VECTORS

1. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|, |\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ is :

(1) $\frac{2}{3}$ (2) 4 (3) 3 (4) $\frac{3}{2}$

2. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to _________.

3. In a triangle ABC, if $|\overrightarrow{BC}|=3$, $|\overrightarrow{CA}|=5$ and $|\overrightarrow{BA}|=7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to

(1) $\frac{19}{2}$ (2) $\frac{13}{2}$ (3) $\frac{11}{2}$ (4) $\frac{15}{2}$

4. For p > 0, a vector $\vec{v}_2=2\hat{i}+(p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1=\sqrt{3}p\hat{i}+\hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta=\frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to _________.

5. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$, and $|\vec{a}|=\sqrt{10}$. Then a possible value of $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{b}\ \vec{d}]+[\vec{a}\ \vec{c}\ \vec{d}]$ is equal to :

(1) – 42 (2) – 40 (3) – 29 (4) – 38

6. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following is **not** true ?

(1) $\vec{a}\times\left((\vec{b}+\vec{c})\times(\vec{b}-\vec{c})\right)=\vec{0}$

(2) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2

(3) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=8$

(4) $|3\vec{a}+\vec{b}-2\vec{c}|^2=51$

7. Let the vectors

$(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k}$,

$(1+b)\hat{i}+2b\hat{j}-b\hat{k}$ and $(2+b)\hat{i}+2b\hat{j}+(1-b)\hat{k}$ a, b, c, ∈ **R**

be co-planar. Then which of the following is true?

(1) 2b = a + c (2) 3c = a + b

(3) a = b + 2c (4) 2a = b + c

8. Let $\vec{p}=2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}+2\hat{j}+\hat{k}$ be two vectors. If a vector $\vec{r}=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$, and $|\vec{r}|=\sqrt{3}$, then $|\alpha|+|\beta|+|\gamma|$ is equal to _______.

9. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co-planar, then c is equal to:

(1) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ (2) $\frac{a+b}{2}$

(3) $\frac{1}{a}+\frac{1}{b}$ (4) $\sqrt{ab}$

10. If $|\vec{a}|=2, |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$, then $|\vec{a}\cdot\vec{b}|$ is equal to :

(1) 6 (2) 4 (3) 3 (4) 5

11. If $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is ________.

12. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product $(\vec{a}+\vec{b})\times\left(\left(\vec{a}\times\left((\vec{a}-\vec{b})\times\vec{b}\right)\right)\times\vec{b}\right)$ is equal to :

(1) $5(34\hat{i}-5\hat{j}+3\hat{k})$ (2) $7(34\hat{i}-5\hat{j}+3\hat{k})$

(3) $7(30\hat{i}-5\hat{j}+7\hat{k})$ (4) $5(30\hat{i}-5\hat{j}+7\hat{k})$

13. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}$ and $\vec{c}=\hat{j}-\hat{k}$ be three vectors such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}\cdot\vec{b}=1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a}\times\vec{c}$ is l, then the value of $3l^2$ is equal to _______.

14. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}=\vec{b}\times(\vec{b}\times\vec{c})$. If magnitudes of the vectors $\vec{a},\vec{b}$ and $\vec{c}$ are $\sqrt{2},1$ and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta \left(0<\theta<\frac{\pi}{2}\right)$, then the value of 1+ tan θ is equal to :

(1) $\sqrt{3}+1$ (2) 2

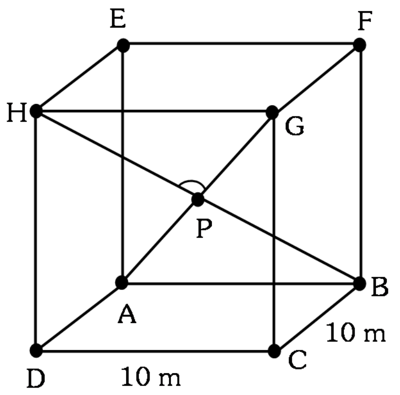
(3) 1 (4) $\frac{\sqrt{3}+1}{\sqrt{3}}$

15. Let $\vec{a}=\hat{i}-\alpha\hat{j}+\beta\hat{k}$, $\vec{b}=3\hat{i}+\beta\hat{j}-\alpha\hat{k}$ and $\vec{c}=-\alpha\hat{i}-2\hat{j}+\hat{k}$, where α and β are integers. If $\vec{a}\cdot\vec{b}=-1$ and $\vec{b}\cdot\vec{c}=10$, then $(\vec{a}\times\vec{b})\cdot\vec{c}$ is equal to____ .

16. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$, then $\vec{a}.(\vec{b}\times\vec{c})$ is equal to:

(1) –2 (2) –6 (3) 6 (4) 2

17. A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :



(1) 5 (2) $2\sqrt{10}$ (3) $5\sqrt{3}$ (4) $5\sqrt{2}$

18. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to ______.

