DIFFERENTIAL EQUATION

1. Let y = y(x) be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy=\left(y\tan\left(\frac{y}{x}\right)-x\right)dx,$$

$-1 \le x \le 1, y\left(\frac{1}{2}\right)=\frac{\pi}{6}$. Then the area of the region bounded by the curves $x=0, x=\frac{1}{\sqrt{2}}$ and y = y (x) in the upper half plane is:

(1) $\frac{1}{8}(\pi-1)$ (2) $\frac{1}{12}(\pi-3)$

(3) $\frac{1}{4}(\pi-2)$ (4) $\frac{1}{6}(\pi-1)$

2. Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}\,dx+\left(\frac{y}{x}\right)dy=0, y(1)=-1.$

Then the value of $(y(3))^2$ is equal to:

(1) $1-4e^3$ (2) $1-4e^6$

(3) $1+4e^3$ (4) $1+4e^6$

3. If [x] denotes the greatest integer less than or equal to x, then the value of the integral

$\int_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$ is equal to :

(1) $-\pi$ (2) π (3) 0 (4) 1

4. If $f:\mathbf{R}\to\mathbf{R}$ is given by $f(x) = x + 1$, then the value of

$$\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+.....+f\left(\frac{5(n-1)}{n}\right)\right],$$

is :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{1}{2}$ (4) $\frac{7}{2}$

5. Let a curve y = y(x) be given by the solution of the differential equation $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx=\sqrt{e^{2x}-1}\,dy$

If it intersects y-axis at y = –1, and the intersection point of the curve with x-axis is (α, 0), then e^{α} is equal to ________.

6. Let y = y(x) be the solution of the differential equation $\text{cosec}^2x\,dy + 2dx = (1 + y\cos 2x)\,\text{cosec}^2x\,dx$, with $y\left(\frac{\pi}{4}\right)=0$. Then, the value of $(y(0) + 1)^2$ is equal to :

(1) $e^{1/2}$ (2) $e^{-1/2}$ (3) e^{-1} (4) e

7. Let y = y(x) be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx = (x+2)\,dy$, y(1) = 1. If the domain of y = y(x) is an open interval (α, β), then |α + β| is equal to _______.

8. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=1+xe^{y-x}, -\sqrt{2}<x<\sqrt{2}, y(0)=0$ then, the minimum value of y(x), $x\in(-\sqrt{2},\sqrt{2})$ is equal to :

(1) $(2-\sqrt{3})-\log_e 2$

(2) $(2+\sqrt{3})+\log_e 2$

(3) $(1+\sqrt{3})-\log_e(\sqrt{3}-1)$

(4) $(1-\sqrt{3})-\log_e(\sqrt{3}-1)$

9. Let y = y(x) be solution of the following differential equation

$$e^y\frac{dy}{dx}-2e^y\sin x+\sin x\cos^2x=0,\quad y\left(\frac{\pi}{2}\right)=0$$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then 4(α + β) is equal to ________.

10. Let y = y (x) be the solution of the differential equation $x\,dy = (y + x^3\cos x)dx$ with y(π) = 0, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(1) $\frac{\pi^2}{4}+\frac{\pi}{2}$ (2) $\frac{\pi^2}{2}+\frac{\pi}{4}$

(3) $\frac{\pi^2}{2}-\frac{\pi}{4}$ (4) $\frac{\pi^2}{4}-\frac{\pi}{2}$

11. Let a curve y = f(x) pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of f(e) is equal to _______.

12. Let y = y(x) be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with y(0) = 0.

If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$, then the value of α is equal to:

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) 2 (4) $-\frac{1}{2}$

13. Let F : [3, 5] → **R** be a twice differentiable function on (3, 5) such that

$$F(x) = e^{-x}\int_3^x (3t^2 + 2t + 4F'(t))\,dt.$$

If $F'(4) = \frac{\alpha e^{\beta} - 224}{(e^{\beta} - 4)^2}$, then $\alpha + \beta$ is equal to _________.

14. If y = y(x), $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$$\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0$$, with y(0) = 0,

then $5y'\left(\frac{\pi}{2}\right)$ is equal to _________.

15. Let y = y(x) be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, x > 2. If y(3) = 3, then y(4) is equal to :

(1) 4 (2) 12 (3) 8 (4) 16

16. Let y= y(x) be the solution of the differential equation $dy = e^{\alpha x + y}\,dx$; $\alpha \in \mathbf{N}$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to_____.

17. Let y = y(x) be a solution curve of the differential equation $(y + 1)\tan^2 x\,dx + \tan x\,dy + y\,dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim_{x \to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :

(1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{4} - 1$ (3) $\frac{\pi}{4} + 1$ (4) $\frac{\pi}{4}$

18. Let y(x) be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0$, $x > 0$. If y(e) = 1, then y(1) is equal to :

(1) 0 (2) 2

(3) $\log_e 2$ (4) $\log_e(2e)$

19. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)\,x - 2\cos x$ such that y(0) = 7. Then y(π) is equal to :

(1) $2e^{\pi^2} + 5$ (2) $e^{\pi^2} + 5$

(3) $3e^{\pi^2} + 5$ (4) $7e^{\pi^2} + 5$

20. Let us consider a curve, y = f(x) passing through the point (–2, 2) and the slope of the tangent to the curve at any point (x, f(x)) is given by $f(x) + xf'(x) = x^2$. Then :

(1) $x^2 + 2xf(x) - 12 = 0$ (2) $x^3 + xf(x) + 12 = 0$

(3) $x^3 - 3xf(x) - 4 = 0$ (4) $x^2 + 2xf(x) + 4 = 0$

21. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) form the line 3x + 4y = 5, is given by :

(1) $10\frac{d^2y}{dx^2} = 11$ (2) $11\frac{d^2x}{dy^2} = 10$

(3) $10\frac{d^2x}{dy^2} = 11$ (4) $11\frac{d^2y}{dx^2} = 10$

22. If the solution curve of the differential equation $(2x - 10y^3)\,dy + y\,dx = 0$, passes through the points $(0, 1)$ and $(2, \beta)$, then β is a root of the equation :

