# PROBABILITY

**1.** Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :

(1) $\frac{1}{66}$ (2) $\frac{1}{11}$ (3) $\frac{1}{9}$ (4) $\frac{2}{11}$

**2.** The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$, is :

(1) $\frac{7}{36}$ (2) $\frac{2}{9}$ (3) $\frac{1}{6}$ (4) $\frac{1}{4}$

**3.** Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is $k^2$, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is :

(1) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

(2) greater than $\frac{1}{2}$

(3) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(4) exactly equal to $\frac{1}{2}$

**4.** Four dice are thrown simultaneously and the numbers shown on these dice are recorded in $2 \times 2$ matrices. The probability that such formed matrices have all different entries and are non-singular, is :

(1) $\frac{45}{162}$ (2) $\frac{23}{81}$ (3) $\frac{22}{81}$ (4) $\frac{43}{162}$

**5.** Let 9 distinct balls be distributed among 4 boxes, $B_1$, $B_2$, $B_3$ and $B_4$. If the probability than $B_3$ contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

(1) $\{x \in \mathbf{R} : |x - 3| < 1\}$

(2) $\{x \in \mathbf{R} : |x - 2| \le 1\}$

(3) $\{x \in \mathbf{R} : |x - 1| < 1\}$

(4) $\{x \in \mathbf{R} : |x - 5| \le 1\}$

**6.** Let x be a random variable such that the probability function of a distribution is given by $P(X = 0) = \frac{1}{2}$, $P(X = j) = \frac{1}{3^j}$ (j = 1, 2, 3, …., ∞). Then the mean of the distribution and P(X is positive and even) respectively are :

(1) $\frac{3}{8}$ and $\frac{1}{8}$ (2) $\frac{3}{4}$ and $\frac{1}{8}$

(3) $\frac{3}{4}$ and $\frac{1}{9}$ (4) $\frac{3}{4}$ and $\frac{1}{16}$

**7.** A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is ________.

**8.** The probability that a randomly selected 2-digit number belongs to the set $\{n \in N : (2^n - 2)$ is a multiple of 3$\}$ is equal to

(1) $\frac{1}{6}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

**9.** A student appeared in an examination consisting of 8 true–false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is :

(1) 5 (2) 6 (3) 3 (4) 4

**10.** Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$. The largest value of p, for which P (exactly one of A, B occurs) = $\frac{5}{9}$, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$ (3) $\frac{4}{9}$ (4) $\frac{5}{12}$

**11.** A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is :

(1) $\frac{125}{216}$ (2) $\frac{11}{36}$ (3) $\frac{5}{6}$ (4) $\frac{25}{36}$

**12.** Two fair dice are thrown. The numbers on them are taken as $\lambda$ and $\mu$, and a system of linear equations

$$x + y + z = 5$$
$$x + 2y + 3z = \mu$$
$$x + 3y + \lambda z = 1$$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :

(1) $p = \frac{1}{6}$ and $q = \frac{1}{36}$ (2) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(3) $p = \frac{5}{6}$ and $q = \frac{1}{36}$ (4) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

**13.** When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6} - x$ and its opposite face occurs with probability $\frac{1}{6} + x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is:

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{1}{9}$ (4) $\frac{1}{12}$

**14.** Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

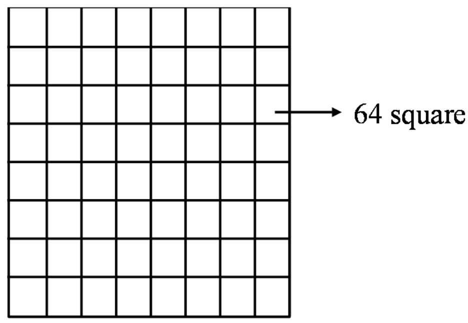
(1) $\frac{1}{8}$ (2) $\frac{5}{8}$ (3) $\frac{5}{16}$ (4) 1

**15.** An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to _________.

**16.** Let S = {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g(1) is :

(1) $\frac{1}{10}$ (2) $\frac{1}{15}$ (3) $\frac{1}{5}$ (4) $\frac{1}{30}$

**17.** Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is :




(1) $\frac{2}{7}$ (2) $\frac{1}{18}$ (3) $\frac{1}{7}$ (4) $\frac{1}{9}$

**18.** Let X be a random variable with distribution.

| x | –2 | –1 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| P(X = x) | $\frac{1}{5}$ | a | $\frac{1}{3}$ | $\frac{1}{5}$ | b |

