STATISTICS

1. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:
(1) 10, 11 (2) 3, 18 (3) 8, 13 (4) 1, 20

2. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a-b|$ is equal to :
(1) 9 (2) 11 (3) 7 (4) 1

3. Consider the following frequency distribution :

Class :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then the value $(a-b)^2$ is equal to _______.

4. Consider the following frequency distribution :

class :	10–20	20–30	30–40	40–50	50–60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha-\beta|$ is equal to _________.

5. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is :
(1) 8 (2) 6 (3) 4 (4) 5

6. If the mean and variance of the following data : 6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to :
(1) 24 (2) 12 (3) 32 (4) 16

7. Let the mean and variance of the frequency distribution

x :	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f :	4	4	α	β

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be :
(1) 4 (2) 5 (3) $\frac{17}{3}$ (4) $\frac{16}{3}$

8. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :
(1) (11, 26) (2) (10.5, 25)
(3) (11, 25) (4) (10.5, 26)

9. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x – y is _______.

10. Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to ________.

11. The probability distribution of random variable X is given by :

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p = P(1 < X < 4 \mid X < 3)$. If $5p = \lambda K$, then λ equal to ________.

12. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to _________.

13. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :
(1) $\frac{92}{5}$ (2) $\frac{134}{5}$ (3) $\frac{536}{25}$ (4) $\frac{112}{5}$

14. If the variance of 10 natural numbers 1, 1, 1,..., 1, k is less than 10, then the maximum possible value of k is ______.

15. Let $X_1, X_2, ..., X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i-\alpha)=36$ and $\sum_{i=1}^{18}(X_i-\beta)^2=90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha-\beta|$ is ______.

16. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _________.

17. Consider three observations a, b and c such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d, then which of the following is true ?

(1) $b^2 = 3(a^2 + c^2) + 9d^2$

(2) $b^2 = a^2 + c^2 + 3d^2$

(3) $b^2 = 3(a^2 + c^2 + d^2)$

(4) $b^2 = 3(a^2 + c^2) - 9d^2$

18. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ______.

19. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is_____.

20. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 425 (2) 650 (3) 250 (4) 925

SOLUTION

1. Official Ans. by NTA (1)

Sol. Let other two numbers be a, (21 – a)

Now,

$$10.25 = \frac{\left(4+16+25+49+a^2+(21-a)^2\right)}{6} - (6.5)^2$$

(Using formula for variance)

$\Rightarrow 6(10.25) + 6(6.5)^2 = 94 + a^2 + (21 - a)^2$

$\Rightarrow a^2 + (21 - a)^2 = 221$

$\therefore$ a = 10 and (21 – a) = 21 – 10 = 11

So, remaining two observations are 10, 11.

$\Rightarrow$ Option (1) is correct.

2. Official Ans. by NTA (4)

Sol. $$10 = \frac{7+10+11+15+a+b}{6}$$

$\Rightarrow a + b = 17$ .....(i)

$$\frac{20}{3} = \frac{7^2+10^2+11^2+15^2+a^2+b^2}{6} - 10^2$$

$a^2 + b^2 = 145$ ......(ii)

Solve (i) and (ii) a = 9, b = 8 or a = 8, b = 9

$|a - b| = 1$

3. Official Ans. by NTA (4)

Sol.

Class	**Frequency**	**x_i**	**$f_i x_i$**
0-6	a	3	3a
6-12	b	9	9b
12-18	12	15	180
18-24	9	21	189
24-30	5	27	135
	N=(26+a+b)		**(504+3a+9b)**

$$\text{Mean} = \frac{3a+9b+180+189+135}{a+b+26} = \frac{309}{22}$$

$\Rightarrow 66a + 198b + 11088 = 309a + 309b + 8034$

$\Rightarrow 243a + 111b = 3054$

$\Rightarrow \boxed{81a+37b=1018} \to (1)$

$$\text{Now, Median} = 12 + \frac{\frac{a+b+26}{2} - (a+b)}{12} \times 6 = 14$$

$$\Rightarrow \frac{13}{2} - \left(\frac{a+b}{4}\right) = 2$$

$$\Rightarrow \frac{a+b}{4} = \frac{9}{2}$$

$\Rightarrow \boxed{a+b=18} \to (2)$

From equation (1) & (2)