19. Let $\vec{a}=\hat{i}+5\hat{j}+\alpha\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _______.

20. The equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to the x-axis is :

(1) $\vec{r}.(\hat{j}-3\hat{k})+6=0$ (2) $\vec{r}.(\hat{i}+3\hat{k})+6=0$

(3) $\vec{r}.(\hat{i}-3\hat{k})+6=0$ (4) $\vec{r}.(\hat{j}-3\hat{k})-6=0$

21. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}|=|3\vec{a}+\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 4 (2) 6 (3) 5 (4) 8

22. Let $\vec{a},\vec{b},\vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude.
If a vector $\vec{r}$ satisfies. $\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$, then $\vec{r}$ is equal to :

(1) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$ (2) $\frac{1}{3}(2\vec{a}+\vec{b}-\vec{c})$

(3) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$ (4) $\frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$

23. Let $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i}+2\hat{j}-\hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to_____.

24. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is :

(1) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$

(2) $\vec{r}.(3\hat{i}+7\hat{j}+3\hat{k})=7$

(3) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=7$

(4) $\vec{r}.(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

25. Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c}=7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is ______.

26. Let $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}\cdot\vec{b}=0$, then $\vec{r}\cdot\vec{a}$ is equal to _________.

27. Let $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to ______:

28. If vectors $\vec{a}_1=x\hat{i}-\hat{j}+\hat{k}$ and $\vec{a}_2=\hat{i}+y\hat{j}+z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i}+y\hat{j}+z\hat{k}$ is

(1) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ (2) $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$

(3) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$ (4) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

29. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) $\vec{0}$ (2) $\frac{1}{2}\left|\vec{a}\right|^4\vec{b}$

(3) $\vec{a}\times\vec{b}$ (4) $\left|\vec{a}\right|^4\vec{b}$

30. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}.\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=-1$, $\alpha\in$ R, then the value of $\alpha+\left|\vec{r}\right|^2$ is equal to :

(1) 9 (2) 15 (3) 13 (4) 11

31. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

If $\vec{c}.\left(\hat{i}+\hat{j}+3\hat{k}\right)=8$ then the value of $\vec{c}.\left(\vec{a}\times\vec{b}\right)$ is equal to _________.

32. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), (0, β) and (0, 0) is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

33. Let O be the origin. Let $\overrightarrow{OP}=x\hat{i}+y\hat{j}-\hat{k}$ and $\overrightarrow{OQ}=-\hat{i}+2\hat{j}+3x\hat{k}$, x, y $\in$ R, x > 0, be such that $\left|\overrightarrow{PQ}\right|=\sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR}=3\hat{i}+z\hat{j}-7\hat{k}$, z $\in$ R, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2+y^2+z^2$ is equal to

(1) 7 (2) 9 (3) 2 (4) 1

34. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i}+2\hat{j}-\hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to ________.

35. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=7\hat{i}+\hat{j}-6\hat{k}$.

If $\vec{r}\times\vec{a}=\vec{r}\times\vec{b}, \vec{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right)=-3$, then $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ is equal to :

(1) 12 (2) 8 (3) 13 (4) 10

36. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

$\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

$\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

such that $\vec{a}\cdot\vec{b}=1$ and $\vec{b}\cdot\vec{c}=-3$, then

$\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to _______.

37. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to:

(1) 1 (2) $-\frac{5}{4}$ (3) $\frac{4}{5}$ (4) –1

38. In a triangle ABC, if $\left|\overrightarrow{BC}\right|=8$, $\left|\overrightarrow{CA}\right|=7$, $\left|\overrightarrow{AB}\right|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

39. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $\left|\vec{a}\right|=\left|\vec{b}\right|$. If $\left|\vec{a}\times\vec{b}\right|=\left|\vec{a}\right|$, then the angle between the vectors $\left(\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

40. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)-b=0$ be (–3, 5, 2). Then the value of |a + b| is equal to _______.

SOLUTION

1. Official Ans. by NTA (4)

Sol. $|\vec{a}| = 3 = a$; $\vec{a}.\vec{c} = c$

Now $|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\Rightarrow c^2 + a^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow c^2 + 9 - 2\,(c) = 8$

$\Rightarrow c^2 - 2c + 1 = 0 \;\Rightarrow c = 1 = |\vec{c}|$

Also, $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

Given $(\vec{a} \times \vec{b}) = |\vec{a} \times \vec{b}||\vec{c}|\sin\frac{\pi}{6}$

$= (3)\,(1)\,(1/2)$

$= 3/2$

2. Official Ans. by NTA (4)

Sol. $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{c})$

$= 3$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$

$\vec{a}.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|\cos\theta$

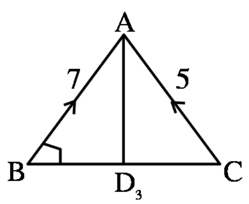
$\Rightarrow 1 = \sqrt{3}\,\cos\theta$

$\Rightarrow \cos 2\theta = -\frac{1}{3}$

$\Rightarrow 36\cos^2 2\theta = \boxed{4}$

3. Official Ans. by NTA (3)

Sol.



Projection of $\overrightarrow{BA}$

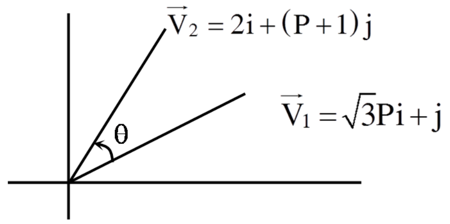
on $\overrightarrow{BC}$ is equal to

$= |\overrightarrow{BA}|\cos\angle ABC$

$= 7\left|\frac{7^2 + 3^2 - 5^2}{2 \times 7 \times 3}\right| = \frac{11}{2}$

4. Official Ans. by NTA (6)

Sol.