(1) $y^5 - 2y - 2 = 0$ (2) $2y^5 - 2y - 1 = 0$

(3) $2y^5 - y^2 - 2 = 0$ (4) $y^5 - y^2 - 1 = 0$

23. Let f be a non–negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt,\ 0 \le x \le 1$ and $f(0) = 0$, then $\lim_{x\to 0} \frac{1}{x^2}\int_0^x f(t)\,dt$:

(1) equals 0 (2) equals 1

(3) does not exist (4) equals $\frac{1}{2}$

24. If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, $y(0) = 1$, then $y(1)$ is equal to :

(1) $\log_2(2 + e)$ (2) $\log_2(1 + e)$

(3) $\log_2(2e)$ (4) $\log_2(1 + e^2)$

25. If $\frac{dy}{dx} = \frac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y}\log_e 2}$, $y(0) = 0$, then for $y = 1$, the value of x lies in the interval :

(1) $(1, 2)$ (2) $\left(\frac{1}{2}, 1\right]$

(3) $(2, 3)$ (4) $\left(0, \frac{1}{2}\right]$

26. If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right], x > 0, \phi > 0$, and $y(1) = -1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to :

(1) $4\,\phi(2)$ (2) $4\,\phi(1)$

(3) $2\,\phi(1)$ (4) $\phi(1)$

27. If $y = y(x)$ is the solution curve of the differential equation $x^2 dy + \left(y - \frac{1}{x}\right)dx = 0$; $x > 0$ and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (2) $3 + \frac{1}{\sqrt{e}}$

(3) $3 + e$ (4) $3 - e$

28. If a curve $y = f(x)$ passes through the point $(1, 2)$ and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)\,dx = \frac{62}{5}$?

(1) 5 (2) 10 (3) $\frac{62}{5}$ (4) $\frac{31}{5}$

29. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

(1) $\log_e 18$ (2) $\log_e 9$

(3) $\frac{1}{2}\log_e 18$ (4) $2\log_e 18$

30. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2 - 4x + y + 8}{x - 2}$, then this curve also passes through the point:

(1) (5, 4) (2) (4, 5) (3) (4, 4) (4) (5, 5)

31. If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + x\,dy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to ______.

32. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line x + 2y = 4 at x = –2, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $\frac{18}{35}$ (2) $-\frac{4}{3}$ (3) $-\frac{18}{19}$ (4) $-\frac{18}{11}$

33. The rate of growth of bacteria in a culture is proportional to the number of bacteris present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4 (2) 8 (3) 2 (4) 16

34. If y = y(x) is the solution of the equaiton $e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x, y(0)=0$;

then $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to

35. The difference between degree and order of a differential equation that represents the family of curves given by $y^2=a\left(x+\frac{\sqrt{a}}{2}\right), a>0$ is

36. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+(\tan x)\,y=\sin x,\ 0\le x\le\frac{\pi}{3}$, with y(0) = 0, then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\frac{1}{4}\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\log_e 2$ (4) $\frac{1}{2}\log_e 2$

37. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2$, x > 0. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1,1), then the area enclosed by the curves C_1 and C_2 is equal to :

(1) $\pi - 1$ (2) $\frac{\pi}{2}-1$ (3) $\pi + 1$ (4) $\frac{\pi}{4}+1$

38. If y = y(x) is the solution of the differential equation, $\frac{dy}{dx}+2y\tan x=\sin x, y\left(\frac{\pi}{3}\right)=0$, then the maximum value of the function y(x) over $\mathbb{R}$ is equal to :

(1) 8 (2) $\frac{1}{2}$ (3) $-\frac{15}{4}$ (4) $\frac{1}{8}$

39. Let y = y(x) be the solution of the differential equation cosx (3sinx + cosx + 3)dy = (1 + y sinx (3sinx + cosx + 3))dx, $0\le x\le\frac{\pi}{2}$, y(0) = 0. Then , $y\left(\frac{\pi}{3}\right)$ is equal to:

(1) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (2) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

(3) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$ (4) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

40. If the curve y = y(x) is the solution of the differential equation $2(x^2+x^{5/4})dy-y(x+x^{1/4})dx=2x^{9/4}dx$, x > 0 which passes through the point $\left(1, 1-\frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to :

(1) $4\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$ (2) $\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$

(3) $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$ (4) $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

41. Which of the following is true for y(x) that satisfies the differential equation

$\frac{dy}{dx} = xy - 1 + x - y\ ;\ y(0) = 0:$

(1) $y(1) = e^{-\frac{1}{2}} - 1$ (2) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

(3) $y(1) = 1$ (4) $y(1) = e^{\frac{1}{2}} - 1$

42. If $[\cdot]$ represents the greatest integer function, then the value of

$\left| \int_0^{\sqrt{\frac{\pi}{2}}} \left[[x^2] - \cos x \right] dx \right|$ is ________ .

43. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

44. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right)$, $0 < x < 2.1$, with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to :

(1) $\frac{-e^{3/2}}{(e^2+1)^2}$ (2) $-\frac{2e^2}{(1+e^2)^2}$

(3) $\frac{e^{5/2}}{(1+e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2+1)^2}$

45. Let y = y(x) be the solution of the differential equation $x\,dy - y\,dx = \sqrt{(x^2 - y^2)}\,dx$, $x \ge 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^{\pi}$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to ____

SOLUTION

1. Official Ans. by NTA (1)

Sol. We have

$$\frac{dy}{dx}=\frac{x\left(\frac{y}{x}\cdot\tan\frac{y}{x}-1\right)}{x\tan\frac{y}{x}}$$

$$\therefore \frac{dy}{dx}=\frac{y}{x}-\cot\left(\frac{y}{x}\right)$$

Put $\frac{y}{x}=v$

$\Rightarrow y = vn$

$$\therefore \frac{dy}{dx}=v+\frac{ndv}{dx}$$

Now, we get

$$v+n\frac{dv}{dx}=v-\cot(v)$$

$$\Rightarrow \int(\tan)dv=-\int\frac{dx}{x}$$

$$\therefore\ \ell n\left|\sec\left(\frac{y}{x}\right)\right|=-\ell n|x|+c$$

$$\text{As}\left(\frac{1}{2}\right)=\left(\frac{y}{x}\right)\Rightarrow \boxed{C=0}$$

$$\therefore\ \sec\left(\frac{y}{x}\right)=\frac{1}{x}$$

$$\Rightarrow \cos\left(\frac{y}{x}\right)=x$$

$$\therefore \boxed{y=x\cos^{-1}(x)}$$

So, required bounded area

$$=\int_0^{1/\sqrt{2}}\underset{(II)}{x}\,(\underset{(I)}{\cos^{-1}x})dx=\left(\frac{\pi-1}{8}\right)$$

(I.B.P.)

