STRAIGHT LINE

1. Consider a triangle having vertices A(–2, 3), B(1, 9) and C(3, 8). If a line L passing through the circum-centre of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is ________.

2. Let the equation of the pair of lines, y = px and y = qx, can be written as (y – px) (y – qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is :

(1) $x^2 - 3xy + y^2 = 0$ (2) $x^2 + 4xy - y^2 = 0$

(3) $x^2 + 3xy - y^2 = 0$ (4) $x^2 - 3xy - y^2 = 0$

3. A ray of light through (2,1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be :

(1) 11x + 7y + 8 = 0 or 11x + 7y – 15 = 0

(2) 11x – 7y – 8 = 0 or 11x + 7y + 15 = 0

(3) 2x – 7y + 29 = 0 or 2x – 7y – 7 = 0

(4) 2x – 7y – 39 = 0 or 2x – 7y – 7 = 0

4. The point P (a,b) undergoes the following three transformations successively :

(a) reflection about the line y = x.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to :

(1) 13 (2) 9 (3) 5 (4) 7

5. Two sides of a parallelogram are along the lines 4x + 5y = 0 and 7x + 2y = 0. If the equation of one of the diagonals of the parallelogram is 11x + 7y = 9, then other diagonal passes through the point :

(1) (1,2) (2) (2,2) (3) (2,1) (4) (1,3)

6. Let ABC be a triangle with A(–3, 1) and $\angle ACB = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is 2x + y – 3 = 0 and the equation of angle bisector of C is 7x – 4y – 1 = 0, then $\tan\theta$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $\frac{4}{3}$ (4) 2

7. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is :

(1) $3x^2 - 2y - 6 = 0$ (2) $3x^2 + 2y - 6 = 0$

(3) $2x^2 + 3y - 9 = 0$ (4) $2x^2 - 3y + 9 = 0$

8. Let A(a, 0), B(b, 2b +1) and C(0, b), $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

(1) $\frac{-2b}{b+1}$ (2) $\frac{2b}{b+1}$ (3) $\frac{2b^2}{b+1}$ (4) $\frac{-2b^2}{b+1}$

9. If p and q are the lengths of the perpendiculars from the origin on the lines,

$x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ and

$x \sin \alpha + y \cos\alpha = k \sin 2\alpha$

respectively, then k^2 is equal to :

(1) $4p^2 + q^2$ (2) $2p^2 + q^2$

(3) $p^2 + 2q^2$ (4) $p^2 + 4q^2$

10. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

(1) $\frac{4}{\sqrt{5}}$ (2) $\frac{16}{\sqrt{5}}$ (3) $\frac{8}{\sqrt{5}}$ (4) $\frac{12}{\sqrt{5}}$

11. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is _______.

12. For which of the following curves, the line $x+\sqrt{3}y=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + y^2 = 7$ (2) $y^2 = \frac{1}{6\sqrt{3}}x$

(3) $2x^2 - 18y^2 = 9$ (4) $x^2 + 9y^2 = 9$

13. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (–5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ________.

14. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1,1), (2, 2) and (4, 4) respectively. Then which of these stones is / are on the path of the man ?

(1) A only (2) C only

(3) All the three (4) B only

15. The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on :

(1) $(x - 2)^2 + (y - 2)^2 = 12$

(2) $(x - 4)^2 + (y + 2)^2 = 16$

(3) $(x - 4)^2 + (y - 4)^2 = 8$

(4) $(x - 2)^2 + (y - 4)^2 = 4$

16. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

(1) Right angled triangle

(2) Equilateral triangle

(3) Isosceles triangle

(4) None of the above

17. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

(1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

18. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin.If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to :

19. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is :

(1) (–1, 0) (2) (–2, –2)

(3) (0, 2) (4) (1, 4)

20. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \leq 100$ and $4x + 3y \leq 75$ for $x \geq 0$ and $y \geq 0$ is _______ .

21. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

(1) 1 (2) 2 (3) 3 (4) 0

22. The equation of one of the straight lines which passes through the point (1,3) and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line $y+1=3\sqrt{2}$, x is

(1) $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

(2) $5\sqrt{2}x + 4y - \left(15 + 4\sqrt{2}\right) = 0$

(3) $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

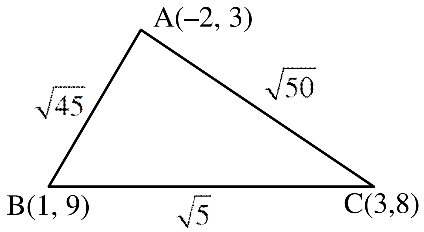
(4) $4\sqrt{2}x - 5y - \left(5 + 4\sqrt{2}\right) = 0$

23. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to :

(1) $\frac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$ (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

SOLUTION

1.



$$\left(\sqrt{50}\right)^2=\left(\sqrt{45}\right)^2+\left(\sqrt{5}\right)^2$$

$\angle B = 90°$

Circum-center = $\left(\frac{1}{2},\frac{11}{2}\right)$

Mid point of BC = $\left(2,\frac{17}{2}\right)$

Line : $\left(y-\frac{11}{2}\right)=2\left(x-\frac{1}{2}\right)\Rightarrow y=2x+\frac{9}{2}$

Passing though $\left(0,\frac{\alpha}{2}\right)$

$$\frac{\alpha}{2}=\frac{9}{2}\Rightarrow \alpha=9$$

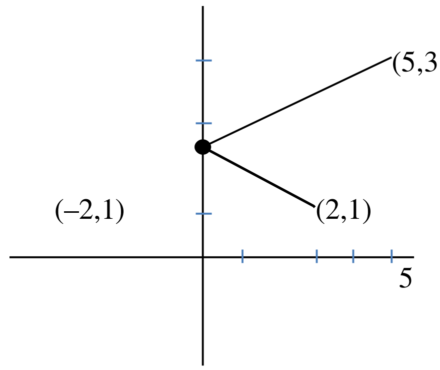
2. $\frac{x^2-y^2}{1-(-5)}=\frac{xy}{-2}$

$$\frac{x^2-y^2}{6}=\frac{xy}{-2}$$

$\Rightarrow x^2-y^2=-3xy$

$\Rightarrow x^2+3xy-y^2=0$

3.



Equation of reflected Ray

$$y-1=\frac{2}{7}(x+2)$$

$7y-7=2x+4$

$2x-7y+11=0$

Let the equation of other directrix is

$2x-7y+\lambda$

Distance of directrix from Focub

$$\frac{a}{e}-ae=\frac{8}{\sqrt{53}}$$

$$3a-\frac{a}{3}=\frac{8}{\sqrt{53}}\ \text{ or }\ a=\frac{3}{\sqrt{53}}$$

Distance from other focus $\frac{a}{e}+ae$

$$3a+\frac{a}{3}=\frac{10a}{3}=\frac{10}{3}\times\frac{3}{\sqrt{53}}=\frac{10}{\sqrt{53}}$$

Distance between two directrix $=\frac{2a}{e}$

$$=2\times3\times\frac{3}{\sqrt{53}}=\frac{18}{\sqrt{53}}$$

$$\left|\frac{\lambda-11}{\sqrt{53}}\right|=\frac{18}{\sqrt{53}}$$

$\lambda-11=18$ or -18

$\lambda=29$ or -7

$2x-7y-7=0$ or $2x-7y+29=0$

4. Image of A(a,b) along y = x is B(b,a). Translating it 2 units it becomes C(b + 2, a).

Now, applying rotation theorem

$$-\frac{1}{2}+\frac{7}{\sqrt{2}}i=((b+2)+ai)\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$\frac{-1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=\left(\frac{b+2}{\sqrt{2}}-\frac{a}{\sqrt{2}}\right)+i\left(\frac{b+2}{\sqrt{2}}+\frac{a}{\sqrt{2}}\right)$$