If the mean of X is 2.3 and variance of X is $\sigma^2$, then $100\sigma^2$ is equal to :

**19.** The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

(1) $\frac{65}{2^7}$ (2) $\frac{65}{2^8}$ (3) $\frac{135}{2^9}$ (4) $\frac{35}{2^7}$

**20.** An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{1}{32}$ (2) $\frac{5}{16}$ (3) $\frac{3}{16}$ (4) $\frac{1}{2}$

**21.** Let $B_i$ ($i$ = 1, 2, 3) be three independent events in a sample space. The probability that only $B_1$ occur is $\alpha$, only $B_2$ occurs is $\beta$ and only $B_3$ occurs is $\gamma$. Let p be the probability that none of the events $B_i$ occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0,1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to______.

**22.** When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

(1) $\frac{1}{27}$ (2) $\frac{3}{4}$ (3) $\frac{1}{8}$ (4) $\frac{3}{8}$

**23.** The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:

(1) $\frac{1}{72}$ (2) $\frac{5}{216}$ (3) $\frac{1}{36}$ (4) $\frac{1}{54}$

**24.** In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is:

(1) $\frac{7}{45}$ (2) $\frac{14}{45}$ (3) $\frac{28}{45}$ (4) $\frac{8}{45}$

**25.** Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

(1) $\frac{2}{9}$ (2) $\frac{122}{297}$ (3) $\frac{97}{297}$ (4) $\frac{1}{5}$

**26.** A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$ (2) $\frac{1}{7}$ (3) $\frac{3}{7}$ (4) $\frac{4}{7}$

**27.** A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{14}}$

**28.** Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

(1) $\frac{9}{56}$ (2) $\frac{4}{9}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

**29.** A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$ (2) $\frac{52}{867}$ (3) $\frac{39}{50}$ (4) $\frac{22}{425}$

**30.** Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

**31.** Let there be three independent events $E_1$, $E_2$ and $E_3$. The probability that only $E_1$ occurs is $\alpha$, only $E_2$ occurs is $\beta$ and only $E_3$ occurs is $\gamma$. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to ______.

**32.** Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{32}{625}$ (2) $\frac{80}{243}$ (3) $\frac{40}{243}$ (4) $\frac{128}{625}$

## SOLUTION

# PROBABILITY

**1. Official Ans. by NTA (2)**

**Sol.** AAEIIMNNOTX

--------M----------

Total words with M at fourth Place = $\frac{10!}{2!2!2!}$

Total words = $\frac{11!}{2!2!2!}$

Required probability = $\frac{10!}{11!} = \frac{1}{11}$

**2. Official Ans. by NTA (2)**

**Sol.** $D < 0$

$\Rightarrow 4(a + 4)^2 - 4(-5a + 64) < 0$

$\Rightarrow a^2 + 16 + 8a + 5a - 64 < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\Rightarrow (a + 16)(a - 3) < 0$

$\Rightarrow a \in (-16, 3)$

$\therefore$ Possible a : $\{-5, -4, ......., 3\}$

$\therefore$ Required probability = $\frac{8}{36}$

$= \frac{2}{9}$

**3. Official Ans. by NTA (2)**

**Sol.** $P(\bar{A}\cap B) + P(A\cap\bar{B}) = 1-k$

$P(\bar{A}\cap C) + P(A\cap\bar{C}) = 1-2k$

$P(\bar{B}\cap C) + P(B\cap\bar{C}) = 1-k$

$P(A\cap B\cap C) = k^2$

$P(A) + P(B) - 2\,P(A\cap B) = 1-k$ .....(i)

$P(B) + P(C) - 2P(B\cap C) = 1-k$ .....(ii)

$P(C) + P(A) - 2P(A\cap C) = 1-2k$ .....(iii)

(1) + (2) + (3)