$\therefore$ option (1) is correct.

2. Official Ans. by NTA (2)

Sol. $e^x\sqrt{1-y^2}dx+\frac{y}{x}dy=0$

$$\Rightarrow e^x\sqrt{1-y^2}dx+\frac{-y}{x}dy$$

$$\Rightarrow \int\frac{-y}{\sqrt{1-y^2}}\,dy=\int\underset{II}{e^x}\ \underset{1}{x}\,dx$$

$$\Rightarrow \sqrt{1-y^2}=e^x(x-1)+c$$

Given : At x = 1, y = – 1

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$$\therefore \sqrt{1-y^2}=e^x(x-1)$$

At x = 3 $\quad 1 - y^2 = (e^3\,2)^2 \Rightarrow y^2 = 1 - 4e^6$

3. Official Ans. by NTA (1)

Sol. $I=\int_{-\pi/2}^{\pi/2}([x]+[-\sin x])dx$ .....(i)

$I=\int_{-\pi/2}^{\pi/2}([-x]+[\sin x])dx$ ….(2)

(King property)

$$2I=\int_{-\pi/2}^{\pi/2}\left(\underbrace{[x]+[-x]}_{-1}\right)+\left(\underbrace{[\sin x]+[-\sin x]}_{-1}\right)dx$$

$$2I=\int_{-\pi/2}^{\pi/2}(-2)dx=-2(\pi)$$

$I=-\pi$

4. Official Ans. by NTA (4)

Sol. $I=\sum_{r=0}^{n-1}f\left(\frac{5r}{n}\right)\frac{1}{n}$

$$I=\int_0^1 f(5x)dx$$

$$I=\int_0^1(5x+1)dx$$

$$I=\left[\frac{5x^2}{2}+x\right]_0^1$$

$$I=\frac{5}{2}+1=\frac{7}{2}$$

5. Official Ans. by NTA (2)

Sol. $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}\,dy$

Put $\cos^{-1}(e^{-x})\ \theta$, $\theta \in [0, \pi]$

$\cos\theta = e^{-x} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 = e^{-x}$

$\cos\frac{\theta}{2} = \sqrt{\frac{e^{-x}+1}{2}} = \sqrt{\frac{e^{x}+1}{2c^{x}}}$

$\sqrt{\frac{e^{x}+1}{2c^{x}}}dx = \sqrt{e^{2x}-1}\,dy$

$\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{e^{x}}\sqrt{e^{x}-1}} = \int dy$

Put $e^x = t$, $\frac{dt}{dx} = e^x$

$\frac{1}{\sqrt{2}}\int\frac{dt}{e^{x}\sqrt{e^{x}}\sqrt{e^{x}-1}} = \int dy$

$\int\frac{dt}{t\sqrt{t^2-t}} = \sqrt{2}y$

Put $t = \frac{1}{z}, \frac{dt}{dz} = -\frac{1}{z^2}$

$\int\frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}-\frac{1}{z}}} = \sqrt{2}y$

$-\int\frac{dz}{\sqrt{1-z}} = \sqrt{2}y$

$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$

$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}y + c$

$2(1-e^{-x})^{1/2} = \sqrt{2y} + c \xrightarrow{(0,-1)} \Rightarrow c = \sqrt{2}$

$2(1-e^{-x})^{1/2} = \sqrt{2}(y+1)$, passes through $(\alpha, 0)$

$2(1-e^{-\alpha})^{1/2} = \sqrt{2}$

$\sqrt{1-e^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1-e^{-\alpha} = \frac{1}{2}$

$e^{-\alpha} = \frac{1}{2} \Rightarrow e^{\alpha} = 2$

6. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} + 2\sin^2 x = 1 + y\cos 2x$

$\Rightarrow \frac{dy}{dx} + (-\cos 2x)y = \cos 2x$

$\text{I.F.} = e^{\int -\cos 2x\,dx} = e^{-\frac{\sin 2x}{2}}$

Solution of D.E.

$y\left(e^{-\frac{\sin 2x}{2}}\right) = \int(\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx + c$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + c$

Given

$y\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 0 = -e^{-1/2} + c \Rightarrow \boxed{c = e^{-1/2}}$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + e^{-1/2}$

at $x = 0$

$y = -1 + e^{-1/2}$

$\Rightarrow y(0) = -1 + e^{-1/2} \Rightarrow (y(0)+1)^2 = e^{-1}$

7. Official Ans. by NTA (4)

Sol. $y + 1 = Y \Rightarrow dy = dY$

$x + 2 = X \Rightarrow dx = dX$

$$\Rightarrow \left(Xe^{\frac{Y}{X}} + Y\right)dX = XdY$$

$$\Rightarrow XdY - YdX = Xe^{Y/X}dX$$

$$\Rightarrow d\left(\frac{Y}{X}\right)e^{-\frac{Y}{X}} = \frac{dX}{X}$$

$$-e^{-Y/X} = \ell|X| + c$$

$$(3, 2) \rightarrow -e^{-2/3} = \ell|3| + c$$

$$-e^{-\frac{Y}{X}} = \ell n|X| - e^{-\frac{2}{3}} - \ell n3$$

$$e^{-\frac{Y}{X}} = e^{2/3} + \ell n3 - \ell n|X| > 0$$

$$\ell n|X| < \left(e^{2/3} + \ell n3\right)$$

Let $\lambda = \left(e^{2/3} + \ell n3\right)$

$|x + 2| < e^{\lambda}$

$-e^{\lambda} < x + 2 < e^{\lambda}$

$-e^{\lambda} - 2 < x < e^{\lambda} - 2$

$\alpha \qquad\qquad \beta$

$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$

Although x = –2 should be excluded from domain but according to the given problem it will the most appropriate solution.