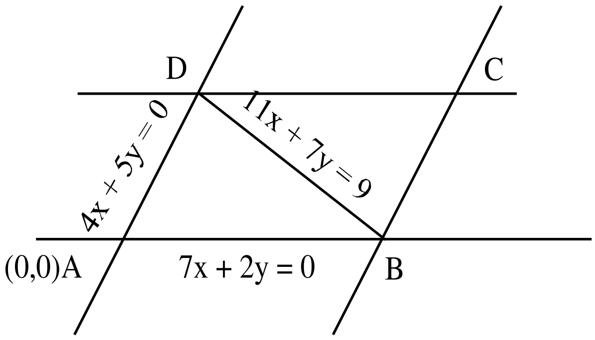
$\Rightarrow b - a + 2 = -1$ ....(i)

and $b + 2 + a = 7$ ....(ii)

$\Rightarrow a = 4;\ b = 1$

$\Rightarrow 2a + b = 9$

5. Both the lines pass through origin.



point D is equal of intersection of 4x + 5y = 0 & 11x + 7y = 9

So, coordinates of point $D=\left(\frac{5}{3},-\frac{4}{3}\right)$

Also, point B is point of intersection of 7x + 2y = 0 & 11x + 7y = 9

So, coordinates of point B = $\left(-\frac{2}{3},\frac{7}{3}\right)$

diagonals of parallelogram intersect at middle

let middle point of B,D

$$\Rightarrow\left(\frac{\frac{5}{3}-\frac{2}{3}}{2},\frac{\frac{-4}{3}+\frac{7}{3}}{2}\right)=\left(\frac{1}{2},\frac{1}{2}\right)$$

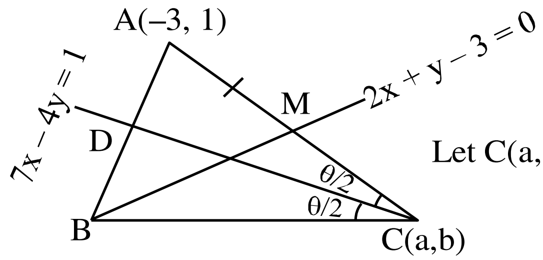
equation of diagonal AC

$$\Rightarrow(y-0)=\frac{\frac{1}{\alpha}-0}{\frac{1}{\alpha}-0}(\pi-0)$$

y = x

diagonal AC passes through (2, 2).

6.



Let C(a, b)

$\therefore$ $M\left(\frac{a-3}{2},\frac{b+1}{2}\right)$ lies on 2x + y – 3 = 0

$\Rightarrow$ 2a + b = 11 .... (i)

$\because$ C lies on 7x – 4y = 1

$\Rightarrow$ 7a – 4b = 1 ......(ii)

$\therefore$ by (i) and (ii) : a = 3, b = 5

$\Rightarrow$ C(3, 5)

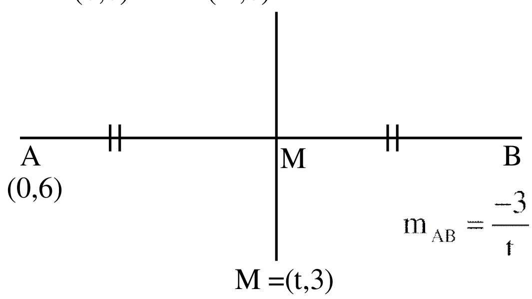
$\therefore$ $m_{AC} = 2/3$

Also, $m_{CD} = 7/4$

$$\Rightarrow\ \tan\frac{\theta}{2}=\left|\frac{\frac{2}{3}-\frac{4}{4}}{1+\frac{14}{12}}\right|\Rightarrow\ \tan\frac{\theta}{2}=\frac{1}{2}$$

$$\Rightarrow\ \tan\theta=\frac{2.\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$$

7. A(0,6) and B(2t,0)



Perpendicular bisector of AB is

$$(y-3)=\frac{t}{3}(x-t)$$

So, $C=\left(0,3-\frac{t^2}{3}\right)$

Let P be (h,k)

$$h=\frac{t}{2};k=\left(3-\frac{t^2}{6}\right)$$

$$\Rightarrow k=3-\frac{4h^2}{6}\Rightarrow 2x^2+3y-9=0$$ option (3)

