RELATION

1. Let **N** be the set of natural numbers and a relation R on **N** be defined by

$R = \{(x,y) \in \mathbf{N} \times \mathbf{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$.

Then the relation R is :

(1) symmetric but neither reflexive nor transitive

(2) reflexive but neither symmetric nor transitive

(3) reflexive and symmetric, but not transitive

(4) an equivalence relation

2. Which of the following is **not** correct for relation R on the set of real numbers ?

(1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.

(2) $(x, y) \in R \Leftrightarrow 0 < |x-y| \leq 1$ is symmetric and transitive.

(3) $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.

(4) $(x, y) \in R \Leftrightarrow |x-y| \leq 1$ is reflexive and symmetric.

3. Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by $(a, b) \simeq (c, d)$, if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 5 (2) 6 (3) 8 (4) 7

4. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive,

(2) R is reflexive, symmetric but not transitive

(3) R is an equivalence relation

(4) R is reflexive, transitive but not symmetric

SOLUTION

1. Official Ans. by NTA (2)

Sol. $x^3 - 3x^2y - xy^2 + 3y^3 = 0$

$\Rightarrow x(x^2 - y^2) - 3y\,(x^2 - y^2) = 0$

$\Rightarrow (x - 3y)\,(x - y)\,(x + y) = 0$

Now, $x = y \;\; \forall (x,y) \in N \times N$ so reflexive

But not symmetric & transitive

See, (3,1) satisfies but (1,3) does not. Also (3,1) & (1,–1) satisfies but (3, –1) does not

2. Official Ans. by NTA (2)

Sol. Note that (1,2) and (2,3) satisfy $0 < |x - y| \leq 1$ but (1,3) does not satisfy it so

$0 \leq |x - y| \leq 1$ is symmetric but not transitive

So, (2) is correct.

3. Official Ans by NTA (4)

Sol. $A = \{2, 3, 4, 5, \ldots, 30\}$

$(a, b) \simeq (c, d) \;\Rightarrow\; ad = bc$

$(4, 3) \simeq (c, d) \;\Rightarrow\; 4d = 3c$

$\Rightarrow \frac{4}{3} = \frac{c}{d}$

$\frac{c}{d} = \frac{4}{3}$ & $c, d \in \{2, 3, \ldots, 30\}$

$(c, d) = \{(4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)\}$

No. of ordered pair = 7

4. Official Ans. by NTA (3)

Sol. A and B are matrices of $n \times n$ order & ARB iff there exists a non singular matrix $P(\det(P) \neq 0)$ such that $PAP^{-1} = B$

For reflexive

$ARA \Rightarrow PAP^{-1} = A$ ...(1) must be true

for P = I, Eq.(1) is true so 'R' is reflexive

For symmetric

$ARB \Leftrightarrow PAP^{-1} = B$ ...(1) is true

for BRA iff $PBP^{-1} = A$ ...(2) must be true

$\because PAP^{-1} = B$

$P^{-1}PAP^{-1} = P^{-1}B$

$IAP^{-1}P = P^{-1}BP$

$A = P^{-1}BP$ ...(3)

from (2) & (3) $PBP^{-1} = P^{-1}BP$

can be true some $P = P^{-1} \Rightarrow P^2 = I \; (\det(P) \neq 0)$

So 'R' is symmetric

For trnasitive

$ARB \Leftrightarrow PAP^{-1} = B$... is true

$BRC \Leftrightarrow PBP^{-1} = C$... is true

now $PPAP^{-1}P^{-1} = C$

$P^2A(P^2)^{-1} = C \Rightarrow ARC$

So 'R' is transitive relation

$\Rightarrow$ Hence R is equivalence