Mathematical Reasoning

1. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $p \Rightarrow q$

(3) $\sim q \Rightarrow p$ (4) $p \Rightarrow \sim q$

2. Consider the following three statements :

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 = 17.

Then, which of the following statements is correct ?

(1) (A) is false, but (B) and (C) are true

(2) (A) and (C) are true while (B) is false

(3) (A) is true while (B) and (C) are false

(4) (A) and (B) are false while (C) is true

3. Which of the following Boolean expressions is **not** a tautology ?

(1) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(2) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

(3) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

(4) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

4. The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) ~q (2) q (3) p (4) ~p

5. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following:

(1) The match will not be played and weather is not good and ground is wet.

(2) If the match will not be played, then either weather is not good or ground is wet.

(3) The match will be played and weather is not good or ground is wet.

(4) The match will not be played or weather is good and ground is not wet.

6. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to:

(1) $P \vee Q$

(2) $P \wedge \sim Q$

(3) $\sim (P \Rightarrow Q)$

(4) $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

7. Which of the following is the negation of the statement "for all M > 0, there exists $x \in S$ such that $x \geq M$" ?

(1) there exists M > 0, such that x < M for all $x \in S$

(2) there exists $M > 0$, there exists $x \in S$ such that $x \geq M$

(3) there exists $M > 0$, there exists $x \in S$ such that $x < M$

(4) there exists $M > 0$, such that $x \geq M$ for all $x \in S$

8. If the truth value of the Boolean expression $((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)) \rightarrow (p \wedge q)$ is false, then the truth values of the statements p, q, r respectively can be :

(1) T F T (2) F F T

(3) T F F (4) F T F

9. Consider the two statements :

$(S1): (p \rightarrow q) \vee (\sim q \rightarrow p)$ is a tautology.

$(S2): (p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

Then :

(1) only (S1) is true.

(2) both (S1) and (S2) are false.

(3) both (S1) and (S2) are true.

(4) only (S2) is true.

10. The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is :

(1) a tautology (2) equivalent to $p \rightarrow \sim r$

(3) a fallacy (4) equivalent to $q \rightarrow \sim r$

11. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to :

(1) $(p \wedge q) \Rightarrow (r \wedge q)$ (2) $(q \wedge r) \Rightarrow (p \wedge q)$

(3) $(p \wedge q) \Rightarrow (r \vee q)$ (4) $(p \wedge r) \Rightarrow (p \wedge q)$

12. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \,\square\, q)$ is a tautology. Then :

(1) $* = \vee, \square = \vee$ (2) $* = \wedge, \square = \wedge$

(3) $* = \wedge, \square = \vee$ (4) $* = \vee, \square = \wedge$

13. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is :

(1) $p \wedge \sim q \wedge \sim r$ (2) $\sim p \wedge q \wedge \sim r$

(3) $\sim p \wedge q \wedge r$ (4) $p \wedge q \wedge r$

14. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

(1) $\sim (q \rightarrow p)$ (2) $\sim p \rightarrow \sim q$

(3) $\sim (p \rightarrow \sim q)$ (4) $\sim (p \rightarrow q)$

15. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.

(2) (a) and (b) both are tautologies.

(3) (a) is a tautology but not (b).

(4) (b) is a tautology but not (a).

16. The statement among the following that is a tautology is :

(1) $A \vee (A \wedge B)$

(2) $A \wedge (A \vee B)$

(3) $B \rightarrow [A \wedge (A \rightarrow B)]$

(4) $[A \wedge (A \rightarrow B)] \rightarrow B$

17. The contrapositive of the statement "If you will work, you will earn money" is :

(1) You will earn money, if you will not work

(2) If you will earn money, you will work

(3) If you will not earn money, you will not work

(4) To earn money, you need to work

18. Let $F_1(A,B,C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

(1) F_1 and F_2 both are tautologies

(2) F_1 is a tautology but F_2 is not a tautology

(3) F_1 is not tautology but F_2 is a tautology

(4) Both F_1 and F_2 are not tautologies

19. The negative of the statement $\sim p \wedge (p \vee q)$ is

(1) $\sim p \vee q$ (2) $p \vee \sim q$

(3) $\sim p \wedge q$ (4) $p \wedge \sim q$

20. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to :