8. $\left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| = 1$

$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 2$

$\Rightarrow a\,(2b + 1 - b) - 0 + 1\,(b^2 - 0) = \pm 2$

$\Rightarrow a = \frac{\pm 2 - b^2}{b+1}$

$\therefore\ a = \frac{2-b^2}{b+1}$ and $a = \frac{-2-b^2}{b+1}$

sum of possible values of 'a' is

$= \frac{-2b^2}{a+1}$ Ans.

9. First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

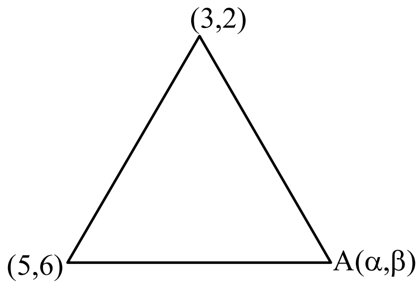
$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$ …(i)

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$ …(ii)

Hence $4p^2 + q^2 = k^2$ (From (i) & (ii))

10.



$\left|\frac{1}{2}\begin{vmatrix} 5 & 6 & 1 \\ 3 & 2 & 1 \\ \alpha & \beta & 1 \end{vmatrix}\right| = 12$

$4\alpha - 2\beta = \pm 24 + 8$

$\Rightarrow 4\alpha - 2\beta = +24 + 8 \Rightarrow 2\alpha - \beta = 16$

$2x - y - 16 = 0$ …(1)

$\Rightarrow 4\alpha - 2\beta = -24 + 8 \Rightarrow 2\alpha - \beta = -8$

$2x - y + 8 = 0$ …(2)

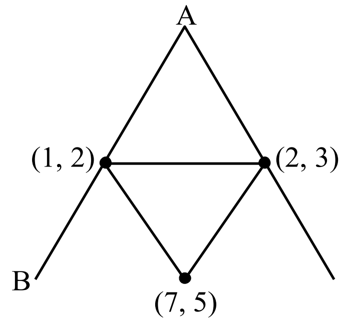
perpendicular distance of (1) from (0, 0)

$\left|\frac{0-0-16}{\sqrt{5}}\right| = \frac{16}{\sqrt{5}}$

perpendicular distance of (2) from (0, 0) is

$\left|\frac{0-0+8}{\sqrt{5}}\right| = \frac{8}{\sqrt{5}}$

11. intersection point of give lines are (1, 2), (7, 5), (2,3)



$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & 5 & 1 \\ 2 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}[1(5-3) - 2(7-2) + 1(21-10)]$

$= \frac{1}{2}[2 - 10 + 11]$

$\Delta DEF = \frac{1}{2}(3) = \frac{3}{2}$

$\Delta ABC = 4\ \Delta DEF = 4\left(\frac{3}{2}\right) = 6$

12. $m = -\frac{1}{\sqrt{3}}, c = 2$

(1) $c = a\sqrt{1+m^2}$

$c = \sqrt{7}\,\frac{2}{\sqrt{3}}$ (incorrect)

(2) $c = \frac{a}{m} = \frac{\frac{1}{24\sqrt{3}}}{\frac{-1}{\sqrt{3}}} = -\frac{1}{24}$ (incorrect)

(3) $c = \sqrt{a^2m^2 - b^2}$

$c = \sqrt{\frac{9}{2}\cdot\frac{1}{3} - \frac{1}{2}} = 1$ (incorrect)

(4) $c = \sqrt{a^2m^2 + b^2}$

$c = \sqrt{9\cdot\frac{1}{3} + 1} = 2$ (correct)