$|\vec{V_1}| = |\vec{V_2}|$

$3P^2 + 1 = 4 + (P+1)^2$

$2P^2 - 2P - 4 = 0 \Rightarrow P^2 - P - 2 = 0$

$P = 2, -1$ (rejected)

$\cos\theta = \frac{\vec{V_1}\cdot\vec{V_2}}{|\vec{V_1}||\vec{V_2}|} = \frac{2\sqrt{3}P + (P+1)}{\sqrt{(P+1)^2 + 4}\sqrt{3P^2+1}}$

$\cos\theta = \frac{4\sqrt{3}+3}{\sqrt{13}\sqrt{13}} = \frac{4\sqrt{3}+3}{13}$

$\tan\theta = \frac{\sqrt{112 - 24\sqrt{3}}}{4\sqrt{3}+3} = \frac{6\sqrt{3}-2}{4\sqrt{3}+3} = \frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$

$\Rightarrow \alpha = 6$

5. Official Ans. by NTA (1)

Sol. $\vec{a} = \lambda\vec{b} + \mu\vec{c} = \hat{i}(2\lambda+\mu) + \hat{j}(\lambda-\mu) + \hat{k}(\lambda+\mu)$

$\vec{a}\cdot\vec{d} = 0 = 3(2\lambda+\mu) + 2(\lambda-\mu) + 6(\lambda+\mu)$

$\Rightarrow 14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda$

$\Rightarrow \vec{a} = (0)\hat{i} - 3\lambda\hat{j} + (-\lambda)\hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{10}\,|\lambda| = \sqrt{10} \;\Rightarrow |\lambda| = 1$

$\Rightarrow \lambda = 1$ or -1

$[\vec{a}\,\vec{b}\,\vec{c}] = 0$

$[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{b}\,\vec{d}] + [\vec{a}\,\vec{c}\,\vec{d}] = [\vec{a}\;\;\vec{b}+\vec{c}\;\;\vec{d}]$

$$= \begin{vmatrix} 0 & -3\lambda & \lambda \\ 3 & 0 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$= 3\lambda(12) + \lambda(6) = 42\lambda = -42$

6. Official Ans. by NTA (4)

Sol. (1) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))$

$=\vec{a}(-\vec{b}\times\vec{c}+\vec{c}\times\vec{b})=-2(\vec{a}\times(\vec{b}\times\vec{c}))$

$=-2(\vec{a}\times\vec{a})=\vec{0}$

(2) Projection of $\vec{a}$ on $\vec{b}\times\vec{c}$

$=\dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}=\dfrac{\vec{a}\cdot\vec{a}}{|\vec{a}|}=|\vec{a}|=2$

(3) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=2[\vec{a}\ \vec{b}\ \vec{c}]=2\vec{a}\cdot(\vec{b}\times\vec{c})$

$=2\vec{a}.\vec{a}=2|\vec{a}|^2=8$

(4) $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow$ $\vec{a},\vec{b},\vec{c}$ are mutually $\perp$ vectors.

$\therefore$ $|\vec{a}\times\vec{b}|=|\vec{c}|\Rightarrow|\vec{a}||\vec{b}|=|\vec{c}|\Rightarrow|\vec{b}|=|\vec{c}|/2$

Also, $|\vec{b}\times\vec{c}|=|\vec{a}|\Rightarrow|\vec{b}||\vec{c}|=2$ $\Rightarrow$ $|\vec{c}|=2$ & $|\vec{b}|=1$

$|3\vec{a}+\vec{b}-2\vec{c}|^2=(3\vec{a}+\vec{b}-2\vec{c})\cdot(3\vec{a}+\vec{b}-2\vec{c})$

$=9|\vec{a}|^2+|\vec{b}|^2+4|\vec{c}|^2$

$=(9\times4)+1+(4\times4)$

$=36+1+16=53$

7. Official Ans. by NTA (1)

Sol. If the vectors are co-planar,

$$\begin{vmatrix} a+b+2 & a+2b+c & -b-c \\ b+1 & 2b & -b \\ b+2 & 2b & 1-b \end{vmatrix}=0$$

Now $R_3\to R_3-R_2$, $R_1\to R_1-R_2$

So $\begin{vmatrix} a+1 & a+c & -c \\ b+1 & 2b & -b \\ 1 & 0 & 1 \end{vmatrix}=0$

$=(a+1)\,2b-(a+c)(2b+1)-c(-2b)$

$=2ab+2b-2ab-a-2bc-c+2bc$

$=2b-a-c=0$

8. Official Ans. by NTA (3)

Sol. $\vec{p}=2\hat{i}+3\hat{j}+\hat{k}$ (Given)

$\vec{q}=\hat{i}+2\hat{j}+\hat{k}$

Now $(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix}$

$=-2\hat{i}-2\hat{j}-2\hat{k}$

$\Rightarrow\vec{r}=\pm\sqrt{3}\dfrac{((\vec{p}+\vec{q})\times(\vec{p}-\vec{q}))}{|(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})|}=\pm\dfrac{\sqrt{3}(-2\hat{i}-2\hat{j}-2\hat{k})}{\sqrt{2^2+2^2+2^2}}$

