HYPERBOLA

1. Let a line L : $2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to :

(1) 12 (2) –12 (3) 24 (4) –24

2. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

(1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$
(2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$
(3) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$
(4) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

3. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to :

(1) $4\sqrt{3}$ (2) 6 (3) $6\sqrt{3}$ (4) $3\sqrt{6}$

4. Let A $(\sec\theta, 2\tan\theta)$ and B $(\sec\phi, 2\tan\phi)$, where $\theta + \phi = \pi/2$, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to ___________.

5. If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ intersect each other at an angle of 90°, then which of the following relations is TRUE?

(1) $a + b = c + d$ (2) $a - b = c - d$
(3) $a - c = b + d$ (4) $ab = \frac{c+d}{a+b}$

6. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$.

Then the composition function $f(g(x))$ is :

(1) onto but not one-one
(2) both one-one and onto
(3) one-one but not onto
(4) neither one-one nor onto

7. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is ___________.

8. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities in one, then the equation of the hyperbola is :

(1) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
(3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

9. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$
(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$
(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$
(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

10. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

11. Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P\left(4,\sqrt{6}\right)$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6}-1$

(3) $\frac{7}{\sqrt{6}}-2$ (4) $4\sqrt{6}-1$

SOLUTION

1. Official Ans. by NTA (4)

Sol. Tangent to hyperbola of

Slope m = –2 (given)

$$y=-2x\pm\sqrt{3(3)}$$

$$\left(y=mx\pm\sqrt{a^2m^2-b^2}\right)$$

$\Rightarrow y + 2x = \pm 3 \Rightarrow 2x + y = 3 \quad (k > 0)$

For parabola $y^2 = \alpha x$

$$y = mx + \frac{\alpha}{4m}$$

$$\Rightarrow y = -2x + \frac{\alpha}{-8}$$

$$\Rightarrow \frac{\alpha}{-8}=3$$

$\Rightarrow \alpha = -24.$

2. Official Ans. by NTA (1)

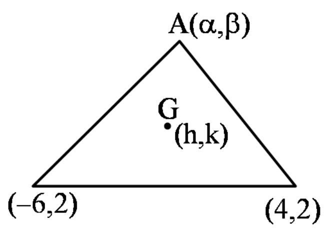
Sol. Given hyperbola is

$16(x + 1)^2 - 9(y - 2)^2 = 164 + 16 - 36 = 144$

$$\Rightarrow \frac{(x+1)^2}{9}-\frac{(y-2)^2}{16}=1$$

Eccentricity, $e=\sqrt{1+\frac{16}{9}}=\frac{5}{3}$

$\Rightarrow$ foci are (4, 2) and (–6, 2)



Let the centroid be (h, k)

& A(α, β) be point on hyperbola

So $h=\frac{\alpha-6+4}{3}, k=\frac{\beta+2+2}{3}$

$\Rightarrow \alpha = 3h + 2, \beta = 3k - 4$

(α, β) lies on hyperbola so

$16(3h + 2 + 1)^2 - 9(3k - 4 - 2)^2 = 144$

$\Rightarrow 144(h + 1)^2 - 81(k - 2)^2 = 144$

$\Rightarrow 16(h^2 + 2h + 1) - 9(k^2 - 4k + 4) = 16$

$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$

3. Official Ans. by NTA (3)

Sol. $P(-2\sqrt{6},\sqrt{3})$ lies on hyperbola

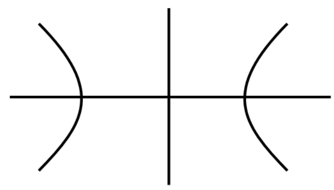
$$\Rightarrow \frac{24}{a^2}-\frac{3}{b^2}=1 \quad ....(i)$$

$$e=\frac{\sqrt{5}}{2} \Rightarrow b^2=a^2\left(\frac{5}{4}-1\right) \Rightarrow 4b^2=a^2$$

Put in (i) $\Rightarrow \frac{6}{b^2}-\frac{3}{b^2}=1 \Rightarrow b=\sqrt{3}$

$\Rightarrow a=\sqrt{12}$

$$\frac{x^2}{12}-\frac{y^2}{3}=1$$



Tangent at P :

$$\frac{-x}{\sqrt{6}}-\frac{y}{\sqrt{3}}=1 \Rightarrow Q\ (0, \sqrt{3})$$

Slope of $T=-\frac{1}{\sqrt{2}}$

Normal at P :

$$y-\sqrt{3}=\sqrt{2}(x+2\sqrt{6})$$

$\Rightarrow R=(0,5\sqrt{3})$

QR = $6\sqrt{3}$

4. Official Ans. by NTA (36)

ALLEN Ans. (Bonus)

Sol. Since, point A ($\sec\theta$, $2\tan\theta$)

lies on the hyperbola

$2x^2 - y^2 = 2$

Therefore, $2\sec^2\theta - 4\tan^2\theta = 2$

$\Rightarrow 2 + 2\tan^2\theta - 4\tan^2\theta = 2$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$

Similarly, for point B, we will get $\phi = 0$.

but according to question $\theta + \phi = \frac{\pi}{2}$

which is not possible.