(1) $A \rightarrow (A \wedge B)$ (2) $A \rightarrow (A \rightarrow B)$

(3) $A \rightarrow (A \leftrightarrow B)$ (4) $A \rightarrow (A \vee B)$

21. Which of the following Boolean expression is a tautology ?

(1) $(p \wedge q) \vee (p \vee q)$ (2) $(p \wedge q) \vee (p \rightarrow q)$

(3) $(p \wedge q) \wedge (p \rightarrow q)$ (4) $(p \wedge q) \rightarrow (p \rightarrow q)$

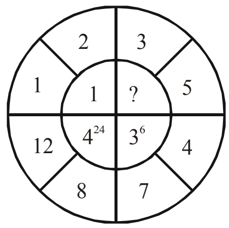
22. If the Boolean expression $(p \wedge q) \circledast (p \otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \vee$ (3) $\vee, \rightarrow$ (4) $\wedge, \rightarrow$

23. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $\sim q \Rightarrow p$ (3) $p \Rightarrow \sim q$ (4) $p \Rightarrow q$

24. The missing value in the following figure is



25. If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

SOLUTION

1. Official Ans. by NTA (2)

Sol.

p	q	~p	~q	$p\wedge\sim q$	$q\vee\sim p$	$(p\wedge\sim q)\Rightarrow(q\vee\sim p)$	$p\Rightarrow q$
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
T	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T

$\therefore\ (p\wedge\sim q)\ \Rightarrow(q\vee\sim p)$

$\equiv p\Rightarrow q$

So, option (2) is correct.

2. Official Ans. by NTA (2)

Sol. Truth Table

P	q	P→q
T	T	T
T	F	F
F	T	T
F	F	T

3. Official Ans. by NTA (4)

Sol. (1) $(p\to q)\vee(\sim q\to p)$

$=(\sim p\vee q)\vee(q\vee p)$

$=(\sim p\vee p)\vee q$

$=t\vee q=t$

(2) $(q\to p)\vee(\sim q\to p)$

$=(\sim q\vee p)\vee(q\vee p)$

$=(\sim q\vee q)\vee p$

$=t\vee p=t$

(3) $(p\to\sim q)\vee(\sim q\to p)$

$=(\sim p\vee\sim q)\vee(q\vee p)$

$=(\sim p\vee p)\vee(\sim q\vee q)$

$=t\vee t=t$

(4) $(\sim q\to q)\vee(\sim q\to p)$

$=(p\vee q)\vee(q\vee p)$

$=(p\vee p)\vee(q\vee p)$

$=p\vee q$

Which is not a tautology.

4. Official Ans. by NTA (4)

Sol. $(p\to q)\wedge(q\to\sim p)$

$\equiv(\sim p\vee q)\wedge(\sim q\vee\sim p)$ $\{p\to q\equiv\sim p\vee q\}$

$\equiv(\sim p\vee q)\wedge(\sim p\vee\sim q)$ {commutative property}

$\equiv\sim p\vee(q\wedge\sim q)$ {distributive property}

$\equiv\sim p$

5. Official Ans. by NTA (3)

Sol. p : weather is food

q : ground is not wet

$\sim(p\wedge q)\equiv\sim p\vee\sim q$

≡ weather is not good or ground is wet

6. Official Ans. by NTA (4)

Sol. Using Truth Table

P	Q	$P\vee Q$	~P	$(P\vee Q)\wedge P$	$(P\vee Q)\wedge\sim P\to Q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

P	Q	~Q	$P\wedge\sim Q$	$P\to Q$	$\sim(P\to Q)$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	F	F	T	F
F	F	T	F	T	F

$\sim(P\to Q)$	$P\wedge\sim Q$	$\sim(P\to Q)\Leftrightarrow P\wedge\sim Q$
F	F	T
T	T	T
F	F	T
F	F	T

7. Official Ans. by NTA (1)

Sol. P : for all $M>0$, there exists $x\in S$ such that $x\geq M$.

$\sim P$: there exists $M>0$, for all $x\in S$

Such that $x<m$

Negation of 'there exsits' is 'for all'.