$\vec{r}=\pm(-\hat{i}-\hat{j}-\hat{k})$

According to question

$\vec{r}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

So $|\alpha|=1$, $|\beta|=1$, $|\gamma|=1$

$\Rightarrow|\alpha|+|\beta|+|\gamma|=3$

9. Official Ans. by NTA (4)

Sol. Because vectors are coplanar

Hence $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}=0$

$\Rightarrow c^2=ab\Rightarrow c=\sqrt{ab}$

10. Official Ans. by NTA (1)

Sol. $|\vec{a}|=2$, $|\vec{b}|=5$

$|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta=\pm8$

$\sin\theta=\pm\dfrac{4}{5}$

$\therefore$ $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$=10.\left(\pm\dfrac{3}{5}\right)=\pm6$

$|\vec{a}.\vec{b}|=6$

11. Official Ans. by NTA (60)

Sol. $(\vec{a}+3\vec{b}) \perp (7\vec{a}-5\vec{b})$

$(\vec{a}+3\vec{b}).(7\vec{a}-5\vec{b})=0$

$7|\vec{a}|^2-15|\vec{b}|^2+16\vec{a}.\vec{b}=0 \quad \ldots(1)$

$(\vec{a}-4\vec{b}).(7\vec{a}-2\vec{b})=0$

$7|\vec{a}|^2+8|\vec{b}|^2-30\vec{a}.\vec{b}=0 \quad \ldots(2)$

from (1) & (2)

$|\vec{a}| = |\vec{b}|$

$\cos\theta = \dfrac{|\vec{b}|}{2|\vec{a}|} \quad \therefore \theta = 60°$

12. Official Ans. by NTA (2)

Sol. $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

$\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}+\vec{b}=3\hat{j}+5\hat{k}$ **;** $\vec{a}.\vec{b}=-1+2+6=7$

$((\vec{a}\times((\vec{a}-\vec{b})\times\vec{b}))\times\vec{b})$

$((\vec{a}\times(\vec{a}\times\vec{b}-\vec{b}\times\vec{b}))\times\vec{b})$

$(\vec{a}\times(\vec{a}\times\vec{b}-0))\times\vec{b}$

$(\vec{a}\times(\vec{a}\times\vec{b}))\times\vec{b}$

$((\vec{a}.\vec{b})\vec{a}-(\vec{a}.\vec{a})\vec{b})\times\vec{b}$

$(\vec{a}.\vec{b})\vec{a}\times\vec{b}-(\vec{a}.\vec{a})(\vec{b}\times\vec{b})$

$(\vec{a}.\vec{b})(\vec{a}\times\vec{b})$

$$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix}=-\hat{i}-5\hat{j}+3\hat{k}$$

$\therefore 7(-\hat{i}-5\hat{j}+3\hat{k})$

$(\vec{a}+\vec{b})\times(7(-\hat{i}-5\hat{j}+3\hat{k}))$

$7(0\hat{i}+3\hat{j}+5\hat{k})\times(-\hat{i}-5\hat{j}+3\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 5 \\ -1 & -5 & 3 \end{vmatrix}$$

$\Rightarrow 34\hat{i}-(5)\hat{j}+(3\hat{k})$

$\Rightarrow 34\hat{i}-5\hat{j}+3\hat{k}$

$\therefore 7(34\hat{i}-5\hat{j}+3\hat{k})$

13. Official Ans. by NTA (2)

Sol. $\vec{a}\times\vec{b}=c$

Take Dot with $\vec{c}$

$(\vec{a}\times\vec{b}).\vec{c}=|\vec{c}|^2=2$

Projection of $\vec{b}$ or $\vec{a}\times\vec{c}=\ell$

$\dfrac{|\vec{b}.(\vec{a}\times\vec{c})|}{|\vec{a}\times\vec{c}|}=\ell$

$\therefore \ell=\dfrac{2}{\sqrt{6}} \Rightarrow \ell^2=\dfrac{4}{6}$

$3\ell^2=2$

14. Official Ans. by NTA (2)

Sol. $\vec{a}=(\vec{b}\cdot\vec{c})\vec{b}-(\vec{b}.\vec{b})\vec{c}$

$=1.2\cos\theta\vec{b}-\vec{c}$

$\Rightarrow \vec{a}=2\cos\theta\vec{b}-\vec{c}$

$|\vec{a}|^2=(2\cos\theta)^2+2^2-2.2\cos\theta\vec{b}\cdot\vec{c}$

$\Rightarrow 2=4\cos^2\theta+4-4\cos\theta\cdot2\cos\theta$

$\Rightarrow -2=-4\cos^2\theta$

$\Rightarrow \cos^2\theta=\dfrac{1}{2}$

$\Rightarrow \sec^2\theta=2$

$\Rightarrow \tan^2\theta=1$

$\Rightarrow \theta=\dfrac{\pi}{4}$

1 + tan θ = 2.