8. Official Ans. by NTA (4)

Sol. $\frac{dy - dx}{e^{y-x}} = x\,dx$

$$\Rightarrow \frac{dy - dx}{e^{y-x}} = x\,dx$$

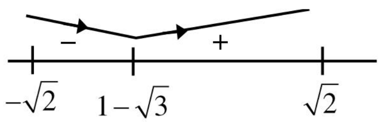
$$\Rightarrow -e^{x-y} = \frac{x^2}{2} + c$$

At $x = 0,\ y = 0 \Rightarrow c = -1$

$$\Rightarrow e^{x-y} = \frac{2 - x^2}{2}$$

$$\Rightarrow y = x - \ell n\left(\frac{2 - x^2}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{2x}{2 - x^2} = \frac{2 + 2x - x^2}{2 - x^2}$$



So minimum value occurs at $x = 1 - \sqrt{3}$

$$y\left(1 - \sqrt{3}\right) = \left(1 - \sqrt{3}\right) - \ell n\left(\frac{2 - \left(4 - 2\sqrt{3}\right)}{2}\right)$$

$$= \left(1 - \sqrt{3}\right) - \ell n\left(\sqrt{3} - 1\right)$$

9. Official Ans. by NTA (4)

Sol. Let $e^y = t$

$\Rightarrow \frac{dt}{dx} - (2\sin x)t = -\sin x \cos^2 x$

I.F. $= e^{2\cos x}$

$\Rightarrow t \cdot e^{2\cos x} = \int e^{2\cos x}.(-\sin x \cos^2 x)\,dx$

$\Rightarrow e^y \cdot e^{2\cos x} = \int e^{2z} \cdot z^2 dz, z = e^{2\cos x}$

$\Rightarrow e^y \cdot e^{2\cos x} = \frac{1}{2}.\cos^2 x \cdot e^{2\cos x} - \frac{1}{2}\cos x \cdot e^{2\cos x} + \frac{e^{2\cos x}}{4} + C$

at $x = \frac{\pi}{2}, y = 0 \Rightarrow C = \frac{3}{4}$

$\Rightarrow e^y = \frac{1}{2}\cos^2 x - \frac{1}{2}\cos x + \frac{1}{4} + \frac{3}{4} \cdot e^{-2\cos x}$

$\Rightarrow y = \log\left[\frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + \frac{3}{4}e^{-2\cos x}\right]$

Put x = 0

$\Rightarrow y = \log\left[\frac{1}{4} + \frac{3}{4}e^{-2}\right] \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$

10. Official Ans. by NTA (1)

Sol. $xdy = (y + x^3\cos x)dx$

$xdy = ydx + x^3\cos x dx$

$\frac{xdy - ydx}{x^2} = \frac{x^3 \cos x\, dx}{x^2}$

$\frac{d}{dx}\left(\frac{y}{x}\right) = \int x \cos x\, dx$

$\Rightarrow \frac{y}{x} = x\sin x - \int 1.\sin x\, dx$

$\frac{y}{x} = x\sin x + \cos x + C$

$\Rightarrow 0 = -1 + C \Rightarrow C = 1, x = \pi, y = 0$

so $\frac{y}{x} = x\sin x + \cos x + 1$

$y = x^2\sin x + x\cos x + x \qquad x = \frac{\pi}{2}$

$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

11. Official Ans. by NTA (1)

Sol. $y' = \frac{2y}{x\ell nx}$

$\Rightarrow \frac{dy}{y} = \frac{2dx}{x\ell nx}$

$\Rightarrow \ell n|y| = 2\ell n|\ell nx| + C$

put $x = 2, y = (\ell n2)^2$

$\Rightarrow c = 0$

$\Rightarrow y = (\ell nx)^2$

$\Rightarrow f(e) = 1$

12. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = e^{3x}.e^{4y} \Rightarrow \int e^{-4y}dy = \int e^{3x}dx$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C \Rightarrow -\frac{1}{4} - \frac{1}{3} = C \Rightarrow C = -\frac{7}{12}$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow e^{-4y} = \frac{4e^{3x} - 7}{-3}$

$e^{4y} = \frac{3}{7 - 4e^{3x}} \Rightarrow 4y = \ell n\left(\frac{3}{7 - 4e^{3x}}\right)$

$4y = \ell n\left(\frac{3}{6}\right)$ when $x = -\frac{2}{3}\ell n2$

$y = \frac{1}{4}\ell n\left(\frac{1}{2}\right) = -\frac{1}{4}\ell n2$

13. Official Ans. by NTA (16)

Sol. $F(3) = 0$

$$e^x F(x) = \int_3^x (3t^2 + 2t + 4F'(t))dt$$

$$e^x F(x) + e^x F'(x) = 3x^2 + 2x + 4F'(x)$$

$$(e^x - 4)\frac{dy}{dx} + e^x y = (3x^2 + 2x)$$

$$\frac{dy}{dx} + \frac{e^x}{(e^x - 4)} y = \frac{(3x^2 + 2x)}{(e^x - 4)}$$

$$ye^{\int \frac{e^x}{(e^x-4)}dx} = \int \frac{(3x^2 + 2x)}{(e^x - 4)} e^{\int \frac{e^x}{e^x - 4}dx} dx$$

