100}.101 \Rightarrow n = 100$

$2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 98$

14. Official Ans. by NTA (55)

Sol. ${}^nC_7 2^{n-7}\frac{1}{3^7} = {}^nC_8 2^{n-8}\frac{1}{3^8}$

$\Rightarrow n-7 = 48 \Rightarrow n = 55$

15. Official Ans. by NTA (3)

Sol. Coefficient of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$

${}^{11}C_r (x^2)^{11-r}.\left(\frac{1}{bx}\right)^r$

${}^{11}C_r x^{22-3r}.\frac{1}{b^r}$

$22-3r = 7$

$r = 5$

$\therefore\ {}^{11}C_5.\frac{1}{b^5}.x^7$

Coefficient of x^{-7} in $\left(x-\frac{b}{bx^2}\right)^{11}$

${}^{11}C_r (x)^{11-r}.\left(-\frac{1}{bx^2}\right)^r$

${}^{11}C_r x^{11-3r}.\frac{(-1)^r}{b^r}$

$11-3r = -7 \ \therefore r = 6$

${}^{11}C_6.\frac{1}{b^6}x^{-7}$

${}^{11}C_5.\frac{1}{b^5} = {}^{11}C_6.\frac{1}{b^6}$

Since $b \neq 0 \ \therefore b = 1$

16. Official Ans. by NTA (4)

Sol. ${}^{10}C_8\left(25^{(x-1)}+7\right)\times\left(5^{(x-1)}+1\right)^{-1} = 180$

$\Rightarrow \frac{25^{x-1}+7}{5^{(x-1)}+1} = 4$

$\Rightarrow \frac{t^2+7}{t+1} = 4;$

$\Rightarrow t = 1, 3 = 5^{x-1}$

$\Rightarrow$ x – 1 = 0 (one of the possible value).

$\Rightarrow x = 1$

17. Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{20} r^2.{}^{20}C_r$

$\sum\left(4(r-1)+r\right).{}^{20}C_r$

$\sum r(r-1).\frac{20\times 19}{r(r-1)}.{}^{18}C_r + r.\frac{20}{r}.\sum {}^{19}C_{r-1}$

$\Rightarrow 20 \times 19.2^{18} + 20.2^{19}$

$\Rightarrow 420 \times 2^{18}$

18. Official Ans. by NTA (136)

Sol. ${}^1P_1 + 2\cdot{}^2P_2 + 3\cdot{}^3P_3 + \ldots + 15\cdot{}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + \ldots.15 \times 15!$

$= \sum_{r=1}^{15}(r+1-1)r!$

$= \sum_{r=1}^{15}(r+1)! - (r)!$

$= 16! - 1$

$= {}^{16}P_{16} - 1$

$\Rightarrow q = r = 16, s = 1$

${}^{q+s}C_{r-s} = {}^{17}C_{15} = 136$

19. Official Ans. by NTA (49)

Sol. $A_k = \sum_{i=0}^{9} {}^9C_i\ {}^{12}C_{k-i} + \sum_{i=0}^{8} {}^8C_i\ {}^{13}C_{k-i}$

$A_k = {}^{21}C_k + {}^{21}C_k = 2\cdot{}^{21}C_k$

$A_4 - A_3 = 2\left({}^{21}C_4 - {}^{21}C_3\right) = 2(5985 - 1330)$

$190\,p = 2(5985 - 1330) \Rightarrow p = 49.$

20. Official Ans. by NTA (1)

Sol. Let $t = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots.\infty$

$= \left(2-\frac{1}{2}\right)x^2 + \left(2-\frac{1}{3}\right)x^3 + \left(2-\frac{1}{4}\right)x^4 + \ldots.\infty$

$= 2(x^2 + x^3 + x^4 + \ldots\infty) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\infty\right)$

$= \frac{2x^2}{1-x} - \left(\ell n(1-x) - x\right)$

$\Rightarrow t = \frac{2x^2}{1-x} + x - \ell n(1-x)$

$\Rightarrow t = \frac{x(1+x)}{1-x} - \ell n(1-x)$

21. Official Ans. by NTA (3)

Sol. $\sum_{k=0}^{20} {}^{20}C_k.{}^{20}C_{20-k}$

sum of suffix is const. so summation will be ${}^{40}C_{20}$

22. Official Ans. by NTA (15)

Sol. $3(1 + 6)^{22} + 2\cdot(1 + 9)^{22} - 44 = (3 + 2 - 44) = 18\,.I$

$= -39 + 18.I$

$= (54 - 39) + 18(I - 3)$

$= 15 + 18\,I_1$

$\Rightarrow$ Remainder = 15.

