FUNCTION

1. Let [x] denote the greatest integer $\le$ x, where $x \in \mathbf{R}$. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty,a)\cup[b,c)\cup[4,\infty), a<b<c$, then the value of $a+b+c$ is :

(1) 8 (2) 1 (3) –2 (4) –3

2. Let $f:\mathbf{R}-\left\{\frac{\alpha}{6}\right\}\to\mathbf{R}$ be defined by $f(x)=\frac{5x+3}{6x-\alpha}$. Then the value of α for which (fof)(x) = x, for all $x\in\mathbf{R}-\left\{\frac{\alpha}{6}\right\}$, is :

(1) No such α exists (2) 5
(3) 8 (4) 6

3. Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in \mathbf{R}$ satisfying the equation $[e^x]^2+[e^x+1]-3=0$ lie in the interval :

(1) $\left[0,\frac{1}{e}\right)$ (2) $[\log_e 2, \log_e 3)$
(3) [1, e) (4) $[0, \log_e 2)$

4. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions f : A → A such that $f(1)+f(2)=3-f(3)$ is equal to

5. Let g : **N** → **N** be defined as
g(3n + 1) = 3n + 2,
g(3n + 2) = 3n + 3,
g(3n + 3) = 3n + 1, for all n ≥ 0.
Then which of the following statements is true ?
(1) There exists an onto function f : **N** → **N** such that fog = f
(2) There exists a one–one function f: **N** → **N** such that fog = f
(3) gogog = g
(4) There exists a function f : **N** → **N** such that gof = f

6. If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to :

(1) 0 (2) 4 (3) –2 (4) 2

7. Consider function f : A → B and g : B → C (A, B, C ⊆ **R**) such that $(gof)^{-1}$ exists, then:
(1) f and g both are one-one
(2) f and g both are onto
(3) f is one-one and g is onto
(4) f is onto and g is one-one

8. Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions f : S → S such that f(m·n) = f(m)·f(n) for every m, n ∈ S and m · n ∈ S is equal to______.

9. Let f : **R** → **R** be defined as
$f(x+y)+f(x-y)=2f(x)f(y),\ f\left(\frac{1}{2}\right)=-1.$
Then,
the value of $\sum_{k=1}^{20}\frac{1}{\sin(k)\sin(k+f(k))}$ is equal to :
(1) $\operatorname{cosec}^2(21)\cos(20)\cos(2)$
(2) $\sec^2(1)\sec(21)\cos(20)$
(3) $\operatorname{cosec}^2(1)\operatorname{cosec}(21)\sin(20)$
(4) $\sec^2(21)\sin(20)\sin(2)$

10. The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

(1) $\left(-1,-\frac{1}{2}\right]\cup(0,\infty)$ (2) $\left[-\frac{1}{2},0\right)\cup[1,\infty)$
(3) $\left(-\frac{1}{2},\infty\right)-\{0\}$ (4) $\left[-\frac{1}{2},\infty\right)-\{0\}$

11. Let f : **N** → **N** be a function such that f(m + n) = f(m) + f(n) for every m, n ∈ **N**. If f(6) = 18, then f(2) · f(3) is equal to :

(1) 6 (2) 54 (3) 18 (4) 36

12. The range of the function,

$$f(x)=\log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$$

is :

(1) $\left(0,\sqrt{5}\right)$ (2) $[-2, 2]$

(3) $\left[\frac{1}{\sqrt{5}},\sqrt{5}\right]$ (4) $[0, 2]$

13. Let f(x) be a polynomial of degree 3 such that $f(k)=-\frac{2}{k}$ for k = 2, 3, 4, 5. Then the value of $52 - 10\,f(10)$ is equal to :

14. Let f, g : N → N such that

$f(n + 1) = f(n) + f(1)\ \forall\ n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?