$$y.(e^x - 4) = \int (3x^2 + 2x)dx + c$$

$$y(e^x - 4) = x^3 + x^2 + c$$

Put $x = 3 \Rightarrow c = -36$

$$F(x) = \frac{(x^3 + x^2 - 36)}{(e^x - 4)}$$

$$F'(x) = \frac{(3x^2 + 2x)(e^x - 4) - (x^3 + x^2 - 36)e^x}{(e^x - 4)^2}$$

Now put value of $x = 4$ we will get $\alpha = 12$ & $\beta = 4$

14. Official Ans. by NTA (2)

Sol. $\sec y \frac{dy}{dx} = 2\sin x \cos y$

$$\sec^2 y dy = 2\sin x dx$$

$$\tan y = -2\cos x + c$$

$c = 2$

$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \frac{\pi}{2}$

$\tan y = 2$

$$\sec^2 y \frac{dy}{dx} = 2\sin x$$

$$5\frac{dy}{dx} = 2$$

15. Official Ans. by NTA (2)

Sol. $(x - x^3)dy = (y + yx^2 - 3x^4)dx$

$\Rightarrow xdy - ydx = (yx^2 - 3x^4)dx + x^3dy$

$$\Rightarrow \frac{xdy - ydx}{x^2} = (ydx + xdy) - 3x^2dx$$

$$\Rightarrow d\left(\frac{y}{x}\right) = d(xy) - d(x^3)$$

Integrate

$$\Rightarrow \frac{y}{x} = xy - x^3 + c$$

given $f(3) = 3$

$$\Rightarrow \frac{3}{3} = 3 \times 3 - 3^3 + c$$

$\Rightarrow c = 19$

$$\therefore \frac{y}{x} = xy - x^3 + 19$$

at $x = 4$, $\frac{y}{4} = 4y - 64 + 19$

$15y = 4 \times 45$

$\Rightarrow y = 12$

16. Official Ans. by NTA (2)

Sol. $\int e^{-y}dy = \int e^{\alpha x} dx$

$$\Rightarrow e^{-y} = \frac{e^{\alpha x}}{\alpha} + c \quad ...(i)$$

Put $(x,y) = (\ell n2, \ell n2)$

$$\frac{-1}{2} = \frac{2^\alpha}{\alpha} + C \quad ...(ii)$$

Put $(x,y) \equiv (0, -\ell n2)$ in (i)

$$-2 = \frac{1}{\alpha} + C \quad ...(iii)$$

(ii) – (iii)

$$\frac{2^\alpha - 1}{\alpha} = \frac{3}{2}$$

$\Rightarrow \alpha = 2$ (as $\alpha \in \mathbb{N}$)

17. Official Ans. by NTA (4)

Sol. $(y + 1)\tan^2 x\, dx + \tan x\, dy + y\, dx = 0$

or $\frac{dy}{dx} + \frac{\sec^2 x}{\tan x}.y = -\tan x$

$IF = e^{\int \frac{\sec^2 x}{\tan x} dx} = e^{\ln \tan x} = \tan x$

$\therefore\ y \tan x = -\int \tan^2 x dx$

or $y \tan x = -\tan x + x + C$

or $y = -1 + \frac{x}{\tan x} + \frac{C}{\tan x}$

or $\lim_{x\to 0} xy = -x + \frac{x^2}{\tan x} + \frac{Cx}{\tan x} = 1$

or $C = 1$

$y(x) = \cot x + x \cot x - 1$

$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

18. Official Ans. by NTA (3)

Sol. $2x^2 dy + (e^y - 2x)dx = 0$

$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$

$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow$ Put $e^{-y} = z$

$\frac{-dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$

$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$

$xe^{-y} = \frac{1}{2}\log_e x + c$, passes through $(e,1)$

$\Rightarrow C = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

19. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} - 2xy = 2(2\sin x - 5)x - 2\cos x$

$IF = e^{-x^2}$

so,

$y.e^{-x^2} = \int e^{-x^2}(2x(2\sin x - 5) - 2\cos x)dx$

$\Rightarrow y.e^{-x^2} = e^{-x^2}(5 - 2\sin x) + c$

$\Rightarrow y = 5 - 2\sin x + c.e^{x^2}$

Given at $x = 0, y = 7$

$\Rightarrow 7 = 5 + c \Rightarrow c = 2$

So, $y = 5 - 2\sin x + 2e^{x^2}$

Now at $x = \pi$,

$y = 5 + 2e^{\pi^2}$

20. Official Ans. by NTA (3)

Sol. $y + \frac{xdy}{dx} = x^2$ (given)

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$

If $= e^{\int \frac{1}{x} dx} = x$

Solution of DE

$\Rightarrow y.x = \int x.x\ dx$

$\Rightarrow xy = \frac{x^3}{3} + \frac{c}{3}$

Passes through $(-2,2)$, so

$-12 = -8 + c \Rightarrow c = -4$

$\therefore\ 3xy = x^3 - 4$

ie. $3x.f(x) = x^3 - 4$

21. Official Ans. by NTA (4)

Sol. $\alpha.\ R = \frac{|3(2)+4(-3)-5|}{5} = \frac{11}{5}$

$(x-h)^2 = \frac{11}{5}(y-k)$

differentiate w.r.t 'x' : –

$2(x-h) = \frac{11}{5}\frac{dy}{dx}$

again differentiate

$2 = \frac{11}{5}\frac{d^2y}{dx^2}$

$\frac{11d^2y}{dx^2} = 10$.

22. Official Ans. by NTA (4)

Sol. $(2x - 10y^3)\ dy + ydx = 0$

$\Rightarrow \frac{dx}{dy} + \left(\frac{2}{y}\right)x = 10y^2$

I. F. $= e^{\int \frac{2}{y}dy} = e^{2\ell n(y)} = y^2$

Solution of D.E. is

$\therefore \quad x.\ y = \int (10y^2)\, y^2.dy$

$xy^2 = \frac{10y^5}{5} + C \Rightarrow xy^2 = 2y^5 + C$

It passes through (0, 1) $\rightarrow$ $0 = 2 + C \Rightarrow C = -2$

$\therefore$ Curve is $\boxed{xy^2 = 2y^5 - 2}$

Now, it passes through $(2,\beta)$

$2\beta^2 = 2\beta^5 - 2 \Rightarrow \beta^5 - \beta^2 - 1 = 0$

$\therefore$ β is root of an equation $\boxed{y^5 - y^2 - 1 = 0}$ Ans.

23. Official Ans. by NTA (4)

Sol. $\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)\, dt \quad 0 \le x \le 1$

differentiating both the sides

$\sqrt{1-(f'(x))^2} = f(x)$

$\Rightarrow 1 - (f'(x))^2 = f^2(x)$

$\frac{f'(x)}{\sqrt{1-f^2(x)}} = 1$

$\sin^{-1} f(x) = x + C$

$\because f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$

Now $\lim_{x\to 0} \frac{\int_0^x \sin t\, dt}{x^2} \left(\frac{0}{0}\right) = \frac{1}{2}$

24. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2^y}$

$2^y \frac{dy}{dx} = 2^x (2^y - 1)$

$\int \frac{2^y}{2^y - 1} dy = \int 2^x\, dx$

$\frac{\ell n(2^y - 1)}{\ell n 2} = \frac{2^x}{\ell n 2} + C$

$\Rightarrow \log_2(2^y - 1) = 2^x \log_2 e + C$

$\because y(0) = 1 \Rightarrow 0 = \log_2 e + C$

$C = -\log_2 e$

$\Rightarrow \log_2(2^y - 1) = (2^x - 1)\log_2 e$

put x = 1, $\log_2(2^y - 1) = \log_2 e$

$2^y = e + 1$

$y = \log_2(e + 1)$ Ans.

25. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}=\frac{2^x(y+2^y)}{2^x(1+2^y\ell n2)}$

$\Rightarrow \int\frac{(1+2^y)\ell n2}{(y+2^y)}dy=\int dx$

$\Rightarrow \ell n|y+2^y|=x+c$

$x=0;\ y=0\Rightarrow c=0$

$\Rightarrow x=\ell n|y+2^y|$

$\Rightarrow$ at $y=1,\ x=\ell n3$

$\because 3\in(e,e^2)\Rightarrow x\in(1,2)$

26. Official Ans. by NTA (2)

Sol. Let, $y = tx$

$\frac{dy}{dx}=t+x\frac{dt}{dx}$

$\therefore\ tx\left(t+x\frac{dt}{dx}\right)=x\left(t^2+\frac{\varphi(t^2)}{\varphi'(t^2)}\right)$

$t^2+xt\frac{dt}{dx}=t^2+\frac{\varphi(t^2)}{\varphi'(t^2)}$

$\int\frac{t\varphi'(t^2)}{\varphi(t^2)}dt=\int\frac{dx}{x}$

Let $\varphi(t^2)=p$

$\therefore \varphi'(t^2)2tdt=dp$

$\Rightarrow \int\frac{dy}{2p}=\int\frac{dx}{x}$

$\frac{1}{2}\ell n\varphi(t^2)=\ell nx+\ell nc$

$\varphi(t^2)=x^2k$

$\varphi\left(\frac{y^2}{x^2}\right)=kx^2,\ \varphi(1)=k$

$\varphi\left(\frac{y^2}{4}\right)=4\varphi(1)$

27. Official Ans. by NTA (4)

Sol. $x^2dy+\left(y-\frac{1}{x}\right)dx=0\ :\ x>0,\ y(1)=1$

$x^2dy+\frac{(xy-1)}{x}dx=0$

$x^2dy=\frac{(xy-1)}{x}dx$

$\frac{dy}{dx}=\frac{1-xy}{x^3}$

$\frac{dy}{dx}=\frac{1}{x^3}-\frac{y}{x^2}$

$\frac{dy}{dx}=\frac{1}{x^2}.y=\frac{1}{x^3}$

If $e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$

$ye^{-\frac{1}{x}}=\int\frac{1}{x^3}.e^{-\frac{1}{x}}$

$ye^{-\frac{1}{x}}=e^{-x}\left(1+\frac{1}{x}\right)+C$

$1.e^{-1}=e^{-1}(2)+C$

$C=-e^{-1}=-\frac{1}{e}$

$ye^{-\frac{1}{x}}=e^{-\frac{1}{x}}\left(1+\frac{1}{x}\right)-\frac{1}{e}$

$y\left(\frac{1}{2}\right)=3-\frac{1}{e}\times e^2$

$y\left(\frac{1}{2}\right)=3-e$

28. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}+\frac{y}{x}=bx^3$

I.F. $= e^{\int\frac{1}{x}dx}=x$

So, solution of D.E. is given by

$y.x=\int b.x^3.x\,dx+c$

$y=\frac{c}{x}+\frac{bx^4}{5}$

Passes through (1, 2)

$2=c+\frac{b}{5}$ ...(1)

$\int_1^2 f(x)dx=\frac{62}{5}$

$\left[c\ln x+\frac{bx^5}{25}\right]_1^2=\frac{62}{5}$

$c\ln 2+\frac{31\,b}{25}=\frac{62}{5}$ ...(2)

By equation (1) & (2)

c = 0 and b = 10

29. Official Ans. by NTA (4)

Sol. $\frac{dP}{dt}=0.5P-450$

$\Rightarrow \int_0^t\frac{dp}{P-900}=\int_0^t\frac{dt}{2}$

$\Rightarrow \left[\ell n|P(t)-900|\right]_0^t=\left[\frac{t}{2}\right]_0^t$

$\Rightarrow \ell n|P(t)-900|-\ell n|P(0)-900|=\frac{t}{2}$

$\Rightarrow \ell n|P(t)-900|-\ell n|50|=\frac{t}{2}$

for P(t) = 0

$\Rightarrow \ell n\left|\frac{900}{50}\right|=\frac{t}{2}\Rightarrow t=2\ell n18$

30. Official Ans. by NTA (4)

Sol. Given

y (0) = 0

& $\frac{dy}{dx}=\frac{(x-2)^2+y+4}{x-2}$

$\Rightarrow \frac{dy}{dx}-\frac{y}{x-2}=(x-2)+\frac{4}{x-2}$

$\Rightarrow$ I.F. $= e^{-\int\frac{1}{x-2}dx}=\frac{1}{x-2}$

Solution of L.D.E.

$\Rightarrow y\frac{1}{x-2}=\int\frac{1}{x-2}\left((x-2)+\frac{4}{x-2}\right).dx$

$\Rightarrow \frac{y}{x-2}=x-\frac{4}{x-2}+C$

Now, at x = 0, y = 0 $\Rightarrow$ C = –2

y = x (x – 2) – 4 – 2 (x – 2)

$\Rightarrow y = x^2 - 4x$

This curve passes through (5, 5)

31. Official Ans. by NTA (1)

Sol. $(2xy^2 - y)dx + xdy = 0$

$2xy^2\,dx - y\,dx + x\,dy = 0$

$2x\,dx=\frac{y\,dx-x\,dy}{y^2}=d\left(\frac{x}{y}\right)$

Now integrate

$x^2=\frac{x}{y}+c$

Now point of intersection of lines are (2, 1)

$4=\frac{2}{1}+c$ $\Rightarrow c=2$

$x^2=\frac{x}{y}+2$

Now y(1) = –1

$\Rightarrow |y(1)| = 1$

32. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} = \frac{xy^2 + y}{x}$

$\frac{xdy - ydx}{y^2} = xdx$

$-d\left(\frac{x}{y}\right) = xdx$

$-\frac{x}{y} = \frac{x^2}{2} + c$

$\because$ curve intersects the line x + 2y = 4 at x = –2

$\Rightarrow$ point of intersection is (–2, 3)