23. Official Ans. by NTA (55)

Sol. $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$

$T_{r+1} = (-1)^r.{}^{12}C_r\left(\frac{x}{4}\right)^{12-r}\left(\frac{12}{x^2}\right)^r$

$T_{r+1} = (-1)^r.{}^{12}C_r\left(\frac{1}{4}\right)^{12-r}(12)^r.(x)^{12-3r}$

Term independent of x $\Rightarrow 12 - 3r = 0 \Rightarrow r = 4$

$T_5 = (-1)^4.{}^{12}C_4\left(\frac{1}{4}\right)^8(12)^4 = \frac{3^6}{4^4}.k$

$\Rightarrow k = 55$

24. Official Ans. by NTA (315)

Sol. $\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha}(2b)^{\beta}.(4ab)^{\gamma}$

$\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha+\gamma}.b^{\beta+\gamma}.2^{\beta}.4^{\gamma}$

$\alpha+\beta+\gamma=10$ ...(1)

$\alpha+\gamma=7$ ...(2)

$\beta+\gamma=8$ ...(3)

$(2)+(3)-(1)\Rightarrow\gamma=5$

$\alpha=2$

$\beta=3$

so coefficients $=\frac{10!}{2!3!5!}2^3.2^{10}$

$=\frac{10\times9\times8\times7\times6\times5}{2\times3\times2\times5!}\times2^{13}$

$=315\times2^{16}\Rightarrow k=315$

25. Official Ans. by NTA (924)

Sol. $(x+y)^n\Rightarrow2^n=4096$ $\quad 2^{10}=1024\times2$

$\Rightarrow2^n=2^{12}$ $\quad 2^{11}=2048$

$n=12$ $\quad 2^{12}=\underline{4096}$

${}^{12}C_6=\frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}$

$=11\times3\times4\times7$

$=924$

26. Official Ans. by NTA (2)

Sol. ${}^{n+1}C_2+2\left({}^2C_2+{}^3C_2+{}^4C_2+........+{}^nC_2\right)$

${}^{n+1}C_2+2\left({}^3C_3+{}^3C_2+{}^4C_2+........+{}^nC_2\right)$

$\left\{\text{use } {}^nC_{r+1}+{}^nC_r={}^{n+1}C_r\right\}$

$={}^{n+1}C_2+2\left({}^4C_3+{}^4C_2+{}^5C_3+.......+{}^nC_2\right)$

$={}^{n+1}C_2+2\left({}^5C_3+{}^5C_2+.......+{}^nC_2\right)$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$={}^{n+1}C_2+2\left({}^nC_3+{}^nC_2\right)$

$={}^{n+1}C_2+2.{}^{n+1}C_3$

$=\frac{(n+1)n}{2}+2.\frac{(n+1)(n)(n-1)}{2.3}$

$=\frac{n(n+1)(2n+1)}{6}$

27. Official Ans. by NTA (BONUS)

Sol. **Bonus**

$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$

${}^{25}C_k+{}^{25}C_{k+1}$

${}^{26}C_{k+1}$

as ${}^nC_r$ is defined for all values of n as will as r

so ${}^{26}C_{k+1}$ always exists

Now k is unbounded so maximum value is not defined.

28. Official Ans. by NTA (2)

Sol. $(-{}^{15}C_1+2.{}^{15}C_2-3.{}^{15}C_3+......-15.{}^{15}C_{15})$

$+({}^{14}C_1+{}^{14}C_3+...+{}^{14}C_{11})$

$=\sum_{r=1}^{15}(-1)^r.r\,{}^{15}C_r+\left({}^{14}C_1+{}^{14}C_3+...+{}^{14}C_{11}+{}^{14}C_{13}\right)-{}^{14}C_3$

$=\sum_{r=1}^{15}(-1)^r15.{}^{14}C_{r-1}+2^{13}-14$

$=15(-{}^{14}C_0+{}^{14}C_1........-{}^{14}C_{14})+2^{13}-14$

29. Official Ans. by NTA (1)

Sol. $x = 4k + 3$

$\therefore (2020 + x)^{2022} = (2020 + 4k + 3)^{2022}$

$= (4(505 + k) + 3)^{2022}$

$= (4\lambda + 3)^{2022} = (16\lambda^2 + 24\lambda + 9)^{1011}$

$= (8(2\lambda^2 + 3\lambda + 1) + 1)^{1011}$

$= (8p + 1)^{1011}$

$\therefore$ Remainder when divided by 8 = 1

$= 2^{13} - 14$

30. Official Ans. by NTA (45)

Sol. $30({}^{30}C_0) + 29({}^{30}C_1) + ... + 2({}^{30}C_{28}) + 1({}^{30}C_{29})$