(1) If fog is one-one, then g is one-one

(2) If f is onto, then $f(n) = n\ \forall\ n\in N$

(3) f is one-one

(4) If g is onto, then fog is one-one

15. A function f(x) is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series

$$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+......+f\left(\frac{39}{20}\right)$$ is equal to :

(1) $\frac{19}{2}$ (2) $\frac{49}{2}$ (3) $\frac{29}{2}$ (4) $\frac{39}{2}$

16. Let A = {1, 2, 3, ..., 10} and f : A → A be defined as $f(k)=\begin{cases}k+1 & \text{if } k \text{ is odd}\\ k & \text{if } k \text{ is even}\end{cases}$

Then the number of possible functions g : A → A such that $go f = f$ is

(1) 10^5 (2) $^{10}C_5$ (3) 5^5 (4) 5!

17. Let $f(x) = \sin^{-1}x$ and $g(x)=\frac{x^2-x-2}{2x^2-x-6}$. If $g(2)=\lim_{x\to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty, -2]\cup\left[-\frac{3}{2}, \infty\right)$

(2) $(-\infty, -2]\cup[-1, \infty)$

(3) $(-\infty, -2]\cup\left[-\frac{4}{3}, \infty\right)$

(4) $(-\infty, -1]\cup[2, \infty)$

18. Let f be any function defined on R and let it satisfy the condition :

$|f(x) - f(y)| \le |(x - y)^2|,\ \forall\ (x,y) \in R$

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0,\ \forall\ x \in R$

(3) $f(x) = 0,\ \forall\ x \in R$

(4) $f(x) > 0,\ \forall\ x \in R$

19. If $a + \alpha = 1$, $b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \ne 0$, then the value of expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is _______.

20. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is :

(1) 3 (2) 4 (3) 2 (4) 5

21. The inverse of $y = 5^{\log x}$ is :

(1) $x = 5^{\log y}$ (2) $x = y^{\log 5}$

(3) $x = y^{\frac{1}{\log 5}}$ (4) $x = 5^{\frac{1}{\log y}}$

22. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :

$f+g$, $f-g$, f/g, g/f, $g-f$ where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$

(1) $0 \le x \le 1$ (2) $0 \le x < 1$
(3) $0 < x < 1$ (4) $0 < x \le 1$

23. Let $f : R - \{3\} \rightarrow R - \{1\}$ be defined by $f(x)=\frac{x-2}{x-3}$. Let $g : R \rightarrow R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(1) 7 (2) 2 (3) 5 (4) 3

SOLUTION

1. Official Ans. by NTA (3)

Sol. For domain,

$$\frac{|[x]|-2}{|[x]|-3} \geq 0$$

Case I : When $|[x]|-2 \geq 0$

and $|[x]|-3 > 0$

$\therefore$ $x \in (-\infty, -3) \cup [4, \infty)$ .....(1)

Case II : When $|[x]|-2 \leq 0$

and $|[x]|-3 < 0$

$\therefore$ $x \in [-2, 3)$ .....(2)

So, from (1) and (2)

we get

Domain of function

$= (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$\therefore$ $(a + b + c) = -3 + (-2) + 3 = -2$ $(a < b < c)$

$\Rightarrow$ Option (3) is correct.

2. Official Ans. by NTA (2)

Sol. $f(x) = \dfrac{5x+3}{6x-\alpha} = y$ ......(i)

$5x + 3 = 6xy - \alpha y$

$x(6y - 5) = \alpha y + 3$

$x = \dfrac{\alpha y+3}{6y-5}$

$f^{-1}(x) = \dfrac{\alpha x+3}{6x-5}$ .....(ii)

$f o f(x) = x$

$f(x) = f^{-1}(x)$

From eq^n (i) & (ii)

Clearly $(\alpha = 5)$

3. Official Ans. by NTA (4)

Sol. $\left[e^x\right]^2 + \left[e^x + 1\right] - 3 = 0$

$\Rightarrow \left[e^x\right]^2 + \left[e^x\right] + 1 - 3 = 0$

Let $\left[e^x\right] = t$

$\Rightarrow t^2 + t - 2 = 0$

$\Rightarrow t = -2, 1$

$\left[e^x\right] = -2$ (Not possible)

or $\left[e^x\right] = 1$ $\therefore$ $1 \leq e^x < 2$

$\Rightarrow$ $\ln(1) \leq x < \ln(2)$

$\Rightarrow$ $0 \leq x < \ln(2)$

$\Rightarrow$ $x \in [0, \ln 2)$

4. Official Ans. by NTA (720)

Sol. $f(1) + f(2) = 3 - f(3)$

$\Rightarrow f(1) + f(2) = 3 + f(3) = 3$

The only possibility is : 0 + 1 + 2 = 3

$\Rightarrow$ Elements 1, 2, 3 in the domain can be mapped with 0, 1, 2 only.