$\therefore$ curve passes through (–2, 3)

$\Rightarrow \frac{2}{3} = 2 + c \Rightarrow c = -\frac{4}{3}$

$\Rightarrow \frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$

Now put (3, y)

$\Rightarrow \frac{-3}{y} = \frac{19}{6}$

$\Rightarrow y = \frac{-18}{19}$

33. Official Ans. by NTA (1)

Sol. $\frac{dB}{dt} = \lambda B \Rightarrow \int_{1000}^{1200} \frac{dB}{B} = \lambda \int_0^2 dt \Rightarrow \lambda = \frac{1}{2}\ell n\left(\frac{6}{5}\right)$

$\int_{1000}^{2000} \frac{dB}{B} = \frac{1}{2}\ell n\left(\frac{6}{5}\right)\int_0^T dt \Rightarrow T = \frac{2\ell n2}{\ell n\left(\frac{6}{5}\right)}$

$\Rightarrow k = 2\ell n2$

34. Official Ans. by NTA (1)

Sol. Put $e^{\sin y} = t$

$\Rightarrow e^{\sin y}\cos y\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow$ D.E is $\frac{dt}{dx} + t\cos x = \cos x$

I.F. $= e^{\int \cos x dx} = e^{\sin x}$

$\Rightarrow$ solution is $t.e^{\sin x} = \int \cos x e^{\sin x}$

$\Rightarrow e^{\sin y} e^{\sin x} = e^{\sin x} + c$

$\because x = 0, y = 0 \Rightarrow c = 0$

$\Rightarrow e^{\sin y} = 1$

$\Rightarrow y = 0$

$\Rightarrow 1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right) =$

35. Official Ans. by NTA (2)

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) = ax + \frac{a^{3/2}}{2}$ ...(1)

$\Rightarrow 2yy' = a$

put in equation (1)

$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$

$(y^2 - 2xyy') = \frac{(2yy')^{3/2}}{2}$

squaring

$(y^2 - 2xyy')^2 = \frac{y^3(y')^3}{2}$

$\therefore$ order = 1

degree = 3

Degree – order = 3 – 1 = 2

36. Official Ans by NTA (2)

Sol. $\frac{dy}{dx}+(\tan x)y=\sin x\,;\ 0\le x\le\frac{\pi}{3}$

$\text{I.F.}=e^{\int \tan x\,dx}=e^{\ell n \sec x}=\sec x$

$y\sec x=\int \tan x dx$

$y\sec x=\int \tan x dx$

$y\sec x=\ell n|\sec x|+C$

$x = 0, y = 0 \Rightarrow \quad \therefore c = 0$

$y\sec x=\ell n|\sec x|$

$y=\cos x\cdot \ell n|\sec x|$

$y|_{x=\frac{\pi}{4}}=\left(\frac{1}{\sqrt{2}}\right)\cdot \ell n\sqrt{2}$

$y|_{x=\frac{\pi}{4}}=\frac{1}{2\sqrt{2}}\log_e 2$

37. Official Ans by NTA (2)

Sol. $\frac{dy}{dx}=\frac{y^2-x^2}{2xy},\quad x\in(0,\infty)$

put $y = vx$

$x\frac{dv}{dx}+v=\frac{v^2-1}{2v}$

$\frac{2v}{v^2+1}dv=-\frac{dx}{x}$

Integrate,

$ln(v^2 + 1) = -lnx + C$

$ln\left(\frac{y^2}{x^2}+1\right)=-lnx+C$

put $x = 1, y = 1, C = ln2$

$ln\left(\frac{y^2}{x^2}+1\right)=-lnx+ln2$

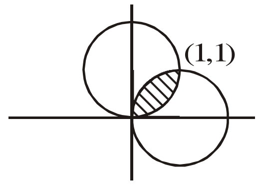
$\Rightarrow x^2 + y^2 - 2x = 0$ (Curve C_1)

Similarly,

$\frac{dy}{dx}=\frac{2xy}{x^2-y^2}$

Put $y = vx$

$x^2 + y^2 - 2y = 0$



required area $=2\int_0^1\left(\sqrt{2x-x^2}-x\right)dx=\frac{\pi}{2}-1$

38. Official Ans. by NTA (4)

Sol. $\frac{dy}{dx}+2y\tan x=\sin x$

$\text{I.F.}=e^{\int 2\tan x dx}=e^{2\ell n \sec x}$

$\text{I.F.} = \sec^2 x$

$y.(\sec^2 x)=\int \sin x.\sec^2 x dx$

$y.(\sec^2 x)=\int \sec x\tan x dx$

$y.(\sec^2 x)=\sec x+C$

$x=\frac{\pi}{3};\ y=0$

$\Rightarrow \quad C=-2$

$\Rightarrow \quad y=\frac{\sec x-2}{\sec^2 x}=\cos x-2\cos^2 x$

$y = t - 2t^2 \Rightarrow \frac{dy}{dt}=1-4t=0\Rightarrow t=\frac{1}{4}$

$\therefore \quad \max=\frac{1}{4}-\frac{1}{8}=\frac{2-1}{8}=\frac{1}{8}$

39. Official Ans. by NTA (2)

Sol. $\cos x(3\sin x+\cos x+3)dy$

$$= (1+y\sin x(3\sin x+\cos x+3))dx$$

$$\frac{dy}{dx}-(\tan x)y=\frac{1}{(3\sin x+\cos x+3)\cos x}$$

$$\text{I.F.}=e^{\int -\tan x\,dx}=e^{\ell n|\cos x|}=|\cos x|$$

$$=\cos x\ \forall\ x\in\left[0,\frac{\pi}{2}\right)$$

Solution of D.E.

$$y(\cos x)=\int(\cos x)\cdot\frac{1}{\cos x(3\sin x+\cos x+3)}dx+C$$

$$y(\cos x)=\int\frac{dx}{3\sin x+\cos x+3}dx+C$$

$$y(\cos x)=\int\frac{\left(\sec^2\frac{x}{2}\right)}{2\tan^2\frac{x}{2}+6\tan\frac{x}{2}+4}dx+C$$

Now

$$\text{Let } I_1=\int\frac{\left(\sec^2\frac{x}{2}\right)}{2\left(\tan^2\frac{x}{2}+3\tan\frac{x}{2}+2\right)}dx+C$$