$= 30({}^{30}C_{30}) + 29({}^{30}C_{29}) + ... + 2({}^{30}C_2) + 1({}^{30}C_1)$

$$= \sum_{r=1}^{30} r\left({}^{30}C_r\right)$$

$$= \sum_{r=1}^{30} r\left(\frac{30}{r}\right)\left({}^{29}C_{r-1}\right)$$

$$= 30\sum_{r=1}^{30} {}^{29}C_{r-1}$$

$$= 30\left({}^{29}C_0 + {}^{29}C_1 + {}^{29}C_2 + ... + {}^{29}C_{29}\right)$$

$$= 30\left(2^{29}\right) = 15(2)^{30} = n(2)^m$$

$\therefore n = 15, m = 30$

31. Official Ans. by NTA (2)

Sol. Term independent of t will be the middle term due to exect same magnitude but opposite sign powers of t in the binomial expression given

$$\text{so } T_6 = {}^{10}C_5\left(tx^2 5\right)^5\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^5$$

$T_6 = f(x) = {}^{10}C_5\left(x\sqrt{1-x}\right)$; for maximum

$$f'(x) = 0 \Rightarrow x = \frac{2}{3} \ \& \ f''\left(\frac{2}{3}\right) < 0$$

$$\text{so } f(x)_{max.} = {}^{10}C_5\left(\frac{2}{3}\right).\frac{1}{\sqrt{3}}$$

32. Official Ans by NTA (6)

Sol. $$A = \sum_{k=0}^{n} {}^nC_k\left[\left(-\frac{1}{2}\right)^k + \left(\frac{-3}{4}\right)^k + \left(\frac{-7}{8}\right)^k + \left(\frac{-15}{16}\right)^k + \left(\frac{-37}{32}\right)^k\right]$$

$$A = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \left(1-\frac{15}{16}\right)^n + \left(1-\frac{31}{32}\right)^n$$

$$A = \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \frac{1}{16^n} + \frac{1}{32^n}$$

$$A = \frac{1}{2^n}\left(\frac{1-\left(\frac{1}{2^n}\right)^5}{1-\frac{1}{2^n}}\right) \Rightarrow A = \frac{\left(1-\frac{1}{2^{5n}}\right)}{(2^n-1)}$$

$(2^n - 1)A = 1 - \frac{1}{2^{5n}}$, Given $63A = 1 - \frac{1}{2^{30}}$

Clearly $5n = 30$

$n = 6$

33. Official Ans. by NTA (1)

Sol. $(3^{1/4} + 5^{1/8})^{60}$

${}^{60}C_r(3^{1/4})^{60-r}.(5^{1/8})^r$

$${}^{60}C_r(3)^{\frac{60-r}{4}}.5^{\frac{r}{8}}$$

For rational terms.

$\frac{r}{8} = k$; $0 \le r \le 60$

$0 \le 8k \le 60$

$0 \le k \le \frac{60}{8}$

$0 \le k \le 7.5$

$k = 0, 1, 2, 3, 4, 5, 6, 7$

$\frac{60-8k}{4}$ is always divisible by 4 for all value of k.

Total rational terms = 8

Total terms = 61

irrational terms = 53

$n - 1 = 53 - 1 = 52$

52 is divisible by 26.

34. Official Ans. by NTA (3)

Sol. $(1-x+x^3)^n=\sum_{j=0}^{3n}a_j x^j$

$(1-x+x^3)^n = a_0 + a_1x + a_2x^2 \ldots\ldots\ldots + a_{3n}x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}$ = Sum of $a_0 + a_2 + a_4 \ldots\ldots\ldots$

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j}+1$ = Sum of $a_1 + a_3 + a_5 \ldots\ldots.$

put x = 1

$1 = a_0 + a_1 + a_2 + a_3 \ldots\ldots\ldots + a_{3n}$ ....(A)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 \ldots\ldots\ldots + (-1)^{3n}a_{3n}$ ....(B)

Solving (A) and (B)

$a_0 + a_2 + a_4 \ldots\ldots = 1$

$a_1 + a_3 + a_5 \ldots\ldots. = 0$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$

35. Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r}$

$= {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + \ldots\ldots + {}^6C_6 \cdot {}^6C_0$

Now,

$(1+x)^6(1+x)^6$

$=\left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \ldots\ldots + {}^6C_6x^6\right)$

$\left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \ldots\ldots + {}^6C_6x^6\right)$

Comparing coefficeint of x^6 both sides

${}^6C_0 \cdot {}^6C_6 + {}^6C_1 + {}^6C_5 + \ldots\ldots. + {}^6C_6 \cdot {}^6C_0 = {}^{12}C_6$

$= 924$

Ans.(4)

36. Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^nC_r(x)^{n-r}\left(\frac{a}{x^2}\right)^r$

$= {}^nC_r\, a^r x^{n-3r}$

${}^nC_2\, a^2 : {}^nC_3\, a^3 : {}^nC_4\, a^4 = 12 : 8 : 3$

After solving

$n = 6,\ a = \frac{1}{2}$

For term independent of 'x' $\Rightarrow n = 3r$

$r = 2$

$\therefore$ Coefficient is ${}^6C_2\left(\frac{1}{2}\right)^2 = \frac{15}{4}$

Nearest integer is 4.