So number of bijective functions.

$= \lfloor 3 \times \lfloor 5 = 720$

5. Official Ans. by NTA (1)

Sol. $g : N \to N \quad g(3n+1) = 3n+2$

$g(3n+2) = 3n+3$

$g(3n+3) = 3n+1$

$$g(x) = \begin{bmatrix} x+1 & x=3k+1 \\ x+1 & x=3k+2 \\ x-2 & x=3k+3 \end{bmatrix}$$

$$g(g(x)) = \begin{bmatrix} x+2 & x=3k+1 \\ x-1 & x=3k+2 \\ x-1 & x=3k+3 \end{bmatrix}$$

$$g(g(g(x))) = \begin{bmatrix} x & x=3k+1 \\ x & x=3k+2 \\ x & x=3k+3 \end{bmatrix}$$

If $f : N \to N$, f is a one-one function such that

$f(g(x)) = f(x) \Rightarrow g(x) = x$, which is not the case

If f $f : N \to N$ f is an onto function

such that $f(g(x)) = f(x)$,

one possibility is

$$f(x) = \begin{bmatrix} n & x=3n+1 \\ n & x=3n+2 \quad n \in N_0 \\ n & x=3n+3 \end{bmatrix}$$

Here $f(x)$ is onto, also $f(g(x)) = f(x)\ \forall\ x \in N$

6. Official Ans. by NTA (2)

Sol. $\sum_{n=8}^{100}\left[\frac{(-1)^n.n}{2}\right]$

$= 4 - 5 + 5 - 6 + 6 + \ldots - 50 + 50 = 4$

7. Official Ans. by NTA (3)

Sol. $\therefore (gof)^{-1}$ exist $\Rightarrow$ gof is bijective

$\Rightarrow$ 'f' must be one-one and 'g' must be ONTO

8. Official Ans. by NTA (490)

Sol. F(mn) = f(m). f(n)

Put m = 1 **f(n) = f(1).** f(n) $\Rightarrow$ f(1) = 1

Put m = n = 2

$$f(4) = f(2).f(2)\begin{cases} f(2)=1 \Rightarrow f(4)=1 \\ \text{or} \\ f(2)=2 \Rightarrow f(4)=4 \end{cases}$$

Put m = 2, n = 3

$$f(6) = f(2).f(3)\begin{cases} \text{when } f(2)=1 \\ f(3)=1 \text{ to } 7 \\ \\ f(2)=2 \\ f(3)=1 \text{ or } 2 \text{ or } 3 \end{cases}$$

f(5), f(7) can take any value

Total = $(1 \times 1 \times 7 \times 1 \times 7 \times 1 \times 7) + (1 \times 1 \times 3 \times 1 \times 7 \times 1 \times 7)$

= 490

9. Official Ans. by NTA (3)

Sol. f(x) = cos λx

$\because f\left(\frac{1}{2}\right) = -1$

So, $-1 = \cos\frac{\lambda}{2}$

$\Rightarrow \lambda = 2\pi$

Thus f(x) = cos 2πx

Now k is natural number

Thus f(k) = 1

$$\sum_{k=1}^{20}\frac{1}{\sin k \sin(k+1)} = \frac{1}{\sin 1}\sum_{k=1}^{20}\left[\frac{\sin((k+1)-k)}{\sin k \cdot \sin(k+1)}\right]$$

$$= \frac{1}{\sin 1}\sum_{k=1}^{20}(\cot k - \cot(k+1))$$

$$= \frac{\cot 1 - \cot 21}{\sin 1} = \operatorname{cosec}^2 1 \operatorname{cosec}(21) \cdot \sin 20$$

