LIMIT

1. If the value of $\lim\limits_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to ________.

2. If $\lim\limits_{x\to 0}\dfrac{\alpha x e^x-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2 x}=10,\ \alpha,\beta,\gamma\in\mathbf{R}$, then the value of $\alpha+\beta+\gamma$ is ________.

3. The value of $\lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{j=1}^{n}\dfrac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

(1) $5+\log_e\left(\frac{3}{2}\right)$ (2) $2-\log_e\left(\frac{2}{3}\right)$

(3) $3+2\log_e\left(\frac{2}{3}\right)$ (4) $1+2\log_e\left(\frac{3}{2}\right)$

4. The value of $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to :

(1) 0 (2) 4 (3) – 4 (4) –1

5. $\lim\limits_{x\to 2}\left(\sum\limits_{n=1}^{9}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to :

(1) $\frac{9}{44}$ (2) $\frac{5}{24}$ (3) $\frac{1}{5}$ (4) $\frac{7}{36}$

6. If α, β are the distinct roots of $x^2+bx+c=0$, then $\lim\limits_{x\to\beta}\dfrac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to:

(1) b^2+4c (2) $2(b^2+4c)$

(3) $2(b^2-4c)$ (4) b^2-4c

7. If $0<x<1$ and $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\dots$, then the value of e^{1+y} at $x=\frac{1}{2}$ is:

(1) $\frac{1}{2}e^2$ (2) 2e (3) $\frac{1}{2}\sqrt{e}$ (4) $2e^2$

8. If $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is:

(1) $\left(1,\frac{1}{2}\right)$ (2) $\left(1,-\frac{1}{2}\right)$

(3) $\left(-1,\frac{1}{2}\right)$ (4) $\left(-1,-\frac{1}{2}\right)$

9. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to :

(1) π^2 (2) $2\pi^2$ (3) $4\pi^2$ (4) 4π

10. If $\alpha=\lim\limits_{x\to\pi/4}\dfrac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ and $\beta=\lim\limits_{x\to 0}(\cos x)^{\cot x}$ are the roots of the equation, $ax^2+bx-4=0$, then the ordered pair (a, b) is :

(1) (1, –3) (2) (–1, 3)

(3) (–1, –3) (4) (1, 3)

11. Let $f:\mathbf{R}\to\mathbf{R}$ be a continuous function. Then $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\frac{\pi}{4}\int_{2}^{\sec^2 x}f(x)\,dx}{x^2-\frac{\pi^2}{16}}$ is equal to :

(1) $f(2)$ (2) $2f(2)$

(3) $2f\left(\sqrt{2}\right)$ (4) $4f(2)$

12. Let $f(x)=x^6+2x^4+x^3+2x+3,\ x\in\mathbf{R}$. Then the natural number n for which $\lim\limits_{x\to 1}\dfrac{x^n f(1)-f(x)}{x-1}=44$ is________.

13. $\lim\limits_{n\to\infty}\tan\left\{\sum\limits_{r=1}^{n}\tan^{-1}\left(\dfrac{1}{1+r+r^2}\right)\right\}$ is equal to ____.

14. $\lim\limits_{n\to\infty}\left(1+\dfrac{1+\frac{1}{2}+\dots\dots+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$ (2) 0 (3) $\frac{1}{e}$ (4) 1

15. If $\lim_{x\to 0}\frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of a – 2b is ______.

16. The value of

$$\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$$ is

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

17. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases}\frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x\neq 0\\ \alpha, & x=0\end{cases}$$

is continuous at x = 0, where {x} = x – [x], [x] is the greatest integer less than or equal to x. Then :

(1) $\alpha=\frac{\pi}{\sqrt{2}}$ (2) $\alpha = 0$

(3) no such α exists (4) $\alpha=\frac{\pi}{4}$

18. Let $f:(0,2)\to\mathbb{R}$ be defined as

$$f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right).$$

Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f(1)\right)$ is equal to _______ .

19. If $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then a + b + c is equal to _______.

20. The value of $\lim_{n\to\infty}\frac{[r]+[2r]+.....+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to :

(1) $\frac{r}{2}$ (2) r (3) 2r (4) 0

21. The value of the limit $\lim_{\theta\to 0}\frac{\tan\left(\pi\cos^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$ is equal to :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) 0 (4) $\frac{1}{4}$

22. The value of

$$\lim_{x\to 0^+}\frac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3},$$ where [x] denotes the greatest integer $\le x$ is :

(1) π (2) 0 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

23. If $\lim_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$ (3) 6 (4) 2

SOLUTION

1. Official Ans. by NTA (3)

Sol. $\lim_{x\to 0}\left(2-\cos x\sqrt{\cos x}\right)^{\frac{x+2}{x^2}}$

form: 1^{∞}

$= e^{\lim_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)\times(x+2)}$

Now $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$= \lim_{x\to 0}\frac{\sin x\sqrt{\cos 2x}-\cos x\times\frac{1}{2\sqrt{\cos 2x}}\times(-2\sin 2x)}{2x}$

