LOGARITHM

1. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$, $x>0$, is

2. The sum of the roots of the equation $x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$, is :

(1) $\log_2 14$ (2) $\log_2 11$

(3) $\log_2 12$ (4) $\log_2 13$

SOLUTION

1. $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$

$\log_{(x+1)}(2x+5)(x+1)+2\log_{(2x+5)}(x+1)=4$

$\log_{(x+1)}(2x+5)+1+2\log_{(2x+5)}(x+1)=4$

Put $\log_{(x+1)}(2x+5)=t$

$t+\frac{2}{t}=3 \Rightarrow t^2-3t+2=0$

$t=1, 2$

$\log_{(x+1)}(2x+5)=1$ & $\log_{(x+1)}(2x+5)=2$

$x+1=2x+3$ & $2x+5=(x+1)^2$

$x=-4$ (rejected)

$x^2=4 \Rightarrow x=2, -2$ (rejected)

So, $x=2$

No. of solution = 1

2. $x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$

$\log_2(2^{x+1})-\log_2(3+2^x)^2+\log_2(10-2^{-x})=0$

$\log_2\left(\frac{2^{x+1}.(10-2^{-x})}{(3+2^x)^2}\right)=0$

$\frac{2(10.2^x-1)}{(3+2^x)^2}=1$

$\Rightarrow 20.2^x-2=9+2^{2x}+6.2^x$

$\therefore (2^x)^2-14(2^x)+11=0$

Roots are 2^{x_1} & 2^{x_2}

$\therefore 2^{x_1}.2^{x_2}=11$

$x_1+x_2=\log_2(11)$