37. Official Ans. by NTA (2)

Sol. n(E) = 5 + 4 + 4 + 3 + 1 = 17

So, P (E) = $\frac{17}{36}$

38. Official Ans. by NTA (1)

Sol. ${}^7C_3x^4\ \ x^{(3\log_2^x)} = 4480$

$\Rightarrow\ x^{(4+3\log_2^x)} = 2^7$

$\Rightarrow\ (4+3t)t = 7; t = \log_2^x$

$\Rightarrow\ t = 1, \frac{-7}{3} \Rightarrow x = 2$

39. Official Ans. by NTA (1)

Sol. $y = 4 + \frac{1}{\left(5+\frac{1}{y}\right)}$

$y - 4 = \frac{y}{(5y+1)}$

$5y^2 - 20y - 4 = 0$

$y = \frac{20+\sqrt{480}}{10}$

$y = \frac{20-\sqrt{480}}{10} \rightarrow$ rejected

$y = 2 + \sqrt{\frac{480}{100}}$

Correct with Option (A)

40. Official Ans. by NTA (4)

Sol. $(2023-2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$ (as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17-1)^{940}$

$= 17k_2 + 4\,(17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder $= 4$

41. Official Ans. by NTA (2)

Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1x + ... + a_{40}x^{40}$

put $x = 1, -1$

$\Rightarrow a_0 + a_1 + a_2 + ... + a_{40} = 2^{20}$

$a_0 - a_1 + a_2 + ... + a_{40} = 2^{20}$

$\Rightarrow a_1 + a_3 + ... + a_{39} = \dfrac{4^{20} - 2^{20}}{2}$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$

here $a_{39} = \dfrac{20!(2)^{19} \times 1}{19!} = 20 \times 2^{19}$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{19}(2^{20} - 1 - 20)$

$= 2^{19}(2^{20} - 21)$

42. Official Ans. by NTA (160)

Sol. $\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3) - 9(r+1) + 8\}$

$= \sum_{r=1}^{10}\left[\{(r+3)! - (r+1)!\} - 8\{(r+1)! - r!\}\right]$

$= (13! + 12! - 2! - 3!) - 8(11! - 1)$

$= (12.13 + 12 - 8).11! - 8 + 8$

$= (160)(11)!$

Hence $\alpha = 160$

43. Official Ans. by NTA (8)

Sol. Let $p'(x) = a(x-1)(x+1) = a(x^2 - 1)$

$p(x) = a\int (x^2 - 1)\,dx + c$

$= a\left(\dfrac{x^3}{3} - x\right) + c$

Now $p(-3) = 0$

$\Rightarrow a\left(-\dfrac{27}{3} + 3\right) + c = 0$

$\Rightarrow -6a + c = 0$ ...(1)

Now $\int_{-1}^{1}\left(a\left(\dfrac{x^3}{3} - x\right) + c\right)dx = 18$

$= 2c = 18 \Rightarrow c = 9$ ...(2)

$\Rightarrow$ from (1) & (2) $\Rightarrow -6a + 9 = 0 \Rightarrow a = \dfrac{3}{2}$

$\Rightarrow p(x) = \dfrac{3}{2}\left(\dfrac{x^3}{3} - x\right) + 9$

sum of coefficient

$= \dfrac{1}{2} - \dfrac{3}{2} + 9$

$= 8$

44. Official Ans. by NTA (19)

Allen Answer (Bonus)

Sol. BONUS

Instead of ${}^nC_k$ it must be ${}^{10}C_k$ i.e.

$\sum_{k=0}^{10}(2^2 + 3k)\,{}^{10}C_k = \alpha.3^{10} + \beta.2^{10}$

LHS $= 4\sum_{k=0}^{10}{}^{10}C_k + 3\sum_{k=0}^{10}k.\dfrac{10}{k}.{}^{9}C_{k-1}$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + \beta.2^{10}$

$\Rightarrow \alpha = 0, \beta = 19 \Rightarrow \alpha + \beta = 19$