Hence it must be a 'BONUS'.

5. Official Ans. by NTA (2)

Sol. For orthogonal curves $a - c = b - d$

$\Rightarrow a - b = c - d$

6. Official Ans. by NTA (3)

Sol. $f(g(x)) = 2g(x) - 1 = 2\left(\frac{2x-1}{2(x-1)}\right) - 1$

$$= \frac{x}{x-1} = 1 + \frac{1}{x-1}$$

Range of $f(g(x) = \mathbb{R} - \{1\}$

Range of $f(g(x))$ is not onto

& $f(g(x))$ is one-one

So $f(g(x))$ is one-one but not onto.

7. Official Ans. by NTA (2)

Sol. $K = \frac{4\sqrt{3}}{\sqrt{3}x + y} = \frac{\sqrt{3}x - y}{4\sqrt{3}}$

$\Rightarrow 3x^2 - y^2 = 48$

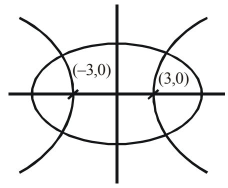
$\Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$

Now, $48 = 16(e^2 - 1)$

$\Rightarrow e = \sqrt{4} = 2$

8. Official Ans. by NTA (2)

Sol.



For ellipse $e_1 = \sqrt{1 - \frac{b^2}{a^2}} = \frac{3}{5}$

for hyperbola $e_2 = \frac{5}{3}$

Let hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\because$ it passes through (3,0) $\Rightarrow \frac{9}{a_2} = 1$

$\Rightarrow a^2 = 9$

$\Rightarrow b^2 = a^2(e^2 - 1)$

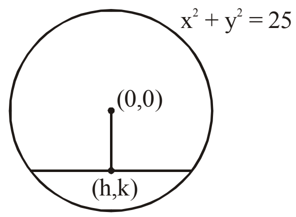
$= 9\left(\frac{25}{9} - 1\right) = 16$

$\therefore$ Hyperbola is

$\frac{x^2}{9} - \frac{y^2}{16} = 1$ ... option 2.

9. Official Ans. by NTA (4)

Sol.



Equation of chord

$y - k = -\frac{h}{k}(x - h)$

$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k}$

tangent to $\frac{x^2}{9} - \frac{y^2}{16} = 1$

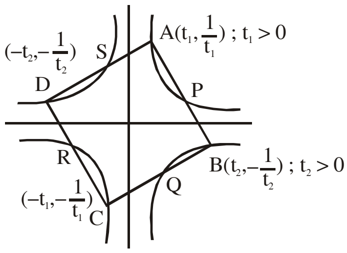
$c^2 = a^2m^2 - b^2$

$\left(\frac{h^2 + k^2}{k}\right)^2 = 9\left(-\frac{h}{k}\right)^2 - 16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

10. Official Ans. by NTA (80)

Sol. $xy = 1, -1$



$$\frac{t_1+t_2}{2}\cdot\frac{\frac{1}{t_1}-\frac{1}{t_2}}{2}=1$$

$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$

$$\frac{1}{t_1^2}\times\left(-\frac{1}{t_2^2}\right)=-1\Rightarrow t_1t_2=1$$

$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$

$t_1^2 - t_2^2 = 4$

$$\Rightarrow t_1^2+t_2^2=\sqrt{4^2+4}=2\sqrt{5}$$

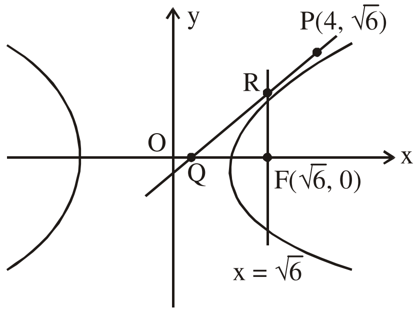
$$\Rightarrow t_1^2 = 2+\sqrt{5} \Rightarrow \frac{1}{t_1^2}=\sqrt{5}-2$$

$$AB^2 = (t_1-t_2)^2 + \left(\frac{1}{t_1}+\frac{1}{t_2}\right)^2$$

$$= 2\left(t_1^2+\frac{1}{t_1^2}\right)=4\sqrt{5} \Rightarrow \text{Area}^2 = 80$$

11. Official Ans. by NTA (3)

Sol.



$$\frac{x^2}{4}-\frac{y^2}{2}=1$$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$$

$\therefore$ Focus F(ae, 0) $\Rightarrow F\left(\sqrt{6},0\right)$

equation of tangent at P to the hyperbola is

$2x - y\sqrt{6} = 2$

tangent meet x-axis at Q(1, 0)

& latus rectum $x = \sqrt{6}$ at $R\left(\sqrt{6},\frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)\right)$

$$\therefore \text{Area of } \Delta_{QFR} = \frac{1}{2}\left(\sqrt{6}-1\right).\frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)$$

$$=\frac{7}{\sqrt{6}}-2$$