INVERSE TRIGONOMETRY FUNCTION

1. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is :

(1) 1 (2) 2 (3) 4 (4) 0

2. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to :

(1) $\frac{-181}{69}$ (2) $\frac{220}{21}$ (3) $\frac{-291}{76}$ (4) $\frac{151}{63}$

3. If the domain of the function $f(x)=\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha+\beta$ is equal to :

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{1}{2}$ (4) 1

4. If $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2}=p$, then the value of tan p is :

(1) $\frac{101}{102}$ (2) $\frac{50}{51}$ (3) 100 (4) $\frac{51}{50}$

5. If $(\sin^{-1}x)^2-(\cos^{-1}x)^2=a;\ 0<x<1,\ a\neq 0$, then the value of $2x^2-1$ is :

(1) $\cos\left(\frac{4a}{\pi}\right)$ (2) $\sin\left(\frac{2a}{\pi}\right)$

(3) $\cos\left(\frac{2a}{\pi}\right)$ (4) $\sin\left(\frac{4a}{\pi}\right)$

6. Let M and m respectively be the maximum and minimum values of the function $f(x)=\tan^{-1}(\sin x+\cos x)$ in $\left[0,\frac{\pi}{2}\right]$, Then the value of $\tan(M-m)$ is equal to:

(1) $2+\sqrt{3}$ (2) $2-\sqrt{3}$

(3) $3+2\sqrt{2}$ (4) $3-2\sqrt{2}$

7. The domain of the function

$$f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)\text{ is :}$$

(1) $\left[0,\frac{1}{4}\right]$ (2) $[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$

(3) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$ (4) $\left[0,\frac{1}{2}\right]$

8. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$ is equal to :

(The inverse trigonometric functions take the principal values)

(1) $3\pi-11$ (2) $4\pi-9$

(3) $4\pi-11$ (4) $3\pi+1$

9. $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to :

(1) $\frac{56}{33}$ (2) $\frac{65}{56}$ (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

10. If $0<a, b<1$, and $\tan^{-1}a+\tan^{-1}b=\frac{\pi}{4}$, then the value of

$$(a+b)-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)-\left(\frac{a^4+b^4}{4}\right)+\ldots\text{ is:}$$

(1) $\log_e 2$ (2) e^2-1

(3) e (4) $\log_e\left(\frac{e}{2}\right)$

11. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c};0<x<1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1-y^2$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

12. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to:

(1) 2 (2) 1 (3) 3 (4) 0

13. Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty}S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$ (2) $\frac{\pi}{2}$

(3) $\cot^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}(3)$

14. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for $x\in[-1, 1]$, and [x] denotes the greatest integer less than or equal to x, is :

(1) 2 (2) 0

(3) 4 (4) Infinite

15. If $\cot^{-1}(\alpha)=\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+\ldots$ upto 100 terms, then α is :

(1) 1.01 (2) 1.00 (3) 1.02 (4) 1.03

16. The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is :

(1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$ (3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

17. The real valued function $f(x)=\frac{\operatorname{cosec}^{-1}x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :

(1) all reals except integers

(2) all non-integers except the interval [–1, 1]

(3) all integers except 0, –1, 1

(4) all reals except the Interval [–1,1]

SOLUTION

1. Official Ans. by NTA (4)

Sol. $\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$

For equation to be defined,

$x^2+x\ge 0$

$\Rightarrow \quad x^2+x+1\ge 1$

$\therefore$ only possibility that the equation is defined

$x^2+x=0 \quad \Rightarrow x=0;\ x=-1$

None of these values satisfy

$\therefore$ No of roots = 0

2. Official Ans. by NTA (2)

Sol. $\underbrace{\tan^{-1}\frac{3}{5}+\tan^{-1}\frac{3}{5}}_{x>0,y>0,xy<1}+\tan^{-1}\frac{5}{12}$

$\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}}=\underbrace{\tan^{-1}\frac{15}{8}+\tan^{-1}\frac{5}{12}}_{x>0,y>0,xy<1}$

$\tan^{-1}\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\cdot\frac{5}{12}}=\tan^{-1}\frac{220}{21}$