a = 8, b = 10

$\therefore (a - b)^2 = (8 - 10)^2$

4. Official Ans. by NTA (164)

Sol. $\because$ Sum of frequencies = 584

$\Rightarrow \alpha + \beta = 390$

Now, Median is at $\frac{584}{2} = 292^{th}$

$\because$ Median = 45 (lies in class 40 – 50)

$\Rightarrow \alpha + 110 + 54 + 15 = 292$

$\Rightarrow \alpha = 113, \beta = 277$

$\Rightarrow |\alpha - \beta| = 164$

5. Official Ans. by NTA (3)

Sol. $n_1 = 100$ m = 250

$\bar{X}_1 = 15$ $\bar{X} = 15.6$

$V_1(x) = 9$ Var(x) = 13.44

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}\left(\bar{x}_1 - \bar{x}_2\right)^2$$

$n_2 = 150$, $\bar{x}_2 = 16$, $V_2(x) = \sigma_2$

$$13.44 = \frac{100\times 9 + 150\times\sigma_2^2}{250} + \frac{100\times 150}{(250)^2}\times 1$$

$\Rightarrow \sigma_2^2 = 16 \Rightarrow \sigma_2 = 4$

6. Official Ans. by NTA (4)

Sol. Mean $= \frac{6+10+7+13+a+12+b+12}{8} = 9$

60 + a + b = 72

a + b = 12 …(1)

variance $= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{37}{4}$

$\Sigma x_i^2 = 6^2 + 10^2 + 7^2 + 13^2 + a^2 + b^2 + 12^2 + 12^2$

$= a^2 + b^2 + 642$

$\frac{a^2+b^2+642}{8} - (9)^2 = \frac{37}{4}$

$\frac{a^2+b^2}{8} + \frac{321}{4} - 81 = \frac{37}{4}$

$\frac{a^2+b^2}{8} = 81 + \frac{37}{4} - \frac{321}{4}$

$\frac{a^2+b^2}{8} = 81 - 71$

$\therefore a^2 + b^2 = 80$ …(2)

From (1) $a^2 + b^2 + 2ab = 144$

$80 + 2ab = 144 \quad \therefore 2ab = 64$

$(a - b)^2 = a^2 + b^2 - 2ab = 80 - 64 = 16$

7. Official Ans. by NTA (3)

Sol. Given $32 + 8\alpha + 9\beta = (8 + \alpha + \beta) \times 6$

$\Rightarrow 2\alpha + 3\beta = 16$ …(i)

Also, $4 \times 16 + 4 \times \alpha + 9\beta = (8 + \alpha + \beta) \times 6.8$

$\Rightarrow 640 + 40\alpha + 90\beta = 544 + 68\alpha + 68\beta$

$\Rightarrow 28\alpha - 22\beta = 96$

$\Rightarrow 14\alpha - 11\beta = 48$ …(ii)

from (i) & (ii)

$\alpha = 5$ & $\beta = 2$

so, new mean $= \frac{32+35+18}{15} = \frac{85}{15} = \frac{17}{3}$

8. Official Ans. by NTA (4)

Sol. Given :

Mean $(\bar{x}) = \frac{\Sigma x_i}{20} = 10$

or $\Sigma x_i = 200$ (incorrect)

or $200 - 25 + 35 = 210 = \Sigma x_i$ (Correct)

Now correct $\bar{x} = \frac{210}{20} = 10.5$

again given S.D = 2.5 (σ)

$\sigma^2 = \frac{\Sigma x_i{}^2}{20} - (10)^2 = (2.5)^2$

or $\Sigma x_i{}^2 = 2125$ (incorrect)

or $\Sigma x_i{}^2 = 2125 - 25^2 + 35^2$

= 2725 (Correct)