15. Official Ans. by NTA (9)

Sol. $\vec{a}=(1,-\alpha,\beta)$

$\vec{b}=(3,\beta,-\alpha)$

$\vec{c}=(-\alpha,-2,1)$; $\alpha,\ \beta \in I$

$\vec{a}.\vec{b}=-1 \Rightarrow 3-\alpha\beta-\alpha\beta=-1$

$\Rightarrow\ \alpha\beta=2$

1	2
2	1
−1	−2
−2	−1

$\vec{b}\cdot\vec{c}=10$

$\Rightarrow\ -3\alpha-2\beta-\alpha=10$

$\Rightarrow\ 2\alpha+\beta+5=0$

$\therefore\ \alpha=-2;\ \beta=-1$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 2 & -1\\ 3 & -1 & 2\\ 2 & -2 & 1\end{vmatrix}$$

$=1(-1+4)-2(3-4)-1(-6+2)$

$=3+2+4=9$

16. Official Ans. by NTA (1)

Sol. $|\vec{a}|=\sqrt{3};\ \vec{a}.\vec{c}=3;\ \ \vec{a}\times\vec{b}=-2\hat{i}+\hat{j}+\hat{k},\ \vec{a}\times\vec{c}=\vec{b}$

Cross with $\vec{a}$.

$\vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{c})\vec{a}-a^2\vec{c}=\vec{a}\times\vec{b}$

$\Rightarrow\ 3\vec{a}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow 3\hat{i}+3\hat{j}+3\hat{k}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow\ \vec{c}=\frac{5\hat{i}}{3}+\frac{2\hat{j}}{3}+\frac{2\hat{k}}{3}$

$\therefore\ \vec{a}.(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b}).\vec{c}=\frac{-10}{3}+\frac{2}{3}+\frac{2}{3}=-2$

17. Official Ans. by NTA (4)

Sol. $A(\hat{j})\ .\ B(10\hat{i})$

H $(h\hat{j}+10\hat{k})$

G $(10\hat{i}+h\hat{j}+10\hat{k})$

$\overrightarrow{AG}=10\hat{i}+h\hat{j}+10\hat{k}$

$\overrightarrow{BH}=-10\hat{i}+h\hat{j}+10\hat{k}$

$\cos\theta=\frac{\overrightarrow{AG}\,\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|}$

$\frac{1}{5}=\frac{h^2}{h^2+200}$

$4h^2=200 \Rightarrow\ h=5\sqrt{2}$

18. Official Ans. by NTA (5)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=1,\ \vec{a}\cdot\vec{b}=12-\lambda$

$(\vec{a}\cdot\vec{b})=|\vec{b}|^2$

$\lambda^2-24\lambda+144=\lambda^2-4\lambda+4+40$

$20\lambda=100 \Rightarrow \lambda=5.$

19. Official Ans. by NTA (90)

Sol. since, $\vec{a}.\vec{b}=0$

$1+15+\alpha\beta=0 \Rightarrow \alpha\beta=-16$ …(1)

Also,

$|\vec{b}\times\vec{c}|^2=75 \Rightarrow (10+\beta^2)14-(5-3\beta)^2=75$

$\Rightarrow 5\beta^2+30\beta+40=0$

$\Rightarrow \beta=-4,-2$

$\Rightarrow \alpha=4,8$

$\Rightarrow |\vec{a}|^2_{max}=(26+\alpha^2)_{max}=90$

20. Official Ans. by NTA (1)

Sol. Equation of planes are

$\vec{r}\,.\left(\hat{i}+\hat{j}+\hat{k}\right)-1=0 \Rightarrow x+y+z-1=0$

and $\vec{r}\,.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0 \Rightarrow 2x+3y-z+4=0$

equation of planes through line of intersection of these planes is :–

$(x + y + z - 1) + \lambda\,(2x + 3y - z + 4) = 0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$

But this plane is parallel to x–axis whose direction are (1, 0, 0)

$\therefore\ (1 + 2\lambda)1 + (1 + 3\lambda)\,0 + (1 - \lambda)\,0 = 0$

$\lambda=-\frac{1}{2}$

$\therefore$ Required plane is

$0\,x+\left(1-\frac{3}{2}\right)y+\left(1+\frac{1}{2}\right)z-1+4\left(\frac{-1}{2}\right)=0$

$\Rightarrow \frac{-y}{2}+\frac{3}{2}z-3=0$

$\Rightarrow y - 3z + 6 = 0$

$\Rightarrow \boxed{\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0}$ Ans.

21. Official Ans. by NTA (3)

Sol. $\left|3\vec{a}+\vec{b}\right|^2=\left|2\vec{a}+3\vec{b}\right|^2$

$\left(3\vec{a}+\vec{b}\right).\left(3\vec{a}+\vec{b}\right)=\left(2\vec{a}+3\vec{b}\right).\left(2\vec{a}+3\vec{b}\right)$

$9\vec{a}.\vec{a}+6\vec{a}.\vec{b}+\vec{b}.\vec{b}=4\vec{a}.\vec{a}+12\vec{a}.\vec{b}+9.\vec{b}.\vec{b}$

$5\left|\vec{a}\right|^2-6\vec{a}.\vec{b}=8\left|\vec{b}\right|^2$

$5(8)^2-6.8.\left|\vec{b}\right|\cos 60^\circ=8\left|\vec{b}\right|^2 \quad \left(\because \frac{1}{8}\left|\vec{a}\right|=1 \Rightarrow \left|\vec{a}\right|=8\right)$

$40-3\left|\vec{b}\right|=\left|\vec{b}\right|^2$

$\Rightarrow \left|\vec{b}\right|^2+3\left|\vec{b}\right|-40=0$

$\left|\vec{b}\right|=-8, \quad \left|\vec{b}\right|=5$

(rejected)