(by L' Hospital Rule)

$\lim_{x\to 0}\frac{\sin x\cos 2x+\sin 2x.\cos x}{2x}$

$= \frac{1}{2}+1=\frac{3}{2}$

So, $e^{\lim_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)(x+2)}$

$= e^{\frac{3}{2}\times 2}=e^3$

$\Rightarrow \boxed{a=3}$

2. Official Ans. by NTA (3)

Sol. $\lim_{x\to 0}\frac{\alpha x\left(1+x+\frac{x^2}{2}\right)-\beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)+\gamma x^2(1-x)}{x^3}$

$\lim_{x\to 0}\frac{x(\alpha-\beta)+x^2\left(\alpha+\frac{\beta}{2}+\gamma\right)+x^3\left(\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right)}{x^3}=10$

For limit to exist

$\alpha-\beta=0,\ \alpha+\frac{\beta}{2}+\gamma=0$

$\frac{\alpha}{2}-\frac{\beta}{3}-\gamma=10$ ……(i)

$\beta=\alpha,\ \gamma=-3\frac{\alpha}{2}$

Put in (i)

$\frac{\alpha}{2}-\frac{\alpha}{3}+\frac{3\alpha}{2}=10$

$\frac{\alpha}{6}+\frac{3\alpha}{2}=10 \Rightarrow \frac{\alpha+9\alpha}{6}=10$

$\Rightarrow \alpha=6$

$\alpha=6,\ \beta=6,\ \gamma=-9$

$\alpha+\beta+\gamma=3$

3. Official Ans. by NTA (4)

Sol. $\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{\left(\frac{2j}{n}-\frac{1}{n}+8\right)}{\left(\frac{2j}{n}-\frac{1}{n}+4\right)}$

$\int_0^1\frac{2x+8}{2x+4}dx=\int_0^1 dx+\int_0^1\frac{4}{2x+4}dx$

$=1+4\frac{1}{2}\left(\ell n|2x+4|\right)\Big|_0^1$

$=1+2\ell n\left(\frac{3}{2}\right)$

4. Official Ans. by NTA (3)

Sol. $\lim_{x\to0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$

$= \lim_{x\to0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$

$= \lim_{x\to0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$

$\left(\frac{\left(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}\right)}{\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}}\right)$

$\left(\frac{\left(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}\right)}{\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}}\right)$

$\left(\frac{\left(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}\right)}{\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}}\right)$

$= \lim_{x\to0}\left(\frac{x}{1-\sin x-(1+\sin x)}\right)$

$\left(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}\right)$

$\left(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}\right)$

$\left(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}\right)$

$=\lim_{x\to0}\frac{x}{(-2\sin x)}\left(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}\right)$

$\left(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}\right)$

$\left(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}\right)$

$= \lim_{x\to0}\left(-\frac{1}{2}\right)(2)\ (2)\ (2)\ \left\{\because \lim_{x\to0}\frac{\sin x}{x}=1\right\} = -4$

5. Official Ans. by NTA (1)

Sol. $S=\lim_{x\to2}\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}$

$S=\sum_{n=1}^{9}\frac{2}{4(n^2+3n+2)}=\frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$S=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{9}{44}$

6. Official Ans. by NTA (3)

Sol. $\lim_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\frac{1\left(1+\frac{2(x^2+bx+c)}{1!}+\frac{2^2(x^2+bx+c)^2}{2!}+...\right)-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\frac{2(x^2+bx+1)^2}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\frac{2(x-\alpha)^2(x-\beta)^2}{(x-\beta)^2}$

$\Rightarrow 2(\beta-\alpha)^2=2(b^2-4c)$

7. Official Ans. by NTA (1)

Sol. $y=\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+.....$

$=(x^2+x^3+x^4+.......)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+....\right)$

$=\frac{x^2}{1-x}+x-\left(x+\frac{x^2}{2}+\frac{x^2}{3}+...\right)$

$=\frac{x}{1-x}+\ell n(1-x)$

$x=\frac{1}{2} \Rightarrow y=1-\ell n2$

$e^{1+y}=e^{1+1-\ell n2}$

$=e^{2-\ell n2}=\frac{e^2}{2}$

8. Official Ans. by NTA (2)

Sol. $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b \qquad (\infty-\infty)$

$\Rightarrow a>0$

Now, $\lim_{x\to\infty}\frac{(x^2-x+1-a^2x^2)}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow 1-a^2=0 \Rightarrow a=1$

Now, $\lim_{x\to\infty}\frac{-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow \frac{-1}{1+a}=b\Rightarrow b=-\frac{1}{2}$