8. Official Ans. by NTA (3)

Sol.

p	q	r	$\underbrace{p\vee q}_{a}$	$\underbrace{q\to r}_{b}$	$a\wedge b$	~r	$\underbrace{a\wedge b\wedge(\sim r)}_{c}$	$\underbrace{p\wedge q}_{d}$	$c\to d$
T	F	T	T	T	T	F	F	F	T
F	F	T	F	T	F	F	F	F	T
T	F	F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	F	F	T

9. Official Ans. by NTA (3)

Sol. $S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 = q \vee (\sim p \vee p) = qvt = t =$ tautology

$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$

= fallacy

10. Official Ans. by NTA (1)

Sol. $\left(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)\right) \rightarrow r$

$\equiv \left(p \wedge (\sim p \vee q) \vee (\sim q \vee r)\right) \rightarrow r$

$\equiv \left((p \wedge q) \wedge (\sim p \vee r)\right) \rightarrow r$

$\equiv (p \wedge q \wedge r) \rightarrow r$

$\equiv \sim (p \wedge q \wedge r) \vee r$

$\equiv (\sim p) \vee (\sim q) \vee (\sim r) \vee r$

$\Rightarrow$ tautology

11. Official Ans. by NTA (1)

Sol. $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge p) \wedge (p \wedge q)$

$\Rightarrow [\sim (p \wedge q) \vee (p \wedge q)] \wedge (\sim (p \wedge q) \vee$

$(r \wedge p)) \Rightarrow t \wedge [\sim (p \wedge q) \vee (r \wedge p)]$

$\Rightarrow \sim (p \wedge q) \vee (r \wedge p)$

$\Rightarrow (p \wedge q) \Rightarrow (r \wedge p)$

Aliter :

given statement says

" if p and q both happen then

p and q and r will happen"

it Simply implies

" If p and q both happen then

'r' too will happen "

i.e.

" if p and q both happen then r and p too will happen

i.e.

$(p \wedge q) \Rightarrow (r \wedge p)$

12. Official Ans. by NTA (3)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$ is tautology

p	q	$\sim q$	$p \wedge \sim q$	$p \vee q$	$(p \wedge \sim q) \rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

13. Official Ans. by NTA (1)

Sol. $\because \sim(A \Rightarrow B) = A \wedge \sim B$

$\therefore \sim((p \vee r) \Rightarrow (q \vee r))$

$= (p \vee r) \wedge (\sim q \wedge \sim r)$

$= ((p \vee r) \wedge (\sim r)) \wedge (\sim q)$

$= p \wedge (\sim r) \wedge (\sim q)$

14. Official Ans. by NTA (4)

Sol.

p	q	$\sim p$	$\sim q$	$p - q$	$\sim (p \rightarrow q)$	$q \rightarrow p$	$\sim (q \rightarrow p)$
T	T	F	F	T	F	T	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	F	T	F

$p \wedge \sim q$	$\sim p \rightarrow \sim q$	$p \rightarrow \sim q$	$\sim (p \rightarrow \sim q)$
F	T	F	T
T	T	T	F
F	F	T	F
F	T	T	F

$p \wedge \sim q \equiv \sim (p \rightarrow q)$

Option (4)

15. Official Ans. by NTA (2)

Sol. (A)

p	q	$\sim q$	$p \rightarrow q$	$\sim p$	$(\sim q \wedge (p \rightarrow q))$	
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

(B)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Both are tautologies

16. Official Ans. by NTA (4)

Sol. $\left(A\wedge(A\rightarrow B)\right)\rightarrow B$

$=\left(A\wedge(\sim A\vee B)\right)\rightarrow B$

$=\left((A\wedge\sim A)\vee(A\wedge B)\right)\rightarrow B$

$=(A\wedge B)\rightarrow B$

$=\sim(A\wedge B)\vee B$

$=(\sim A\vee\sim B)\vee B$

$=T$

17. Official Ans. by NTA (3)

Sol. Constrapositive of $p\rightarrow q$ is $\sim q\rightarrow\sim p$

$\Rightarrow$ If you will not earn money, you will not work. option (3)

18. Official Ans. by NTA (3)

Sol. $F_1 : (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$

$F_2 : (A \vee B) \vee (B \rightarrow \sim A)$

$F_1 : \{(A \wedge \sim B) \vee \sim A\} \vee [(A \vee B) \wedge \sim C]$

$: \{(A \vee \sim A) \wedge (\sim A \vee \sim B)\} \vee [(A \vee B) \wedge \sim C]$