22. Official Ans. by NTA (3)

Sol. Suppose $\vec{r}=x\vec{a}+y\vec{b}+2\vec{c}$

and $|\vec{a}|=|\vec{b}|=|\vec{c}|=k$

$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$

$\Rightarrow k^2\left(\vec{r}-\vec{b}\right)-k^2x\vec{a}+k^2\left(\vec{r}-\vec{c}\right)-k^2y\vec{b}+k^2\left(\vec{r}-\vec{a}\right)-k^2z\vec{c}=\vec{0}$

$\Rightarrow 3\vec{r}-\left(\vec{a}+\vec{b}+\vec{c}\right)-\vec{r}=\vec{0}$

$\Rightarrow \vec{r}=\frac{\vec{a}+\vec{b}+\vec{c}}{2}$

23. Official Ans. by NTA (1494)

Sol. $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$

$\vec{b}=\hat{i}+2\hat{j}-\hat{k}$

$\vec{c}=3\hat{i}+2\hat{j}-\hat{k}$

$\vec{v}=x\vec{a}+y\vec{b} \quad \vec{v}\left(3\hat{i}+2\hat{j}-k\right)=0$

$\vec{v}\cdot\hat{a}=19$

$\vec{v}=\lambda\vec{c}\times\left(\vec{a}\times\vec{b}\right)$

$\vec{v}=\lambda\left[(\vec{c}.\vec{b})\vec{a}-(\vec{c}\cdot\vec{a})\vec{b}\right]$

$=\lambda[(3+4+1)\left(2\hat{i}-\hat{j}+2\hat{k}\right)-\left(\frac{6-2-2}{2}\right)\left(\hat{i}+2\hat{j}+\hat{k}\right)$

$=\lambda[16\hat{i}-8\hat{j}+16\hat{k}-2\hat{i}-4\hat{j}+2\hat{k}]$

$\vec{v}=\lambda\left[14\hat{i}-12\hat{j}+18\hat{k}\right]$

$\lambda[14\hat{i}-12\hat{j}+18\hat{k}]\cdot\frac{\left(2\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{4+1+4}}=19$

$\lambda\frac{[28+12+36]}{3}=19$

$\lambda\left(\frac{76}{3}\right)=19$

$4\lambda=3 \Rightarrow \lambda=\frac{3}{4}$

$|2v^2|=\left|2\times\frac{3}{4}\left(14\hat{i}-12\hat{j}+18\hat{k}\right)\right|^2$

$\frac{9}{4}\times 4\left(7\hat{i}-6\hat{j}+9\hat{k}\right)^2$

$= 9\,(49 + 36 + 81)$

$= 9\,(166)$

$= 1494$

24. Official Ans. by NTA (3)

Sol. $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$

$\vec{r}.(\hat{i}-2\hat{j})=-2$

point (1, 0, 2)

Eq^n of plane

$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1+\lambda\left\{r.(\hat{i}-2\hat{j})+2\right\}=0$

$\vec{r}.\left\{\hat{i}\left(1+\lambda\right)+\hat{j}\left(1-2\lambda\right)+\hat{k}(1)\right\}-1+2\lambda=0$

Point $\hat{i}+0\hat{j}+2\hat{k}=\vec{r}$

$\therefore (\hat{i}+2\hat{k}).\left\{\hat{i}(1+\lambda)+\hat{j}(1-2\lambda)+\hat{k}(1)\right\}-1+2\lambda=0$

$1+\lambda+2-1+2\lambda=0$

$\lambda=-\dfrac{2}{3}$

$\therefore\ \vec{r}.\left[\hat{i}\left(\dfrac{1}{3}\right)+\hat{j}\left(\dfrac{7}{3}\right)+\hat{k}\right]=\dfrac{7}{3}$

$r.\left[\hat{i}+7\hat{j}+3\hat{k}\right]=7$

Ans. 3

25. Official Ans. by NTA (75)

Sol. Let $\vec{c}=\lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$=\lambda\left(\left(\vec{b}.\vec{b}\right)\vec{a}-\left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$=\lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$

$=\lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a}=7\Rightarrow 3\lambda+5\lambda+6\lambda=7$

$\lambda=\dfrac{1}{2}$

$\therefore\ 2\left|\left(\dfrac{-3}{2}-1+2\right)\hat{i}+\left(\dfrac{5}{2}+1\right)\hat{j}+\left(3+1+1\right)\hat{k}\right|^2$

$=2\left(\dfrac{1}{4}+\dfrac{49}{4}+25\right)=25+50=75$

26. Official Ans. by NTA (12)

Sol. $(\vec{r}-\vec{c})\times\vec{a}=0$

$\Rightarrow\ \vec{r}=\vec{c}+\lambda\vec{a}$

Now, $0=\vec{b}.\vec{c}+\lambda\vec{a}.\vec{b}$

$\Rightarrow\ \lambda=\dfrac{-\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}=-\dfrac{2}{-1}=2$

So, $\vec{r}\cdot\vec{a}=\vec{a}\cdot\vec{c}+2a^2=12$

27. Official Ans. by NTA (2)

Sol. $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$

$\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$

area of parallelogram = $\left|\vec{a}\times\vec{b}\right|=8\sqrt{3}$.