10. Official Ans. by NTA (4)

Sol. $\frac{1+x}{x} \in (-\infty,-1] \cup [1,\infty)$

$\frac{1}{x} \in (-\infty,-2] \cup [0,\infty)$

$x \in \left[-\frac{1}{2},0\right) \cup (0,\infty)$

$x \in \left[-\frac{1}{2},\infty\right) - \{0\}$

11. Official Ans. by NTA (2)

Sol. $f(m + n) = f(m) + f(n)$

Put m = 1, n = 1

$f(2) = 2f(1)$

Put m = 2, n = 1

$f(3) = f(2) + f(1) = 3f(1)$

Put m = 3, n = 3

$f(6) = 2f(3) \Rightarrow f(3) = 9$

$\Rightarrow f(1) = 3,\ f(2) = 6$

$f(2).f(3) = 6 \times 9 = 54$

12. Official Ans. by NTA (4)

Sol. $f(x) = \log_{\sqrt{5}}$

$\left(3 + \cos\left(\frac{3\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right) - \cos\left(\frac{3\pi}{4}-x\right)\right)$

$f(x) = \log_{\sqrt{5}}\left[3 + 2\cos\left(\frac{\pi}{4}\right)\cos(x) - 2\sin\left(\frac{3\pi}{4}\right)\sin(x)\right]$

$f(x) = \log_{\sqrt{5}}\left[3 + \sqrt{2}(\cos x - \sin x)\right]$

Since $-\sqrt{2} \le \cos x - \sin x \le \sqrt{2}$

$\Rightarrow \log_{\sqrt{5}}\left[3 + \sqrt{2}(-\sqrt{2}) \le f(x) \le \log_{\sqrt{5}}\left[3 + \sqrt{2}(\sqrt{2})\right]\right]$

$\Rightarrow \log_{\sqrt{5}}(1) \le f(x) \le \log_{\sqrt{5}}(5)$

So Range of f(x) is [0, 2]

Option (4)

13. Official Ans. by NTA (26)

Sol. $k\,f(k) + 2 = \lambda\,(x-2)(x-3)(x-4)(x-5)$ ...(1)

put x = 0

we get $\lambda = \frac{1}{60}$

Now put λ in equation (1)

$\Rightarrow kf(k) + 2 = \frac{1}{60}(x-2)(x-3)(x-4)(x-5)$

Put x = 10

$\Rightarrow 10f(10) + 2 = \frac{1}{60}(8)(7)(6)(5)$

$\Rightarrow 52 - 10f(10) = 52 - 26 = 26$

14. Official Ans. by NTA (4)

Sol. $f(n + 1) - f(n) = f(1)$

$\Rightarrow f(n) = nf(1)$

$\Rightarrow$ f is one-one

Now, Let $f(g(x_2)) = f(g(x_1))$

$\Rightarrow g(x_2) = g(x_1)$ (as f is one-one)

$\Rightarrow x_1 = x_2$ (as fog is one-one)

$\Rightarrow$ g is one-one

Now, f(g(n)) = g(n) f(1)

may be many-one if

g(n) is many-one

15. Official Ans. by NTA (4)

Sol. $f(x) = \frac{5^x}{5^x+5}$ $\qquad f(2-x) = \frac{5}{5^x+5}$

$f(x) + f(2-x) = 1$

$\Rightarrow f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + \ldots + f\left(\frac{39}{20}\right)$

$= \left(f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right)\right) + \ldots + \left(f\left(\frac{19}{20}\right) + f\left(\frac{21}{20}\right) + f\left(\frac{20}{20}\right)\right)$

$= 19 + \frac{1}{2} = \frac{39}{2}$

16. Official Ans. by NTA (1)

Sol. $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x, & \text{if } x \text{ is even} \end{cases}$

$\because$ g : A → A such that $g(f(x)) = f(x)$

⇒ If x is even then g(x) = x ...(1)