$: \{t \wedge (\sim A \vee \sim B)\} \vee [(A \vee B) \wedge \sim C]$

$: (\sim A \vee \sim B) \vee [(A \vee B) \wedge \sim C]$

$: \underbrace{[(\sim A\vee\sim B)\vee(A\vee B)]}_{t}\wedge[(\sim A\vee\sim B)\wedge\sim C]$

$F_1 : (\sim A \vee \sim B) \wedge \sim C \neq t$ (tautology)

$F_2 : (A \vee B) \vee (\sim B \vee \sim A) = t$ (tautology)

19. Official Ans. by NTA (2)

Sol. $\sim(\sim p \wedge (p \vee q))$

$p \vee (\sim p \wedge \sim q)$

$\underbrace{(p\vee\sim p)}_{t}\wedge(p\vee\sim q)$

$p \vee \sim q$

20. Official Ans. by NTA (4)

Sol. $A \rightarrow (B \rightarrow A)$

$\equiv A \rightarrow (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A)$

$\equiv (\sim A \vee A) \vee \sim B$

$\equiv T \vee \sim B \equiv T$

$\therefore T \vee B = T$

$\equiv (\sim A \vee A) \vee B$

$\equiv \sim A \vee (A \vee B)$

$\equiv A \rightarrow (A \vee B)$

21. Official Ans. by NTA (4)

Sol.

p	q	$p\wedge q$	$p\rightarrow q$	$(p\wedge q)\rightarrow(p\rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \rightarrow (p \rightarrow q)$ is tautology

22. Official Ans. by NTA (1)

Sol. **Option (1)**

$(p\wedge q)\longrightarrow(p\rightarrow q)$

$=\sim(p\wedge q)\vee(\sim p\vee q)$

$=(\sim p\vee\sim q)\vee(\sim p\vee q)$

$=\sim p\vee(\sim q\vee q)$

$=\sim p\vee t$

$=t$

Option (2)

$(p\wedge q)\wedge(p\vee q)=(p\wedge q)$ (Not a tautology)

Option (3)

$(p\wedge q)\vee(p\rightarrow q)$

$=(p\wedge q)\vee(\sim p\vee q)$

$=\sim p\vee q$ (Not a tautology)

Option (4)

$=(p\wedge q)\wedge(\sim p\vee q)$

$=p\wedge q$ (Not a tautology)

Option (1)

23. Official Ans. by NTA (1)

Sol. $\because p\rightarrow q\equiv\sim p\vee q$

So, $*\equiv v$

Thus, $p*(\sim q)\equiv pv(\sim q)$

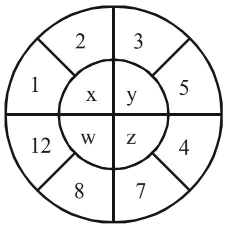
$\equiv q\rightarrow p$

24. Official Ans. by NTA (4 or 16 or 64)

Sol. $x = (2 - 1)^{1!} = 1$

$w = (12 - 8)^{4!} = 4^{24}$

$z = (7 - 4)^{3!} = 3^{6}$



hence $y = (5 - 3)^{2!} = 2^{2}$

25. Official Ans. by NTA (2)

Sol. LHS of all the options are some i.e.

$((P \to Q) \wedge \sim Q)$

$\equiv (\sim P \vee Q) \wedge \sim Q$

$\equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)$

$\equiv \sim P \wedge \sim Q$

(A) $(\sim P \wedge \sim Q) \to Q$

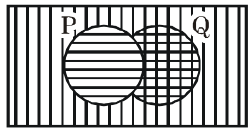
$\equiv \sim(\sim P \wedge \sim Q) \vee Q$

$\equiv (P \vee Q) \vee Q \neq$ tautology

(B) $(\sim P \wedge \sim Q) \to \sim P$

$\equiv \sim(\sim P \wedge \sim Q) \vee \sim P$

$\equiv (P \vee Q) \vee \sim P$



$\Rightarrow$ Tautology

(C) $(\sim P \wedge \sim Q) \to P$

$\equiv (P \vee Q) \vee P \neq$ Tautology

(D) $(\sim P \wedge \sim Q) \to (P \wedge Q)$

$\equiv (P \vee Q) \vee (P \wedge Q) \neq$ Tautology

Aliter :

P	Q	$P \vee Q$	$P \vee Q$	$\sim P$	$(P \vee Q) \vee \sim P$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T