$(a,b)=\left(1,-\frac{1}{2}\right)$

9. Official Ans. by NTA (3)

Sol. $\lim_{x\to 0}\frac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$

$\lim_{x\to 0}\frac{1-\cos\left(2\pi\cos^4 x\right)}{2x^4}$

$\lim_{x\to 0}\frac{1-\cos\left(2\pi-2\pi\cos^4 x\right)}{\left[2\pi\left(1-\cos^4 x\right)\right]^2}4\pi^2.\frac{\sin^4 x}{2x^4}\left(1+\cos^2 x\right)^2$

$=\frac{1}{2}.4\pi^2.\frac{1}{2}(2)^2=4\pi^2$

10. Official Ans. by NTA (4)

Sol. $\alpha=\lim_{x\to\frac{\pi}{4}}\frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$; $\frac{0}{0}$ form

Using L Hopital rule

$\alpha=\lim_{x\to\frac{\pi}{4}}\frac{3\tan^2 x\sec^2 x-\sec^2 x}{-\sin\left(x+\frac{\pi}{4}\right)}$

$\Rightarrow \alpha=-4$

$\beta=\lim_{x\to 0}(\cos x)^{\cot x}=e^{\lim_{x\to 0}\frac{(\cos x-1)}{\tan x}}$

$\beta=e^{\lim_{x\to 0}\frac{-(1-\cos x)}{x^2}.\frac{x^2}{\left(\frac{\tan x}{x}\right)^x}}$

$\beta=e^{\lim_{x\to 0}\left(\frac{-1}{2}\right).\frac{x}{1}}=e^0\Rightarrow\beta=1$

$\alpha=-4$; $\beta=1$

If $ax^2+bx-4=0$ are the roots then

$16a-4b-4=0$ & $a+b-4=0$

$\Rightarrow a=1$ & $b=3$

11. Official Ans. by NTA (2)

Sol. $\lim_{x\to\frac{\pi}{4}}\frac{\frac{\pi}{4}\int_2^{\sec^2 x}f(x)dx}{x^2-\frac{\pi^2}{16}}$

$\lim_{x\to\frac{\pi}{4}}\frac{\pi}{4}.\frac{\left[f\left(\sec^2 x\right).2\sec x.\sec x\tan x\right]}{2x}$

$\lim_{x\to\frac{\pi}{4}}\frac{\pi}{4}f\left(\sec^2 x\right).\sec^3 x.\frac{\sin x}{x}$

$\frac{\pi}{4}f(2).\left(\sqrt{2}\right)^3.\frac{1}{\sqrt{2}}\times\frac{4}{\pi}$

$\Rightarrow 2f(2)$

12. Official Ans. by NTA (7)

Sol. $f(n) = x^6 + 2x^4 + x^3 + 2x + 3$

$$\lim_{x\to 1}\frac{x^n f(1) - f(x)}{x-1} = 44$$

$$\lim_{x\to 1}\frac{9x^n - (x^6 + 2x^4 + x^3 + 2x + 3)}{x-1} = 44$$

$$\lim_{x\to 1}\frac{9nx^{n-1} - (6x^5 + 8x^3 + 3x^2 + 2)}{1} = 44$$

$\Rightarrow 9n - (19) = 44$

$\Rightarrow 9n = 63$

$\Rightarrow n = 7$

13. Official Ans. by NTA (1)

Sol. $$\lim_{n\to \alpha}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r(r+1)}\right)\right)$$

$$= \lim_{n\to\alpha}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)\right)$$

$$= \tan\left(\lim_{n\to\alpha}\sum_{r=1}^{n}\left[\tan^{-1}(r+1) - \tan^{-1}(r)\right]\right)$$

$$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1) - \frac{\pi}{4}\right)\right)$$

$$= \tan\left(\frac{\pi}{4}\right) = 1$$

14. Official Ans. by NTA (4)

Sol. Given limit is of 1^∞ form

So, $l = \exp\left(\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{3}+........+\frac{1}{n}}{n}\right)$

Now,

$$0 \le 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \le 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}}$$

$$\le 2\sqrt{n} - 1$$

So, l = exp (0) (from sandwich theorem)

= 1

15. Official Ans. by NTA (5)

Sol. $$\lim_{x\to 0}\frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0}\frac{ax - (e^{4x} - 1)}{ax \,.\, 4x} \qquad \boxed{\text{Use } \lim_{x\to 0}\frac{e^{4x}-1}{4x} = 1}$$