Put $\tan\frac{x}{2}=t\ \Rightarrow\ \frac{1}{2}\sec^2\frac{x}{2}dx=dt$

$$I_1=\int\frac{dt}{t^3+3t+2}=\int\frac{dt}{(t+2)(t+1)}$$

$$=\int\left(\frac{1}{t+1}-\frac{1}{t+2}\right)dt$$

$$=\ell n\left|\left(\frac{t+1}{t+2}\right)\right|=\ell n\left|\left(\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right)\right|$$

So solution of D.E.

$$y(\cos x)=\ell n\left|\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right|+C$$

$$\Rightarrow y(\cos x)=\ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right)+C\quad\text{for } 0\le x<\frac{\pi}{2}$$

Now, it is given y(0) = 0

$$\Rightarrow 0=\ell n\left(\frac{1}{2}\right)+C\qquad\Rightarrow\ \boxed{C=\ell n2}$$

$$\Rightarrow y(\cos x)=\ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right)+\ell n2$$

For $x=\frac{\pi}{3}$

$$y\left(\frac{1}{2}\right)=\ell n\left(\frac{1+\frac{1}{\sqrt{3}}}{2+\frac{1}{\sqrt{3}}}\right)+\ell n2$$

$$y=2\ell n\left(\frac{2\sqrt{3}+10}{11}\right)$$ **Ans.(2)**

40. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx}-\frac{y}{2x}=\frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}$

$IF=e^{-\int\frac{dx}{2d}}=e^{-\frac{1}{2}\ln x}=\frac{1}{x^{1/2}}$

$y.x^{-1/2}=\int\frac{x^{9/4}\cdot x^{-1/2}}{x^{5/4}\left(x^{3/4}+1\right)}dx$

$\int\frac{x^{1/2}}{(x^{3/4}+1)}dx$

$x = t^4 \Rightarrow dx = 4t^3\,dt$

$\int\frac{t^2\cdot 4t^3\,dt}{(t^3+1)}$

$4\int\frac{t^2\left(t^3+1-1\right)}{(t^3+1)}dt$

$4\int t^2dt-4\int\frac{t^2}{t^3+1}dt$

$\frac{4t^3}{3}-\frac{4}{3}\ln\left(t^3+1\right)+C$

$yx^{-1/2}=\frac{4x^{3/4}}{3}-\frac{4}{3}\ln\left(x^{3/4}+1\right)+C$

$1-\frac{4}{3}\log_e 2=\frac{4}{3}-\frac{4}{3}\log_e 2+C$

$\Rightarrow C=-\frac{1}{3}$

$y=\frac{4}{3}x^{5/4}-\frac{4}{3}\sqrt{x}\ln\left(x^{3/4}+1\right)-\frac{\sqrt{x}}{3}$

$y(16)=\frac{4}{3}\times 32-\frac{4}{3}\times 4\ln 9-\frac{4}{3}$

$=\frac{124}{3}-\frac{32}{3}\ln 3 = 4\left(\frac{31}{3}-\frac{8}{3}\ln 3\right)$

41. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}=(1+y)(x-1)$

$\frac{dy}{(y+1)}=(x-1)dx$

Integrate $\ln(y+1)=\frac{x^2}{2}-x+c$

$(0,0)\Rightarrow c=0\Rightarrow\ y=e^{\left(\frac{x^2}{2}-x\right)}-1$

42. Official Ans. by NTA (1)

Sol. $I=\int_0^{\sqrt{\pi/2}}\left([x^2]+[-\cos x]\right)dx$

$=\int_0^1 0\,dx+\int_1^{\sqrt{\pi/2}}dx+\int_0^{\sqrt{\pi/2}}(-1)\,dx$

$=\sqrt{\frac{\pi}{2}}-1-\sqrt{\frac{\pi}{2}}=-1$

$\Rightarrow |I|=1$

43. Official Ans. by NTA (3)

Sol. $y^2 = 4ax + 4a^2$

differentiate with respect to x

$\Rightarrow 2y\frac{dy}{dx}=4a$

$\Rightarrow a=\left(\frac{y}{2}\frac{dy}{dx}\right)$

so, required differential equation is

$y^2=\left(4\times\frac{y}{2}\frac{dy}{dx}\right)x+4\left(\frac{y}{2}\frac{dy}{dx}\right)^2$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2+2xy\left(\frac{dy}{dx}\right)-y^2=0$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2+2x\left(\frac{dy}{dx}\right)-y=0$

44. Official Ans. by NTA (1)

Sol. Let y + 1 = Y

$\therefore \dfrac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$

Put $-\dfrac{1}{Y} = k$

$\Rightarrow \dfrac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$

I.F. = $e^{-\frac{x^2}{2}}$

$\therefore k = (x+c)e^{x^2/2}$

Put $k = -\dfrac{1}{y+1}$

$\therefore y+1 = -\dfrac{1}{(x+c)e^{x^2/2}}$ ...(i)

when x = 2, y = 0, then $c = -2 - \dfrac{1}{e^2}$

diffentiate equation (i) & put x = 1

we get $\left(\dfrac{dy}{dx}\right)_{x=1} = -\dfrac{e^{3/2}}{(1+e^2)^2}$

45. Official Ans. by NTA (4)

Sol. $xdy - ydx = \sqrt{x^2 - y^2}\,dx$

$\Rightarrow \dfrac{xdy - ydx}{x^2} = \dfrac{1}{x}\sqrt{1 - \dfrac{y^2}{x^2}}\,dx$

$\Rightarrow \int \dfrac{d\left(\frac{y}{x}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \int \dfrac{dx}{x}$

$\Rightarrow \sin^{-1}\left(\dfrac{y}{x}\right) = \ell n|x| + c$

at x = 1, y = 0 ⇒ c = 0

$y = x\sin(\ell nx)$

$A = \int_1^{e^\pi} x\sin(\ell n x)\,dx$

$x = e^t$, $dx = e^t dt \Rightarrow \int_0^\pi e^{2t}\sin(t)\,dt = A$

$\alpha e^{2\pi} + \beta = \left(\dfrac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^\pi = \dfrac{1+e^{2\pi}}{5}$

$\alpha = \dfrac{1}{5}, \beta = \dfrac{1}{5}$ so $10(\alpha + \beta) = 4$