$\therefore$ correct $\sigma^2 = \frac{2725}{20} - (10.5)^2$

$\underline{\sigma}^2 = 26$

or $\sigma = 26$

$\therefore \underline{\alpha} = 10.5, \beta = 26$

9. Official Ans. by NTA (12)

Sol. $5 = \frac{3+7+x+y}{4} \Rightarrow x + y = 10$

Var(x) = 10 = $\frac{3^2+7^2+x^2+y^2}{4} - 25$

$140 = 49 + 9 + x^2 + y^2$

$x^2 + y^2 = 82$

x + y = 10

$\Rightarrow$ (x,y) = (9,1)

Four numbers are 21,9,10,8

Mean = $\frac{48}{4} = 12$

10. Official Ans. by NTA (13)

Sol. $\frac{n^2-1}{12} = 14 \Rightarrow n = 13$

11. Official Ans. by NTA (30)

Sol. $\sum P(X)=1 \Rightarrow k+2k+2k+3k+k=1$

$\Rightarrow k = \frac{1}{9}$

Now, $p = P\left(\frac{kX<4}{X<3}\right) = \frac{P(X=2)}{P(X<3)} = \frac{\frac{2k}{9k}}{\frac{k}{9k}+\frac{2k}{9k}} = \frac{2}{3}$

$\Rightarrow p = \frac{2}{3}$

Now, $5p = \lambda k$

$\Rightarrow (5)\left(\frac{2}{3}\right) = \lambda(1/9)$

$\Rightarrow \lambda = 30$

12. Official Ans. by NTA (25)

Sol. $\sigma_b^2 = 2$ (variance of boys) n_1 = no. of boys

$\bar{x}_b = 12$ n_2 = no. of girls

$\sigma_g^2 = 2$

$\bar{x}_g = \frac{50\times15-12\times\sigma_b}{30} = \frac{750-12\times20}{30} = 17 = \mu$

variance of combined series

$$\sigma^2 = \frac{n_1\sigma_b^2+n_2\sigma_g^2}{n_1+n_2}+\frac{n_1\cdot n_2}{(n_1+n_2)^2}(\bar{x}_b-\bar{x}_g)^2$$

$$\sigma^2 = \frac{20\times2+30\times2}{20+30}+\frac{20\times30}{(20+30)^2}(12-17)^2$$

$\sigma^2 = 8.$

$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$

13. Official Ans. by NTA (3)

Sol. Let 8, 16, x_1, x_2, x_3, x_4, x_5 be the observations.

Now $\frac{x_1+x_2+\ldots+x_5+14}{7} = 8$

$\Rightarrow \sum_{i=1}^{5} x_i = 42$ …(1)

Also $\frac{x_1^2+x_2^2+\ldots x_5^2+8^2+6^2}{7} - 64 = 16$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 560 - 100 = 460$ …(2)

So variance of $x_1, x_2, \ldots, x_5$

$= \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{2300-1764}{25} = \frac{536}{25}$

14. Official Ans. by NTA (11)

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$= \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$90 + 10k^2 - 81 - k^2 - 18k < 1000$

$9k^2 - 18k - 991 < 0$

$k^2 - 2k < \frac{991}{9}$

$(k-1)^2 < \frac{1000}{9}$

$\frac{-10\sqrt{10}}{3} < k-1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

$k \le 11$

Maximum value of k is 11.

15. Official Ans. by NTA (4)

Sol. $\sum_{i=1}^{18}(x_i-\alpha) = 36, \sum_{i=1}^{18}(x_i-\beta)^2 = 90$

$\Rightarrow \sum_{i=1}^{18} x_i = 18(\alpha+2), \sum_{i=1}^{18} x_i^2 - 2\beta\sum_{i=1}^{18} x_i + 18\beta^2 = 90$

Hence $\sum x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$

Given $\frac{\sum x_i^2}{18} - \left(\frac{\sum x_i}{18}\right)^2 = 1$

$\Rightarrow 90 - 18\beta^2 + 36\beta(\alpha+2) - 18(\alpha+2)^2 = 18$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4\alpha - 4 = 1$