Apply L'Hospital Rule

$$= \lim_{x\to 0}\frac{a - 4e^{4x}}{8ax} \quad \left(\frac{a-4}{0}\text{ form}\right)$$

limit exists only when $a - 4 = 0 \Rightarrow a = 4$

$$= \lim_{x\to 0}\frac{4 - 4e^{4x}}{32x}$$

$$= \lim_{x\to 0}\frac{1 - e^{4x}}{8x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0}\frac{-e^{4x}.4}{8} = -\frac{1}{2} \Rightarrow b = -\frac{1}{2}$$

$$a - 2b = 4 - 2\left(-\frac{1}{2}\right)$$

= 5

16. Official Ans. by NTA (1)

Sol. $$L = \lim_{h\to 0} 2\left(\frac{\sqrt{3}\left(\frac{1}{2}\cosh + \frac{\sqrt{3}}{2}\sinh\right) - \left(\frac{\sqrt{3}}{2}\cosh - \frac{\sinh}{2}\right)}{(\sqrt{3}h)(\sqrt{3})}\right)$$

$$L = \lim_{h\to 0}\frac{4\sinh}{3h}$$

$$\Rightarrow L = \frac{4}{3}$$

17. Official Ans by NTA (3)

Sol. $\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-}(x)$

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)\cdot\sin^{-1}(1-x)}{x(1-x)(1+x)}$$

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)}{x\cdot 1\cdot 1}\cdot\frac{\pi}{2}$$

Let $1 - x^2 = \cos\theta$

$$\frac{\pi}{2}\lim_{x\to 0^+}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$\frac{\pi}{2}\lim_{\theta\to 0^+}\frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}} = \frac{\pi}{\sqrt{2}}$$

Now, $\lim_{x\to 0^-}\frac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}\left(-\sin^{-1}x\right)}{(1+x)(2+x)(-x)}$$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}}{1\cdot 2}\cdot\frac{\sin^{-1}x}{x} = \frac{\pi}{4}$$

$\Rightarrow$ RHL $\neq$ LHL

Function can't be continuous

$\Rightarrow$ No value of α exist

18. Official Ans. by NTA (1)

Sol. $E = 2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi x}{4}\right)dx \quad .....(i)$$

replacing $x \to 1 - x$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\frac{1+\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1\left(\ell n2 - \ell n\left(1+\tan\frac{\pi x}{4}\right)\right)dx \quad .....(ii)$$

equation (i) + (ii)

E = 1

19. Official Ans. by NTA (4)

Sol. $\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

$$\Rightarrow \lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}...\right)-b\left(1-\frac{x^2}{2!}+...\right)+c\left(1-x+\frac{x^2}{2!}\right)}{\left(\frac{x\sin x}{x}\right)x} = 2$$

$a - b + c = 0 \quad .....(1)$

$a - c = 0 \quad .....(2)$

& $\frac{a+b+c}{2} = 2$

$\Rightarrow \boxed{a+b+c=4}$

20. Official Ans. by NTA (1)

Sol. We know that

$$r \le [r] < r + 1$$

and

$$2r \le [2r] < 2r + 1$$
$$3r \le [3r] < 3r + 1$$
$$\vdots \quad \vdots \quad \vdots$$
$$nr \le [nr] < nr + 1$$

$r + 2r + ... + nr$

$\le [r] + [2r] + ... + [nr] < (r + 2r + ... + nr) + n$

$$\frac{\frac{n(n+1)}{2}\cdot r}{n^2} \le \frac{[r]+[2r]+......+[nr]}{n^2} < \frac{\frac{n(n+1)}{2}r+n}{n^2}$$

Now,

$$\lim_{n\to\infty}\frac{n(n+1)\cdot r}{2\cdot n^2} = \frac{r}{2}$$

and $$\lim_{n\to\infty}\frac{\frac{n(n+1)\,r}{2}+n}{n^2} = \frac{r}{2}$$

So, by Sandwich Theorem, we can conclude that

$$\lim_{n\to\infty}\frac{[r]+[2r]+......+[nr]}{n^2} = \frac{r}{2}$$

Ans. (1)

21. Official Ans. by NTA (1)

Sol. $$\lim_{\theta\to 0}\frac{\tan\left(\pi\left(1-\sin^2\theta\right)\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$= \lim_{\theta\to 0}\frac{-\tan\left(\pi\sin^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$= \lim_{\theta\to 0} -\left(\frac{\tan\left(\pi\sin^2\theta\right)}{\pi\sin^2\theta}\right)\left(\frac{2\pi\sin^2\theta}{\sin\left(2\pi\sin^2\theta\right)}\right)\times\frac{1}{2}$$

$= \frac{-1}{2}$ **Option (1)**

22. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0^+}\frac{\cos^{-1}x}{(1-x^2)}\times\frac{\sin^{-1}x}{x} = \frac{\pi}{2}$$

23. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0}\frac{\left(x+\frac{x^3}{3!}...\right)-\left(x-\frac{x^3}{3}...\right)}{3x^3} = \frac{1}{6}$$

So $6L + 1 = 2$