$\tan\left(\tan^{-1}\frac{220}{21}\right)=\frac{220}{21}$

3. Official Ans. by NTA (1)

Sol. $O\le x^2-x+1\le 1$

$\Rightarrow x^2-x\le 0$

$\Rightarrow x\in[0,1]$

Also, $0<\sin^{-1}\left(\frac{2x-1}{2}\right)\le\frac{\pi}{2}$

$\Rightarrow 0<\frac{2x-1}{2}\le 1$

$\Rightarrow 0<2x-1\le 2$

$1<2x\le 3$

$\frac{1}{2}<x\le\frac{3}{2}$

Taking intersection

$x\in\left(\frac{1}{2},1\right]$

$\Rightarrow \alpha=\frac{1}{2},\ \beta=1$

$\Rightarrow \alpha+\beta=\frac{3}{2}$

4. Official Ans. by NTA (2)

Sol. $\sum_{r=1}^{50}\tan^{-1}\left(\frac{2}{4r^2}\right)=\sum_{r=1}^{50}\tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$\sum_{r=1}^{50}\tan^{-1}(2r+1)-\tan^{-1}(2r-1)$

$\tan^{-1}(101)-\tan^{-1}1 \Rightarrow \tan^{-1}\frac{50}{51}$

5. Official Ans. by NTA (2)

Sol. Given $a=\left(\sin^{-1}x\right)^2-\left(\cos^{-1}x\right)^2$

$=(\sin^{-1}x+\cos^{-1}x)(\sin^{-1}x-\cos^{-1}x)$

$=\frac{\pi}{2}\left(\frac{\pi}{2}-2\cos^{-1}x\right)$

$\Rightarrow 2\cos^{-1}x=\frac{\pi}{2}-\frac{2a}{\pi}$

$\Rightarrow \cos^{-1}\left(2x^2-1\right)=\frac{\pi}{2}-\frac{2a}{\pi}$

$\Rightarrow 2x^2-1=\cos\left(\frac{\pi}{2}-\frac{2a}{\pi}\right)$ option (2)

6. Official Ans. by NTA (4)

Sol. Let g(x) = sin x + cos x = $\sqrt{2}\ \sin\left(x+\frac{\pi}{4}\right)$

$g(x) \in [1,\sqrt{2}]$ for $x \in [0,\pi/2]$

$f(x) = \tan^{-1}(\sin x + \cos x) \in \left[\frac{\pi}{4}, \tan^{-1}\sqrt{2}\right]$

$\tan\left(\tan^{-1}\sqrt{2} - \frac{\pi}{4}\right) = \frac{\sqrt{2}-1}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 3-2\sqrt{2}$

7. Official Ans. by NTA (3)

Sol. $f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$

$-1 \le \frac{x-1}{x+1} \le 1 \Rightarrow 0 \le x < \infty$ ...(1)

$-1 \le \frac{3x^2+x-1}{(x-1)^2} \le 1 \Rightarrow x \in \left[\frac{-1}{4}, \frac{1}{2}\right] \cup \{0\}$ ...(2)

(1) & (2)

$\Rightarrow$ Domain = $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

8. Official Ans. by NTA (3)

Sol. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$

$\Rightarrow (2\pi - 5) + (6 - 2\pi) - (12 - 4\pi)$

$\Rightarrow 4\pi - 11.$

9. Official Ans. by NTA (2)

Sol. $\operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right]$

$\operatorname{cosec}\left[\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right]$

$= \operatorname{cosec}\left[\tan^{-1}\left(\frac{56}{33}\right)\right] = \frac{65}{56}$ option (2)

10. Official Ans. by NTA (1)

Sol. $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$ $\quad 0 < a, b < 1$

$\Rightarrow \frac{a+b}{1-ab} = 1$

a + b = 1 – ab

(a + 1)(b + 1) = 2

Now $\left[a - \frac{a^2}{2} + \frac{a^3}{3} + ...\right] + \left[b - \frac{b^2}{2} + \frac{b^3}{3} + ...\right]$

$= \log_e(1 + a) + \log_e(1 + b)$

($\because$ expansion of $\log_e(1 + x)$)

$= \log_e[(1 + a)(1 + b)]$

$= \log_e 2$

11. Official Ans. by NTA (3)

Sol. $\frac{\sin^{-1}x}{r} = a,\ \frac{\cos^{-1}x}{r} = b,\ \frac{\tan^{-1}y}{r} = c$

So, $a + b = \frac{\pi}{2r}$

$\cos\left(\frac{\pi c}{a+b}\right) = \cos\left(\frac{\pi \tan^{-1}y}{\frac{\pi}{2r}r}\right)$

$= \cos(2\tan^{-1}y)$, let $\tan^{-1}y = \theta$

$= \cos(2\theta)$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-y^2}{1+y^2}$

12. Official Ans by NTA (3)

Sol. $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$

$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right)=\sin^{-1}x$

$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x$

$x = 0,\ 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$ squaring we get

$16(25-9x^2) = 625+9(25-16x^2)-150\sqrt{25-16x^2}$

$400 = 625 + 225 - 150\sqrt{25-16x^2}$

$\sqrt{25-16x^2} = 3 \Rightarrow 25 - 16x^2 = 9$

$\Rightarrow x^2 = 1$

Put x = 0, 1, –1 in the original equation

We see that all values satisfy the original equation.

