TRIGONOMETRIC EQUATION

1. The number of solutions of $\sin^7x + \cos^7x = 1$, $x \in [0, 4\pi]$ is equal to
(1) 11 (2) 7 (3) 5 (4) 9

2. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to :
(1) 8π (2) 11π (3) 12π (4) 9π

3. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to:
(1) 23 (2) –27 (3) –23 (4) 27

4. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :
(1) $-\frac{11\pi}{30}$ (2) $\frac{\pi}{10}$ (3) $-\frac{7\pi}{30}$ (4) $-\frac{\pi}{15}$

5. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to ________.

6. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \le x \le \frac{\pi}{4}$ is :
(1) 3 (2) 1 (3) 0 (4) 2

7. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :
(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$
(2) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$
(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$
(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

8. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :
(1) $\frac{1}{2}$ (2) $\frac{1+\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

9. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is

10. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

11. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10}n - 1)$, $n > 0$, then the value of n is equal to :
(1) 20 (2) 12 (3) 9 (4) 16

12. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :
(1) 3 (2) 4 (3) 8 (4) 2

13. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

SOLUTION

1. Official Ans. by NTA (3)

Sol. $\sin^7 x \le \sin^2 x \le 1$ ...(1)

and $\cos^7 x \le \cos^2 x \le 1$ ...(2)

also $\sin^2 x + \cos^2 x = 1$

$\Rightarrow$ equality must hold for (1) & (2)

$\Rightarrow \sin^7 x = \sin^2 x$ & $\cos^7 = \cos^2 x$

$\Rightarrow \sin x = 0$ & $\cos x = 1$

or

$\cos x = 0$ & $\sin x = 1$

$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow$ 5 solutions

2. Official Ans. by NTA (4)

Sol. $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{\cos\frac{3x}{2} + \cos\frac{x}{2}\right\} = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{2\cos x\cos\frac{x}{2}\right\} = 0$

$2\sin\frac{5x}{2} = 0 \Rightarrow \frac{5X}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$

$\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

So sum $= 6\pi + \pi + 2\pi = 9\pi$

3. Official Ans. by NTA (3)

Sol. $\sin\theta + \cos\theta = \frac{1}{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$

$\sin 2\theta = -\frac{3}{4}$

Now :

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$= 1 - 2\left(-\frac{3}{4}\right)^2$

$= 1 - 2\times\frac{9}{16} = -\frac{1}{8}$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$= (3 - 4\sin^2 2\theta).\sin 2\theta$

$= \left[3 - 4\left(\frac{9}{16}\right)\right].\left(-\frac{3}{4}\right)$

$\Rightarrow \left[\frac{3}{4}\right]\times\left(-\frac{3}{4}\right) = -\frac{9}{16}$

$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$

$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$

4. Official Ans. by NTA (1)

Sol. $\frac{\cos x}{1+\sin x} = |\tan 2x|$

$\Rightarrow \frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)} = |\tan 2x|$

$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$

$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$

$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$

or sum $= \frac{-11\pi}{6}$.

5. Official Ans. by NTA (56)

Sol. Given equation

$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 1 - \sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 2 - (\sin 2\theta)^2 - \sin 2\theta = 0$

$\Rightarrow (\sin 2\theta)^2 + (\sin 2\theta) - 2 = 0$

$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$

$\Rightarrow \sin 2\theta = 1$ or $\underset{\text{(not possible)}}{\boxed{\sin 2\theta = -2}}$

$$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$\Rightarrow S = \frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4} = 7\pi$$

$$\Rightarrow \frac{8S}{\pi} = \frac{8\times 7\pi}{\pi} = 56.00$$

6. Official Ans. by NTA (2)

Sol. $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$$

$$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$$

In interval $\left[0, \frac{\pi}{4}\right]$ only one solution

7. Official Ans. by NTA (4)

Sol. $\sin 2\theta + \tan 2\theta > 0$

$$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$$

$$\Rightarrow \sin 2\theta \frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0 \Rightarrow \tan 2\theta(2\cos^2\theta) > 0$$

Note : $\cos 2\theta \neq 0$

$$\Rightarrow 1 - 2\sin^2\theta \neq 0 \Rightarrow \sin\theta \neq \pm\frac{1}{\sqrt{2}}$$

Now, $\tan 2\theta\,(1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ (as $\cos 2\theta + 1 > 0$)

$$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

As $\sin\theta \neq \pm\frac{1}{\sqrt{2}}$; which has been already considered

8. Official Ans. by NTA (2)

Sol. $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$

$$\cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)$$

$$+\frac{1}{4}.\cos^2\left(\frac{x-y}{2}\right) + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2}\cos\left(\frac{x-y}{2}\right)\right)^2 + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0 \text{ and } \cos\left(\frac{x+y}{2}\right) = \frac{1}{2}\cos\left(\frac{x-y}{2}\right)$$

$$\Rightarrow x = y \text{ and } \cos x = \frac{1}{2} = \cos y$$

$$\therefore \sin x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin x + \cos y = \frac{1+\sqrt{3}}{2}$$

option (2)

9. Official Ans. by NTA (11)

Sol. 3 sinx + 4 cosx = k + 1

$\Rightarrow k+1 \in \left[-\sqrt{3^2+4^2}, \sqrt{3^2+4^2}\right]$

$\Rightarrow k+1 \in [-5,5]$

$\Rightarrow k \in [-6,4]$

No. of integral values of k = 11

10. Official Ans. by NTA (1)

Sol. $\sqrt{3}(\cos x)^2 - \sqrt{3}\cos x + \cos x - 1 = 0$

$\Rightarrow (\sqrt{3}\cos x + 1)(\cos x - 1) = 0$

$\Rightarrow \cos x = 1$ or $\cos x = -\frac{1}{\sqrt{3}}$ (reject)

$\Rightarrow x = 0$ only

11. Official Ans. by NTA (2)

Sol. $x \in \left(0, \frac{\pi}{2}\right)$

$\log_{10}\sin x + \log_{10}\cos x = -1$

$\Rightarrow \log_{10}\sin x.\cos x = -1$

$\Rightarrow \sin x.\cos x = \frac{1}{10}$ ......(1)

$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \sin x + \cos x = 10^{\left(\log_{10}\sqrt{n} - \frac{1}{2}\right)} = \sqrt{\frac{n}{10}}$

by squaring

$1 + 2\sin x.\cos x = \frac{n}{10}$

$\Rightarrow 1 + \frac{1}{5} = \frac{n}{10} \Rightarrow n = 12$

12. Official Ans. by NTA (2)

Sol. $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$

$(81)^{\sin^2 x} = t$

$t + \frac{81}{t} = 30$

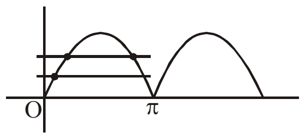
$t^2 - 30t + 81 = 0$

$(t-3)(t-27) = 0$

$(81)^{\sin^2 x} = 3^1$ or $(81)^{\sin^2 x} = 3^3$

$3^{4\sin^2 x} = 3^1$ or $3^{4\sin^2 x} = 3^3$

$\sin^2 x = \frac{1}{4}$ or $\sin^2 x = \frac{3}{4}$



Total sol. = 4

13. Official Ans. by NTA (1)

Sol. If cotx > 0 $\Rightarrow \frac{1}{\sin x} = 0$ (Not possible)

If cotx < 0 $\Rightarrow$ 2cotx + $\frac{1}{\sin x}$ = 0

$\Rightarrow$ 2cosx = –1

$\Rightarrow x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ (reject)