If x is odd then g(x + 1) = x + 1 ...(2)

from (1) and (2) we can say that

g(x) = x if x is even

⇒ If x is odd then g(x) can take any value in set A

so number of g(x) = $10^5 \times 1$

17. Official Ans. by NTA (3)

Sol. Domain of fog(x) = $\sin^{-1}(g(x))$

$\Rightarrow |g(x)| \le 1,\ g(2) = \frac{3}{7}$

$$\left|\frac{x^2-x-2}{2x^2-x-6}\right| \le 1$$

$$\left|\frac{(x+1)(x-2)}{(2x+3)(x-2)}\right| \le 1$$

$$\frac{x+1}{2x+3} \le 1 \text{ and } \frac{x+1}{2x+3} \ge -1$$

$$\frac{x+1-2x-3}{2x+3} \le 0 \text{ and } \frac{x+1+2x+3}{2x+3} \ge 0$$

$$\frac{x+2}{2x+3} \ge 0 \text{ and } \frac{3x+4}{2x+3} \ge 0$$

$$x \in (-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$$

18. Official Ans. by NTA (4)

Sol. $\left|\frac{f(x)-f(y)}{(x-y)}\right| \le |(x-y)|$

x – y = h let ⇒ x = y + h

$$\lim_{x\to 0}\left|\frac{f(y+h)-f(y)}{h}\right| \le 0$$

$\Rightarrow |f'(y)| \le 0 \Rightarrow f'(y) = 0$

⇒ $f(y)$ = k (constant)

and $f(0) = 1$ given

So, $f(y) = 1 \Rightarrow f(x) = 1$

19. Official Ans. by NTA (2)

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ....(1)

replace x by $\frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ ....(2)

(1) + (2)

$$(a+\alpha)f(x) + (a+\alpha)f\left(\frac{1}{x}\right) = x(b+\beta) + (b+\beta)\frac{1}{x}$$

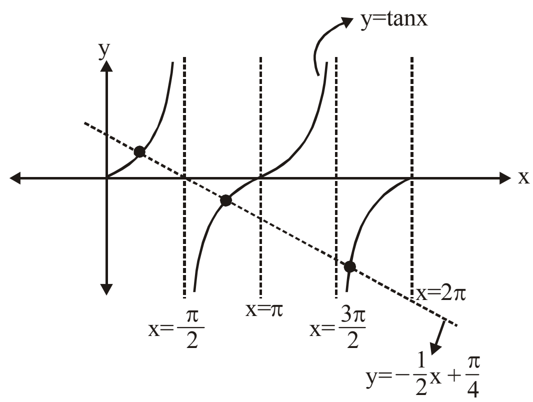
$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = \frac{b+\beta}{a+\alpha} = \frac{2}{1} = 2$$

20. Official Ans. by NTA (1)

Sol. $x + 2\tan x = \frac{\pi}{2}$

$\Rightarrow 2\tan x = \frac{\pi}{2} - x$

$\Rightarrow \tan x = -\frac{1}{2}x + \frac{\pi}{4}$



Number of soluitons of the given eauation is '3'.

Ans. (1)

21. Official Ans. by NTA (3)

Allen Ans. (1 or 2 or 3)

Sol. Given $y = 5^{(\log_a x)} = f(x)$

Interchanging x & y for inverse

$x = 5^{(\log_a y)} = y^{(\log_a 5)}$

option (1) or option (2)

Further, from given relation

$\log_5 y = \log_a x$

$\Rightarrow x = a^{(\log_5 y)} = y^{(\log_5 a)}$

$\Rightarrow x = y^{\left(\frac{1}{\log_a 5}\right)} = f^{-1}(y)$

option (3)

22. Official Ans. by NTA (3)

Sol. $f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$, domain [0, 1]

$f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$, domain [0, 1]

$g(x) - f(x) = \sqrt{1-x} - \sqrt{x}$, domain [0, 1]

$\frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{1-x}}$, domain [0, 1)

$\frac{g(x)}{f(x)} = \frac{\sqrt{1-x}}{\sqrt{x}}$, domain (0, 1]

So, common domain is (0, 1)

23. Official Ans. by NTA (3)

Sol. $f(x) = y = \frac{x-2}{x-3}$

$\therefore x = \frac{3y-2}{y-1}$

$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$

& $g(x) = y = 2x - 3$

$\therefore x = \frac{y+3}{2}$

$\therefore g^{-1}(x) = \frac{x+3}{2}$

$\because f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\therefore x^2 - 5x + 6 = 0 \; \begin{cases} x_1 \\ x_2 \end{cases}$

$\therefore$ sum of roots

$x_1 + x_2 = 5$