Number of solution = 3

13. Official Ans. by NTA (3)

Sol. $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

Divide by 3^{2r}

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r}.2+3}\right)$

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$

Let $\left(\frac{2}{3}\right)^r = t$

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{t-\frac{2t}{3}}{1+t.\frac{2t}{3}}\right)$

$\sum_{r=1}^{k}\left(\tan^{-1}(t)-\tan^{-1}\left(\frac{2t}{3}\right)\right)$

$\sum_{r=1}^{k}\left(\tan^{-1}\left(\frac{2}{3}\right)^r-\tan^{-1}\left(\frac{2}{3}\right)^{r+1}\right)$

$S_k=\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}$

$S_\infty=\lim_{k\to\infty}\left(\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}\right)$

$=\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}(0)$

$\therefore\quad S_\infty=\tan^{-1}\left(\frac{2}{3}\right)=\cot^{-1}\left(\frac{3}{2}\right)$

14. Official Ans. by NTA (2)

Sol. Given equation

$$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$$

Now, $\sin^{-1}\left[x^2+\frac{1}{3}\right]$ is defined if

$$-1\le x^2+\frac{1}{3}<2 \Rightarrow \frac{-4}{3}\le x^2<\frac{5}{3}$$

$$\Rightarrow \boxed{0\le x^2<\frac{5}{3}} \quad ....(1)$$

and $\cos^{-1}\left[x^2-\frac{2}{3}\right]$ is defined if

$$-1\le x^2-\frac{2}{3}<2 \Rightarrow \frac{-1}{3}\le x^2<\frac{8}{3}$$

$$\Rightarrow \boxed{0\le x^2<\frac{8}{3}} \quad ....(2)$$

So, form (1) and (2) we can conclude

$$\boxed{0\le x^2<\frac{5}{3}}$$

Case - I if $0\le x^2<\frac{2}{3}$

$\sin^{-1}(0)+\cos^{-1}(-1)=x^2$

$\Rightarrow x+\pi=x^2$

$\Rightarrow x^2=\pi$

but $\pi\notin\left[0,\frac{2}{3}\right)$

$\Rightarrow$ No value of 'x'

Case - II if $\frac{2}{3}\le x^2<\frac{5}{3}$

$\sin^{-1}(1)+\cos^{-1}(0)=x^2$

$\Rightarrow \frac{\pi}{2}+\frac{\pi}{2}=x^2$

$\Rightarrow x^2=\pi$

but $\pi\notin\left[\frac{2}{3},\frac{5}{3}\right)$

$\Rightarrow$ No value of 'x'

So, number of solutions of the equation is zero.

Ans.(2)

15. Official Ans. by NTA (1)

Sol. $\cot^{-1}(\alpha)=\cot^{-1}(2)+\cot^{-1}(8)+\cot^{-1}(18)+.....$

$$=\sum_{n=1}^{100}\tan^{-1}\left(\frac{2}{4n^2}\right)$$

$$=\sum_{n=1}^{100}\tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$=\sum_{n=1}^{100}\tan^{-1}(2n+1)-\tan^{-1}(2n-1)$$

$=\tan^{-1}201-\tan^{-1}1$

$$=\tan^{-1}\left(\frac{200}{202}\right)$$

$$\therefore\ \cot^{-1}(\alpha)=\cot^{-1}\left(\frac{202}{200}\right)$$

$\alpha=1.01$

16. Official Ans. by NTA (1)

Sol. $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\frac{8}{31}$

Taking tangent both sides :-

$$\frac{(x+1)+(x-1)}{1-(x^2-1)}=\frac{8}{31}$$

$$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$$

$\Rightarrow 4x^2+31x-8=0$

$\Rightarrow x=-8,\ \frac{1}{4}$

But, if $x=\frac{1}{4}$

$\tan^{-1}(x+1)\in\left(0,\frac{\pi}{2}\right)$

& $\cot^{-1}\left(\frac{1}{x-1}\right)\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow$ LHS $>\frac{\pi}{2}$ & RHS $<\frac{\pi}{2}$

(Not possible)

Hence, $x=-8$

17. Official Ans. by NTA (2)

Sol. $f(x)=\frac{\operatorname{cosec}^{-1}x}{\sqrt{\{x\}}}$

Domain $\in(-\infty,-1]\cup[1,\infty)$

$\{x\}\ne 0$ so $x\ne$ integers