$P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C)$

$-P(C\cap A) = \frac{-4k+3}{2}$

So

$P(A\cup B\cup C) = \frac{-4k+3}{2} + k^2$

$P(A\cup B\cup C) = \frac{2k^2 - 4k + 3}{2}$

$= \frac{2(k-1)^2 + 1}{2}$

$P(A\cup B\cup C) > \frac{1}{2}$

**4. Official Ans. by NTA (4)**

**Sol.** $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $\quad |A| = ad - bc$

Total case = $6^4$

For non-singular matrix $|A| \neq 0 \Rightarrow ad - bc \neq 0$

$\Rightarrow ad \neq bc$

And a, b, c, d are all different numbers in the set

$\{1, 2, 3, 4, 5, 6\}$

Now for ad = bc

(i) $6 \times 1 = 2 \times 3$

$\left.\begin{array}{l} \Rightarrow a=6,\ b=2,\ c=3,\ d=1 \\ \text{or } a=1,\ b=2,\ c=3,\ d=6 \\ \vdots \end{array}\right\}$ 8 such cases

(ii) $6 \times 2 = 3 \times 4$

$\left.\begin{array}{l} \Rightarrow a=6,\ b=3,\ c=4,\ d=2 \\ \text{or } a=2,\ b=3,\ c=4,\ d=6 \\ \vdots \end{array}\right\}$ 8 such cases

favourable cases

$= {}^6C_4 \underline{|4} - 16$

required probability

$= \frac{{}^6C_4\underline{|4} - 16}{6^4} = \frac{43}{162}$

**5. Official Ans. by NTA (1)**

**Sol.** required probability = $\frac{{}^9C_3.3^6}{4^9}$

$=\frac{{}^9C_3}{27}.\left(\frac{3}{4}\right)^9$

$=\frac{28}{9}.\left(\frac{3}{4}\right)^9 \Rightarrow k=\frac{28}{9}$

Which satisfies $|x-3|<1$

**6. Official Ans. by NTA (2)**

**Sol.** mean = $\Sigma x_ip_i = \sum_{r=0}^{\infty} r.\frac{1}{3^r}=\frac{3}{4}$

p(x is even) = $\frac{1}{3^2}+\frac{1}{3^4}+...\infty$

$=\frac{\frac{1}{9}}{1-\frac{1}{9}}=\frac{1/9}{8/9}=\frac{1}{8}$

**7. Official Ans. by NTA (4)**

**Sol.** P(Head) = $\frac{1}{2}$

1 – P(All tail) $\geq 0.9$

$1-\left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{10}$

$\Rightarrow n_{min} = 4$

**8. Official Ans. by NTA (3)**

**Sol.** Total number of cases = ${}^{90}C_1 = 90$

Now, $2^n - 2 = (3-1)^n - 2$

${}^nC_0 3^n - {}^nC_1.3^{n-1} + ... + (-1)^{n-1}.{}^nC_{n-1}3 + (-1)^n.{}^nC_n - 2$

$3\left(3^{n-1} - n3^{n-2} + ... + (-1)^{n-1}.n\right) + (-1)^n - 2$

$(2^n - 2)$ is multiply of 3 only when n is odd

Req. Probability $=\frac{45}{90}=\frac{1}{2}$

**9. Official Ans. by NTA (1)**

**Sol.** $P(E) < \frac{1}{2}$

$\Rightarrow \sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^{8-r}\left(\frac{1}{2}\right)^r < \frac{1}{2}$

$\Rightarrow \sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^8 < \frac{1}{2}$

$\Rightarrow {}^8C_n + {}^8C_{n+1} + .... + {}^8C_8 < 128$

$\Rightarrow 256 - \left({}^8C_0 + {}^8C_1 + .... + {}^8C_{n-1}\right) < 128$

$\Rightarrow {}^8C_0 + {}^8C_1 + .... + {}^8C_{n-1} > 128$

$\Rightarrow n-1 \geq 4$

$\Rightarrow n \geq 5$

**10. Official Ans. by NTA (4)**

**Sol.** P(Exactly one of A or B)

$= P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right) = \frac{5}{9}$

$= P(A)P\left(\overline{B}\right) + P\left(\overline{A}\right)P(B) = \frac{5}{9}$