$\Rightarrow (\alpha-\beta)^2 + 4(\alpha-\beta) = 0 \Rightarrow |\alpha-\beta| = 0$ or 4

As α and β are distinct $|\alpha-\beta| = 4$

16. Official Ans by NTA (5)

Sol. $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1n_2}{(n_1+n_2)}(\bar{x}_1 - \bar{x}_2)^2$

$n_1 = 10$, $n_2 = n$, $\sigma_1^2 = 2$, $\sigma_2^2 = 1$

$\bar{x}_1 = 2$, $\bar{x}_2 = 3$, $\sigma^2 = \frac{17}{9}$

$\frac{17}{9} = \frac{10\times 2 + n}{n+10} + \frac{10n}{(n+10)^2}(3-2)^2$

$\Rightarrow \frac{17}{9} = \frac{(n+20)(n+10)+10n}{(n+10)^2}$

$\Rightarrow 17n^2 + 1700 + 340n = 90n + 9(n^2+30n+200)$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$2n^2 - 5n - 25 = 0$

$\Rightarrow (2n+5)(n-5) = 0 \Rightarrow n = \frac{-5}{2}, 5$

$\downarrow$ (Rejected)

Hence n = 5

17. Official Ans. by NTA (4)

Sol. For a, b, c

mean = $\frac{a+b+c}{3} (= \bar{x})$

$b = a + c$

$\Rightarrow \bar{x} = \frac{2b}{3}$ ......(1)

S.D. $(a+2, b+2, c+2)$ = S.D. $(a, b, c) = d$

$\Rightarrow d^2 = \frac{a^2+b^2+c^2}{3} - (\bar{x})^2$

$\Rightarrow d^2 = \frac{a^2+b^2+c^2}{3} - \frac{4b^2}{9}$

$\Rightarrow 9d^2 = 3(a^2+b^2+c^2) - 4b^2$

$\Rightarrow b^2 = 3(a^2+c^2) - 9d^2$

18. Official Ans. by NTA (68)

Sol. Let number be $a_1, a_2, a_3, \ldots\ldots a_{2n}, b_1, b_2, b_3 \ldots b_n$

$\sigma^2 = \frac{\sum a^2 + \sum b^2}{3n} - (5)^2$

$\Rightarrow \sum a^2 + \sum b^2 = 87n$

Now, distribution becomes

$a_1 + 1, a_2 + 1, a_3 + 1, \ldots\ldots a_{2n} + 1, b_1 - 1, b_2 - 1 \ldots b_n - 1$

Variance

$= \frac{\sum(a+1)^2 + \sum(b-1)^2}{3n} - \left(\frac{12n+2n+3n-n}{3n}\right)^2$

$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n}$

$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n} - \left(\frac{16}{3}\right)^2$

$= \frac{87n + 3n + 2(12n) - 2(3n)}{3n} - \left(\frac{16}{3}\right)^2$

$\Rightarrow k = \frac{108}{3} - \left(\frac{16}{5}\right)^2$

$\Rightarrow 9k = 3(108) - (16)^2 = 324 - 256 = 68$

Ans. 68.00

19. Official Ans. by NTA (35)

Sol. $\frac{\sum x_i}{25} = 40$ & $\frac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow 1000 - 60 + N = 975$

$\Rightarrow N = 35$ years

20. Official Ans. by NTA (1)

Sol. Let observations are denoted by x_i for $1 \le i < 2n$

$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a+a+\ldots+a)-(a+a+\ldots+a)}{2n}$

$\Rightarrow \bar{x} = 0$

and $\sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2+a^2+\ldots+a^2}{2n} - 0 = a^2$

$\Rightarrow \sigma_x = a$

Now, adding a constant b then $\bar{y} = \bar{x} + b = 5$

$\Rightarrow b = 5$

and $\sigma_y = \sigma_x$ (No change in S.D.) $\Rightarrow a = 20$

$\Rightarrow a^2 + b^2 = 425$