13. Let point is (h, k)

So, $\sqrt{(h-5)^2 + k^2} = 3\sqrt{(h+5)^2 + k^2}$

$8x^2 + 8y^2 + 100\,x + 200 = 0$

$x^2 + y^2 + \frac{25}{2}x + 25 = 0$

$r^2 = \frac{(25)^2}{4^2} - 25$

$4r^2 = \frac{25^2}{4} - 100$

$4r^2 = 156.25 - 100$

$4r^2 = 56.25$

After round of $4r^2 = 56$

14. Let the line be $y = mx + c$

x-intercept : $-\frac{c}{m}$

y-intercept : c

A.M of reciprocals of the intercepts :

$\frac{-\frac{m}{c} + \frac{1}{c}}{2} = \frac{1}{4} \Rightarrow 2(1-m) = c$

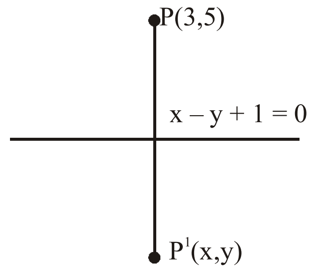
line : $y = mx + 2(1 - m) = c$

$\Rightarrow$ $(y - 2) - m(x - 2) = 0$

$\Rightarrow$ line always passes through (2, 2)

Ans. 4

15.

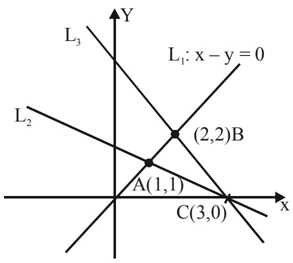


$\frac{x-3}{1} = \frac{y-5}{-1} = -2\left(\frac{3-5+1}{1+1}\right)$

So, $x = 4, y = 4$

Hence, $(x - 2)^2 + (y - 4)^2 = 4$

16.



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : x + y = 6$

on solving L_1 and L_2 :

y = L and x = 1

L_1 and L_3 :

x = 2

y = 2

L_2 and L_3 :

x + y = 3

2x + y = 6

x = 3

y = 0

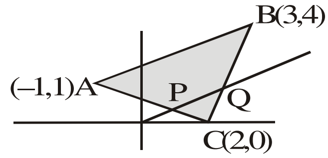
$AC = \sqrt{4+1} = \sqrt{5}$

$BC = \sqrt{4+1} = \sqrt{5}$

$AB = \sqrt{1+1} = \sqrt{2}$

so its an isosceles triangle

17.



$P \equiv (x_1, mx_1)$

$Q \equiv (x_2, mx_2)$

$$A_1 = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{13}{2}$$

$$A_2 = \frac{1}{2}\begin{vmatrix} x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$A_2 = \frac{1}{2}\left|2(mx_1 - mx_2)\right| = m\left|x_1 - x_2\right|$$