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \alpha & 3\\ 3 & -\alpha & 1\end{vmatrix}=\hat{i}(4\alpha)-\hat{j}(-8)+\hat{k}(-4\alpha)$

$\therefore\ \left|\vec{a}\times\vec{b}\right|=\sqrt{64+32\alpha^2}=8\sqrt{3}$

$\Rightarrow 2+\alpha^2=6\Rightarrow\alpha^2=4$

$\therefore\ \vec{a}\cdot\vec{b}=3-\alpha^2+3=2$

28. Official Ans. by NTA (4)

Sol. $\vec{a}_1$ and $\vec{a}_2$ are collinear

so $\dfrac{x}{1}=\dfrac{-1}{y}=\dfrac{1}{z}$

unit vector in direction of

$x\hat{i}+y\hat{j}+z\hat{k}=\pm\dfrac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

29. Official Ans. by NTA (4)

Sol. $\vec{a}.\vec{b}=0$

$\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\left(\vec{a}.\vec{b}\right)\vec{a}-\left(\vec{a}.\vec{a}\right)\vec{b}=-\left|\vec{a}\right|^2\vec{b}$

Now $\vec{a}\times\left(\vec{a}\times\left(-\left|\vec{a}\right|^2\vec{b}\right)\right)$

$=-\left|\vec{a}\right|^2\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)$

$=-\left|\vec{a}\right|^2\left(-\left|\vec{a}\right|^2\vec{b}\right)=\left|\vec{a}\right|^4\vec{b}$

30. Official Ans by NTA (2)

Sol. $\vec{r}\times\vec{a}=\vec{b}\times\vec{r} \Rightarrow \vec{r}\times(\vec{a}+\vec{b})=0$

$\vec{r}=\vec{\lambda}(\vec{a}+\vec{b}) \Rightarrow \vec{r}=\vec{\lambda}(\hat{i}+2\hat{j}-3\hat{k}+2\hat{i}-3\hat{j}+5\hat{k})$

$\vec{r}=\vec{\lambda}(3\hat{i}-\hat{j}+2\hat{k})$ ...(1)

$\vec{r}\cdot(\alpha\hat{i}+2\hat{j}+\hat{k})=3$

Put $\vec{r}$ from (1) $\alpha\lambda=1$ ...(2)

$\vec{r}\cdot(2\hat{i}+5\hat{j}-\alpha\hat{k})=-1$

Put $\vec{r}$ from (1) $2\lambda\alpha-\lambda=1$ ...(3)

Solve (2) & (3)

$\alpha = 1,\ \lambda=1$

$\Rightarrow\ \vec{r}=3\hat{i}-\hat{j}+2\hat{k}$

$|\vec{r}|^2=14\ \&\ \alpha=1$

$\alpha+|\vec{r}|^2=15$

31. Official Ans by NTA (28)

Sol. $\vec{c}=\lambda(\vec{a}\times\vec{b})$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-1\\1&2&1\end{vmatrix}$$

$(\vec{a}\times\vec{b})=3\hat{i}-2\hat{j}+\hat{k}$

$\vec{c}\cdot(\hat{i}+\hat{j}+3\hat{k})=\lambda(3\hat{i}-2\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}+3\hat{k})$

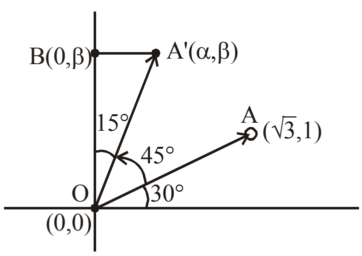
$\Rightarrow \lambda(4)=8 \Rightarrow \lambda=2$

$\vec{c}=2(\vec{a}\times\vec{b})$

$\vec{c}\cdot(\vec{a}\times\vec{b})=2|\vec{a}\times\vec{b}|^2=28$

32. Official Ans. by NTA (1)

Sol.



Area of $\Delta(OA'B)=\frac{1}{2}OA'\cos15°\times OA'\sin15°$

$=\frac{1}{2}(OA')^2\frac{\sin30°}{2}$

$=(3+1)\times\frac{1}{8}=\frac{1}{2}$

33. Official Ans. by NTA (2)

Sol. $\overrightarrow{OP}\perp\overrightarrow{OQ}$

$\Rightarrow -x+2y-3x=0$

$\Rightarrow y=2x$ .....(i)

$|\overrightarrow{PQ}|^2=20$

$\Rightarrow (x+1)^2+(y-2)^2+(1+3x)^2=20$

$\Rightarrow x=1$

$\overrightarrow{OP},\overrightarrow{OQ},\overrightarrow{OR}$ are coplanar.

$$\Rightarrow\begin{vmatrix}x&y&-1\\-1&2&3x\\3&z&-7\end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix}1&2&-1\\-1&2&3\\3&z&-7\end{vmatrix}=0$$

$\Rightarrow 1(-14-3z)-2(7-9)-1(-z-6)=0$

$\Rightarrow z=-2$

$\therefore x^2+y^2+z^2=1+4+4=9$ **Option (2)**

34. Official Ans. by NTA (486)

Sol. Let $\vec{x}=\lambda\vec{a}+\mu\vec{b}$ (λ and μ are scalars)

$\vec{x}=\hat{i}(2\lambda+\mu)+\hat{j}(2\mu-\lambda)+\hat{k}(\lambda-\mu)$

Since $\vec{x}\cdot(3\hat{i}+2\hat{j}-\hat{k})=0$

$3\lambda+8\mu=0$ .....(1)