$\Rightarrow P(A)(1-P(B)) + (1-P(A))P(B) = \frac{5}{9}$

$\Rightarrow p(1-2p) + (1-p)\,2p = \frac{5}{9}$

$\Rightarrow 36p^2 - 27p + 5 = 0$

$\Rightarrow p = \frac{1}{3}$ or $\frac{5}{12}$

$p_{max} = \frac{5}{12}$

**11. Official Ans. by NTA (4)**

**Sol.** $P(x \geq 5 | x > 2) = \dfrac{P(x \geq 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \ldots\ldots + \infty}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \ldots\ldots + \infty}$$

$$\frac{\dfrac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1-\frac{5}{6}}}{\dfrac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1-\frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

**12. Official Ans. by NTA (2)**

**Sol.** $D \neq 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 5$

For no solution $D = 0 \Rightarrow \lambda = 5$

$$D_1 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} \neq 0 \Rightarrow \mu \neq 3$$

$$p = \frac{5}{6}$$

$$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

Option (2)

**13. Official Ans. by NTA (2)**

**Sol.** Probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces

$$= 2\left[\left(\frac{1}{6} - x\right)\left(\frac{1}{6} + x\right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}\right]$$

$$\Rightarrow 2\left[\left(\frac{1}{6} - x\right)\left(\frac{1}{6} + x\right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}\right] = \frac{13}{96}$$

(given)

$$x = \frac{1}{8}$$

**14. Official Ans. by NTA (3)**

**Sol.** C – I '0' Head

T T T $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{1}{64}$

C – II '1' head

H T T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C – III '2' Head

H H T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C–IV '3' Heads

H H H $\left(\frac{1}{8}\right)\left(\frac{1}{8}\right) = \frac{1}{64}$

Total probability $= \dfrac{5}{16}$.

**15. Official Ans. by NTA (28)**

**Sol.** $I_1$ = first unit is functioning

$I_2$ = second unit is functioning

$P(I_1) = 0.9$, $P(I_2) = 0.8$

$P(\bar{I}_1) = 0.1$, $P(\bar{I}_2) = 0.2$

$$P = \frac{0.8 \times 0.1}{0.1 \times 0.2 + 0.9 \times 0.2 + 0.1 \times 0.8} = \frac{8}{28}$$

$$98P = \frac{8}{28} \times 98 = 28$$

**16. Official Ans. by NTA (1)**

**Sol.** g(3) = 2g(1) can be defined in 3 ways

number of onto functions in this condition = $3 \times 4!$

Total number of onto functions = 6!

Required probability = $\frac{3\times4!}{6!} = \frac{1}{10}$

**17. Official Ans. by NTA (2)**

**Sol.** Total ways of choosing square = $^{64}C_2$

$= \frac{64\times63}{2\times1} = 32\times63$

ways of choosing two squares having common side = 2 (7 × 8) = 112

Required probability = $\frac{112}{32\times63} = \frac{16}{32\times9} = \frac{1}{18}$

Ans. (2)

**18. Official Ans. by NTA (781)**

**Sol.**

| | | | | | |
|---|---|---|---|---|---|
| x | –2 | –1 | 3 | 4 | 6 |
| P(X = x) | $\frac{1}{5}$ | a | $\frac{1}{3}$ | $\frac{1}{5}$ | b |

$\overline{X} = 2.3$

$-a + 6b = \frac{9}{10}$ ........ (1)

$\sum P_i = \frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$

$a + b = \frac{4}{15}$ ........ (2)

From equation (1) and (2)

$a = \frac{1}{10}, \quad b = \frac{1}{6}$

$\sigma^2 = \Sigma p_i x_i^2 - (\overline{X})^2$

$\frac{1}{5}(4) + a(1) + \frac{1}{3}(9) + \frac{1}{5}(16) + b(36) - (2.3)^2$

$= \frac{4}{5} + a + 3 + \frac{16}{5} + 36b - (2.3)^2$

$= 4 + a + 3 + 36b - (2.3)^2$

$= 7 + a + 36b - (2.3)^2$

$= 7 + \frac{1}{10} + 6 - (2.3)^2$

$= 13 + \frac{1}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{131}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{1310 - (23)^2}{100}$

$= \frac{1310 - 529}{100}$

$\sigma^2 = \frac{781}{100}$

$100\sigma^2 = 781$

**19. Official Ans. by NTA (3)**

**Sol.** Total subsets = $2^5 = 32$

Probability = $\frac{^5C_2 \times 3^3}{32\times32} = \frac{10\times27}{12^{10}} = \frac{135}{2^9}$