$$A_1 = 3A_2 \Rightarrow \frac{13}{2} = 3m\left|x_1 - x_2\right|$$

$$\Rightarrow \left|x_1 - x_2\right| = \frac{16}{6m}$$

AC : x + 3y = 2

BC : y = 4x – 8

$P : x + 3y = 2 \ \& \ y = mx \Rightarrow x_1 = \dfrac{2}{1+3m}$

$Q : y = 4x - 8 \ \& \ y = mx \Rightarrow x_2 = \dfrac{8}{4-m}$

$$\left|x_1 - x_2\right| = \left|\frac{2}{1+3m} - \frac{8}{4-m}\right|$$

$$= \left|\frac{-26m}{(1+3m)(4-m)}\right| = \frac{26m}{(3m+1)|m-4|}$$

$$= \frac{26m}{(3m+1)(4-m)}$$

$$\left|x_1 - x_2\right| = \frac{13}{6m}$$

$$\frac{26m}{(3m+1)(4-m)} = \frac{13}{6m}$$

$\Rightarrow 12m^2 = -(3m + 1)(m - 4)$

$\Rightarrow 12m^2 = -(3m^2 - 11m - 4)$

$\Rightarrow 15m^2 - 11m - 4 = 0$

$\Rightarrow 15m^2 - 15m + 4m - 4 = 0$

$\Rightarrow (15m + 4)(m - 1) = 0$

$\Rightarrow m = 1$

18. Since orthocentre and circumcentre both lies on y-axis

$\Rightarrow$ Centroid also lies on y-axis

$\Rightarrow \Sigma\cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

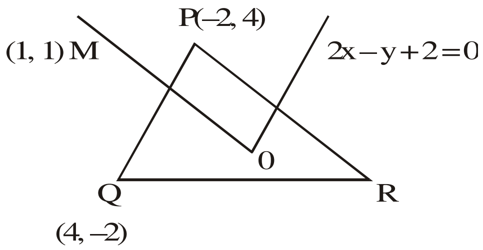
$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$\therefore \dfrac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}$

$= \dfrac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\cos\gamma}$

$= 12$

19.



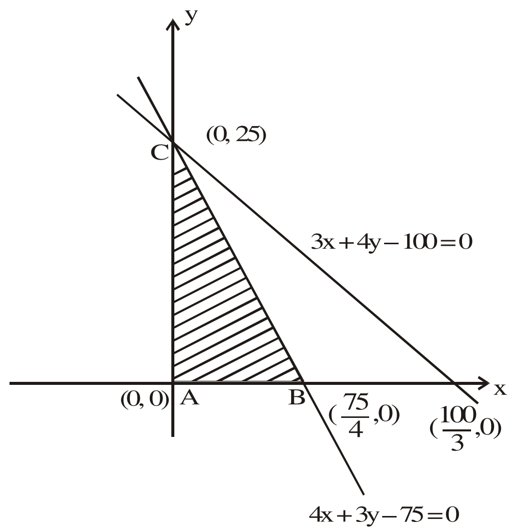
Equation of perpendicular bisector of PR is

$y = x$

Solving with $2x - y + 2 = 0$ will give

$(-2, 2)$

20.



$z = 6xy + y^2 = y(6x + y)$

$3x + 4y \le 100$ .....(i)

$4x + 3y \le 75$ ......(ii)

$x \ge 0$

$y \ge 0$

$x \le \dfrac{75 - 3y}{4}$

$Z = y(6x + y)$

$Z \le y\left(6.\left(\dfrac{75 - 3y}{4}\right) + y\right)$

$Z \le \dfrac{1}{2}(225y - 7y^2) \le \dfrac{(225)^2}{2 \times 4 \times 7}$

$= \dfrac{50625}{56}$

≈ 904.0178

≈ 904.02

It will be attained at $y = \dfrac{225}{14}$

21. $3x + 4y = 9$

$y = mx + 1$

$\Rightarrow 3x + 4mx + 4 = 9$

$\Rightarrow (3 + 4m)x = 5$

$\Rightarrow$ x will be an integer when

$3 + 4m = 5, -5, 1, -1$

$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$

so, number of integral values of m is 2

22. $y = mx + c$

$3 = m + c$

$\sqrt{2} = \left|\frac{m - 3\sqrt{2}}{1 + 3\sqrt{2}m}\right|$

$= 6m + \sqrt{2} = m - 3\sqrt{2}$

$= \sin = -4\sqrt{2} \rightarrow m = \frac{-4\sqrt{2}}{5}$

$= \; = 6m - \sqrt{2} = m - 3\sqrt{2}$

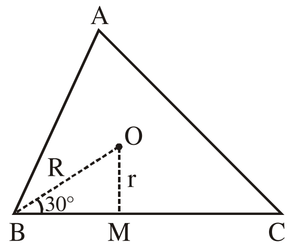
$= 7m - 2\sqrt{2} \rightarrow m = \frac{2\sqrt{2}}{7}$

According to options take $m = \frac{-4\sqrt{2}}{5}$

So $y = \frac{-4\sqrt{2}x}{5} + \frac{3 + 4\sqrt{2}}{5}$

$4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

23.



$r = OM = \frac{3}{\sqrt{2}}$

& $\sin 30° = \frac{1}{2} = \frac{r}{R} \Rightarrow R = \frac{6}{\sqrt{2}}$

$\therefore r + R = \frac{9}{\sqrt{2}}$