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$

$\frac{\vec{x}\cdot\vec{a}}{|\vec{a}|}=\frac{17\sqrt{6}}{2}$

$6\lambda-\mu=51$ .....(2)

From (1) and (2)

$\lambda=8,\ \mu=-3$

$\vec{x}=13\hat{i}-14\hat{j}+11\hat{k}$

$|\vec{x}|^2=486$

35. Official Ans. by NTA (1)

Sol. $\vec{r}\times\vec{a}-\vec{r}\times\vec{b}=0$

$\Rightarrow \vec{r}\times(\vec{a}-\vec{b})=0$

$\Rightarrow \vec{r}=\lambda(\vec{a}-\vec{b})$

$\Rightarrow \vec{r}=\lambda(-5\hat{i}-4\hat{j}+10\hat{k})$

Also $\vec{r}.(\hat{i}+2\hat{j}+\hat{k})=-3$

$\Rightarrow \lambda(-5-8+10)=-3$

$\lambda=1$

Now $\vec{r}=-5\hat{i}-4\hat{j}+10\hat{k}$

$=\vec{r}.(2\hat{i}-3\hat{j}+\hat{k})$

$=-10+12+10=12$

36. Official Ans. by NTA (2)

Sol. $\vec{a}.\vec{b}=1 \Rightarrow -\alpha\beta-\alpha\beta-3=1$

$\Rightarrow -2\alpha\beta=4 \Rightarrow \boxed{\alpha\beta=-2}$ ....(1)

$\vec{b}.\vec{c}=-3 \Rightarrow -\beta+2\alpha+1=-3$

$\boxed{\beta-2\alpha=4}$ ....(2)

Solving (1) & (2), $(\alpha,\beta)=(-1,2)$

$$\frac{1}{3}[\vec{a}\ \vec{b}\ \vec{c}]=\frac{1}{3}\begin{vmatrix}\alpha & \beta & 3\\ -\beta & -\alpha & -1\\ 1 & -2 & -1\end{vmatrix}$$

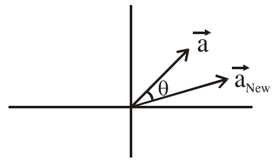
$$=\frac{1}{3}\begin{vmatrix}-1 & 2 & 3\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}$$

$$=\frac{1}{3}\begin{vmatrix}0 & 0 & 2\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}=\frac{1}{3}[2(4-1)]=2$$

37. Official Ans. by NTA (4)

Sol. $\vec{a}_{Old}=3p\hat{i}+\hat{j}$

$\vec{a}_{New}=(p+1)\hat{i}+\sqrt{10}\hat{j}$



$\Rightarrow|\vec{a}_{Old}|=|\vec{a}_{New}|$

$\Rightarrow ap^2+1=p^2+2p+1+10$

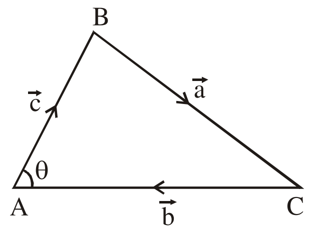
$8p^2-2p-10=0$

$4p^2-p-5=0$

$(4p-5)(p+1)=0\rightarrow p=\frac{5}{4},-1$

38. Official Ans. by NTA (2)

Sol.



$|\vec{a}|=8, |\vec{b}|=7, |\vec{c}|=10$

$$\cos\theta=\frac{|\vec{b}|^2+|\vec{c}|^2-|\vec{a}|^2}{2|\vec{b}||\vec{c}|}=\frac{17}{28}$$

Projection of $\vec{c}$ on $\vec{b}$

$=|\vec{c}|\cos\theta$

$=10\times\frac{17}{28}$

$=\frac{85}{14}$

39. Official Ans. by NTA (2)

Sol. $|\vec{a}|=|\vec{b}|$, $|\vec{a}\times\vec{b}|=|\vec{a}|$, $\vec{a}\perp\vec{b}$

$|\vec{a}\times\vec{b}|=|\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90^\circ=|\vec{a}| \Rightarrow |\vec{b}|=1=|\vec{a}|$

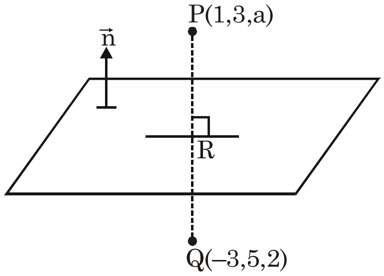
$\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a}=\hat{i}$, $\vec{b}=\hat{j} \Rightarrow \vec{a}\times\vec{b}=\hat{k}$

$$\cos\theta=\frac{(\hat{i}+\hat{j}+\hat{k}).\hat{i}}{\sqrt{3}\ \sqrt{1}}=\frac{1}{\sqrt{3}} \Rightarrow \theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

40. Official Ans. by NTA (1)

Sol.



plane = 2x – y + z = b

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore \; -2-4+\frac{a+2}{2} = b$

$\Rightarrow$ a + 2 = 2b + 12 $\Rightarrow$ a = 2b + 10 ...(i)

<PQ> = <4, –2, a – 2>

$\therefore \; \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$

$\Rightarrow$ a – 2 = 2 $\Rightarrow$ a = 4, b = –3

$\therefore \; |a + b| = 1$