**20. Official Ans. by NTA (4)**

**Sol.** $^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow {}^nC_2 = {}^nC_3$

$\Rightarrow n = 5$

Probability of getting an odd number for odd number of times is

$^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5 = \frac{1}{2^5}(5+10+1)$

$= \frac{1}{2}$

**21. Official Ans. by NTA (6)**

**Sol.** Let $P(B_1) = p_1$, $P(B_2) = p_2$, $P(B_3) = p_3$

given that $p_1(1 - p_2)(1 - p_3) = \alpha$ .....(i)

$p_2(1 - p_1)(1 - p_3) = \beta$ .....(ii)

$p_3(1 - p_1)(1 - p_2) = \gamma$ .....(iii)

and $(1 - p_1)(1 - p_2)(1 - p_3) = p$ .....(iv)

$\Rightarrow \frac{p_1}{1-p_1} = \frac{\alpha}{p}, \frac{p_2}{1-p_2} = \frac{\beta}{p} \ \& \ \frac{p_3}{1-p_3} = \frac{\gamma}{p}$

Also $\beta = \frac{\alpha p}{\alpha + 2p} = \frac{3\gamma p}{p - 2\gamma}$

$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma$

$\Rightarrow \alpha p - 6p\gamma = 5\alpha\gamma$

$\Rightarrow \frac{p_1}{1-p_1} - \frac{6p_3}{1-p_3} = \frac{5p_1p_3}{(1-p_1)(1-p_3)}$

$\Rightarrow p_1 - 6p_3 = 0$

$\Rightarrow \frac{p_1}{p_3} = 6$

**22. Official Ans. by NTA (3)**

**Sol.** Required probability = $\left(\frac{2}{3} \times \frac{3}{4}\right)^3 = \frac{1}{8}$

**23. Official Ans. by NTA (2)**

**Sol.** $ax^2 + bx + c = 0$

For equal roots D = 0

$\Rightarrow b^2 = 4ac$

**Case I :** ac = 1

(a, b, c) = (1, 2, 1)

**Case II :** ac = 4

(a, b, c) = (1, 4, 4)

or (4, 4, 1)

or (2, 4, 2)

**Case III :** ac = 9

(a, b, c) = (3, 6, 3)

Required probability = $\frac{5}{216}$

**24. Official Ans. by NTA (3)**

**Sol.** Consider following events

A : Person chosen is a smoker and non vegetarian.

B : Person chosen is a smoker and vegetarian.

C : Person chosen is a non-smoker and vegetarian.

E : Person chosen has a chest disorder

Given

$P(A) = \frac{160}{400} \ P(B) = \frac{100}{400} \ P(C) = \frac{140}{400}$

$P\left(\frac{E}{A}\right) = \frac{35}{100} \ P\left(\frac{E}{B}\right) = \frac{20}{100} \ P\left(\frac{E}{C}\right) = \frac{10}{100}$

To find

$$P\left(\frac{A}{E}\right) = \frac{P(A)P\left(\frac{E}{A}\right)}{P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right) + P(C).P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{160}{400} \times \frac{35}{100}}{\frac{160}{400} \times \frac{35}{100} + \frac{100}{400} \times \frac{20}{100} + \frac{140}{400} \times \frac{10}{100}}$$

$= \frac{28}{45}$ option (3)

**25. Official Ans. by NTA (3)**

**Sol.** n(s) = n(when 7 appears on thousands place)

+ n(7 does not appear on thousands place)

$= 9 \times 9 \times 9 + 8 \times 9 \times 9 \times 3$

$= 33 \times 9 \times 9$

n(E) = n(last digit 7 & 7 appears once)

+ n(last digit 2 when 7 appears once)

$= 8 \times 9 \times 9 + (9 \times 9 + 8 \times 9 \times 2)$

$\therefore \ P(E) = \frac{8 \times 9 \times 9 + 9 \times 25}{33 \times 9 \times 9} = \frac{97}{297}$

**26. Official Ans. by NTA (3)**

**Sol.** Digits = 3, 3, 4, 4, 4, 5, 5

Total 7 digit numbers = $\frac{7!}{2!2!3!}$

Number of 7 digit number divisible by 2

$\Rightarrow$ last digit = 4

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | 4 |

3 , 3 , 4 , 4 , 5 , 5

Now 7 digit numbers which are divisible by 2

$= \frac{6!}{2!2!2!}$

Required probability = $\frac{\frac{6!}{2!2!2!}}{\frac{7!}{3!2!2!}} = \frac{3}{7}$

