SOLUTION OF TRIANGLE

1. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

(1) $10+6\sqrt{2}$ (2) $8+2\sqrt{2}$

(3) $6+8\sqrt{3}$ (4) $4+2\sqrt{3}$

2. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

(1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{5}-1}{2}$

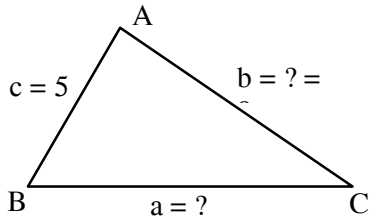
(3) $\frac{\sqrt{2}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

3. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _____.

SOLUTION

1. Official Ans. by NTA (3)

Sol.



As, $\cos B = \frac{3}{5} \Rightarrow \boxed{B = 53°}$

As, $R = 5 \Rightarrow \frac{c}{\sin c} = 2R$

$\Rightarrow \frac{5}{10} = \sin c \Rightarrow \boxed{C = 30°}$

Now, $\frac{b}{\sin B} = 2R \Rightarrow \boxed{b = 2(5)\left(\frac{4}{5}\right) = 8}$

Now, by cosine formula

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$

$\Rightarrow a^2 - 6a - 3g = 0$

$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$

$\Rightarrow \boxed{3 + 4\sqrt{3}}$ (Reject $a = 3 - 4\sqrt{3}$)

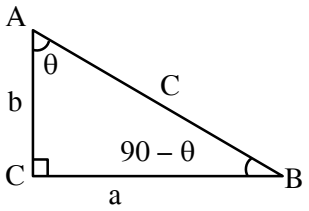
Now, $\Delta = \frac{abc}{4R} = \frac{(3 + 4\sqrt{3})(8)(5)}{4(5)} = 2(3 + 4\sqrt{3})$

$\Rightarrow \Delta = (6 + 8\sqrt{3})$

$\Rightarrow$ Option (3) is correct.

2. Official Ans. by NTA (2)

Sol.



$<A = \theta$

$<B = 90 - \theta$

a = smallest side

$c^2 = a^2 + b^2$

$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$

$\frac{b^2c^2}{a^2} = b^2 + c^2$

Use $a = 2R \sin A = 2R \sin\theta$

$b = 2R \sin B = 2R \sin(90 - \theta) = 2R \cos\theta$

$c = 2R \sin C = 2 \sin 90° = 2R$

$\frac{4R^2\cos^2\theta}{4R^2\sin^2\theta} = 4R^2\cos^2\theta + 4R^2$

$\cos^2\theta = \sin^2\theta \cos^2\theta + \sin^2\theta$

$1 - \sin^2\theta = \sin^2\theta(1 - \sin^2\theta) + \sin^2\theta$

$\sin^2\theta = \frac{3 - \sqrt{5}}{2}$

$\Rightarrow \sin\theta = \frac{\sqrt{5} - 1}{2}$

3. Official Ans by NTA (15)

Sol. $\Delta = \frac{1}{2}.5.12.\sin A = 30$

$\sin A = 1$

$A = 90° \Rightarrow BC = 13$

$BC = 2R = 13$

$r = \frac{\Delta}{S} = \frac{30}{15} = 2$

$2R + r = 15$