**27. Official Ans. by NTA (1)**

**Sol.** Let the coin be tossed n-times

$P(H) = P(T) = \frac{1}{2}$

$P(7 \text{ heads}) = {}^{n}C_7\left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^{7} = \frac{{}^{n}C_7}{2^n}$

$P(9 \text{ heads}) = {}^{n}C_9\left(\frac{1}{2}\right)^{n-9}\left(\frac{1}{2}\right)^{9} = \frac{{}^{n}C_9}{2^n}$

P(7 heads) = P(9 heads)

${}^{n}C_7 = {}^{n}C_9 \Rightarrow n = 16$

$P(2 \text{ heads}) = {}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^{2} = \frac{15\times 8}{2^{16}}$

$P(2 \text{ heads}) = \frac{15}{2^{13}}$

**28. Official Ans by NTA (2)**

**Sol.** Total cases :

$\underline{6}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}$

n(s) = 6 · 6!

Favourable cases :

Number divisible by 3 ≡

Sum of digits must be divisible by 3

**Case-I**

1, 2, 3, 4, 5, 6

Number of ways = 6!

**Case-II**

0, 1, 2, 4, 5, 6

Number of ways = 5·5!

**Case-III**

0, 1, 2, 3, 4, 5

Number of ways = 5·5!

n(favourable) = 6! + 2·5·5!

$P = \frac{6! + 2\cdot 5\cdot 5!}{6\cdot 6!} = \frac{4}{9}$

**29. Official Ans. by NTA (3)**

**Sol.** $E_1$ : Event denotes spade is missing

$P(E_1) = \frac{1}{4};\ P\left(\bar{E}_1\right) = \frac{3}{4}$

A : Event drawn two cards are spade

$$P(A) = \frac{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$= \frac{39}{50}$

**30. Official Ans. by NTA (4)**

**Sol.** $\underset{\text{odd place}}{1}\ \underset{\text{even place}}{0}\ \underset{\text{odd place}}{0}\ \underset{\text{even place}}{1}$

or $\underset{\text{even place}}{1}\ \underset{\text{odd place}}{0}\ \underset{\text{even place}}{0}\ \underset{\text{odd place}}{1}$

$\Rightarrow \left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\right)+\left(\frac{2}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\right)$

$\Rightarrow \frac{1}{9}$

**31. Official Ans. by NTA (6)**

**Sol.** Let $P(E_1) = P_1$ ; $P(E_2) = P_2$ ; $P(E_3) = P_3$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = \alpha = P_1(1-P_2)(1-P_3)$......(1)

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = \beta = (1-P_1)P_2(1-P_3)$......(2)

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = \gamma = (1-P_1)(1-P_2)P_3$......(3)

$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P = (1-P_1)(1-P_2)(1-P_3)$......(4)

Given that, $(\alpha - 2\beta) P = \alpha\beta$

$\Rightarrow (P_1 (1 - P_2) (1 - P_3) - 2 (1 - P_1) P_2 (1 - P_3) P$

$= P_1P_2 (1 - P_1)\ (1 - P_2) (1 - P_3)^2$

$\Rightarrow (P_1 (1 - P_2)\ - 2(1 - P_1) P_2) = P_1P_2$

$\Rightarrow (P_1\ - P_1P_2 - 2P_2 + 2P_1P_2) = P_1P_2$

$\Rightarrow P_1 = 2P_2$ ......(1)

and similarly, $(\beta - 3\gamma)P = 2B\gamma$

$P_2 = 3P_3$ ......(2)

So, $P_1 = 6P_3 \Rightarrow \boxed{\frac{P_1}{P_3} = 6}$

**32. Official Ans. by NTA (1)**

**Sol.** $P(X = 1) = {}^5C_1.p.q^4 = 0.4096$

$P(X = 2) = {}^5C_2.p^2.q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

Now

$P(X = 3) = {}^5C_3.\left(\frac{1}{5}\right)^3.\left(\frac{4}{5}\right)^2 = \frac{10\times16}{125\times25} = \frac{32}{625}$