DETERMINANT

1. Let a, b, c, d be in arithmetic progression with common difference λ. If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$,

then value of λ^2 is equal to ___________.

2. The value of $k \in \mathbf{R}$, for which the following system of linear equations

$3x - y + 4z = 3,$

$x + 2y - 3z = -2,$

$6x + 5y + kz = -3,$

has infinitely many solutions, is :

(1) 3 (2) – 5 (3) 5 (4) –3

3. The values of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solution, are :

(1) $\lambda = 3, \mu = 5$ (2) $\lambda = 3, \mu \neq 10$

(3) $\lambda \neq 2, \mu = 10$ (4) $\lambda = 2, \mu \neq 10$

4. The values of a and b, for which the system of equations

$2x + 3y + 6z = 8$

$x + 2y + az = 5$

$3x + 5y + 9z = b$

has no solution, are :

(1) $a = 3, b \neq 13$ (2) $a \neq 3, b \neq 13$

(3) $a \neq 3, b = 3$ (4) $a = 3, b = 13$

5. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is:

(1) 4 (2) 1 (3) 2 (4) 3

6. Let

$$f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi]$$

Then the maximum value of f(x) is equal to _________.

7. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$(1 + \cos^2\theta)x + \sin^2\theta\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4 \sin 3\theta)\, z = 0$

has a non–trivial solution, then the value of θ is :

(1) $\frac{4\pi}{9}$ (2) $\frac{7\pi}{18}$ (3) $\frac{\pi}{18}$ (4) $\frac{5\pi}{18}$

8. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to _________.

9. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval:

(1) [68, 69) (2) [62, 63)

(3) [65, 66) (4) [60, 61)

10. Let [λ] be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is:

(1) **R** (2) $(-\infty, -9) \cup (-9, \infty)$

(3) [–9, –8) (4) $(-\infty, -9) \cup [-8, \infty)$

11. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a = -\frac{1}{3}, b \neq \frac{7}{3}$ (2) $a \neq \frac{1}{3}, b = \frac{7}{3}$

(3) $a \neq -\frac{1}{3}, b = \frac{7}{3}$ (4) $a = \frac{1}{3}, b \neq \frac{7}{3}$

12. If $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9}$, r = 1, 2, 3,...,

$i = \sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

equal to :

(1) $a_2a_6 - a_4a_8$ (2) a_9

(3) $a_1a_9 - a_3a_7$ (4) a_5

13. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has :

(1) no solution

(2) infinitely many solution

(3) exactly two solutions

(4) a unique solution

14. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in \mathbf{R}$ for which the system is inconsistent and S_2 be the set of all $a \in \mathbf{R}$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

(1) $n(S_1) = 2, n(S_2) = 2$

(2) $n(S_1) = 1, n(S_2) = 0$

(3) $n(S_1) = 2, n(S_2) = 0$

(4) $n(S_1) = 0, n(S_2) = 2$

15. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3, m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3, m \neq \frac{4}{5}$ (4) $k = 3, m \neq \frac{4}{5}$

16. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to ________.

17. The following system of linear equations

$2x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(2) has infinitely many solutions

(3) does not have any solution

(4) has a unique solution

18. Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c,$

where a, b and c are real constants. Then the system of equations :

(1) has a unique solution when $5a = 2b + c$

(2) has infinite number of solutions when $5a = 2b + c$

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

19. For the system of linear equations :

$x - 2y = 1,\ x - y + kz = -2,\ ky + 4z = 6,\ k \in R,$

consider the following statements :

(A) The system has unique solution if $k \neq 2$, $k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct ?

(1) (C) and (D) only (2) (B) and (E) only

(3) (A) and (E) only (4) (A) and (D) only

20. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) $(a + 2)(a + 3)(a + 4)$

(2) -2

(3) $(a + 1)(a + 2)(a + 3)$

(4) 0

21. The maximum value of

$$f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R \text{ is:}$$

(1) $\sqrt{7}$ (2) $\frac{3}{4}$ (3) $\sqrt{5}$ (4) 5

22. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution if k is equal to :

(1) 0 (2) 1

(3) -1 (4) -2

23. The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi)$$

, are

(1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$

(3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{7\pi}{12}, \frac{11\pi}{12}$

24. Let α,β,γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, (a,b,c $\in$ R and a,b $\neq$ 0). If the system of equations (in, u,v,w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is

(1) 5 (2) 3

(3) 1 (4) 0

25. Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0$, $\lambda, \mu \in R$.

has a non-trivial solution. Then which of the following is true ?

(1) $\mu = 6, \lambda \in R$ (2) $\lambda = 2, \mu \in R$

(3) $\lambda = 3, \mu \in R$ (4) $\mu = -6, \lambda \in R$

SOLUTION

1. $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$

$C_2 \to C_2 - C_3$

$\Rightarrow \begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix} = 2$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

$\Rightarrow \lambda \begin{vmatrix} x-2\lambda & 1 & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix} = 2$

$\Rightarrow 1\,(4\lambda^2 - 4\lambda^2 + 2\lambda) = 2$

$\Rightarrow \boxed{\lambda^2 = 1}$

2. $\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & K \end{vmatrix} = 0$

$\Rightarrow 3(2K + 15) + K + 18 - 28 = 0$

$\Rightarrow 7K + 35 = 0 \Rightarrow K = -5$

3. $x + y + z = 6$ ...(i)

$3x + 5y + 5z = 26$ ...(ii)

$x + 2y + \lambda z = \mu$ ...(iii)

$5 \times$ (i) – (ii) $\Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore$ from (i) and (iii)

$y + z = 4$ ...(iv)

$2y + \lambda z = \mu - 2$ ...(v)

(v) – 2 × (iv)

$\Rightarrow (\lambda - 2)z = \mu - 10$

$\Rightarrow z = \dfrac{\mu - 10}{\lambda - 2}$ & $y = 4 - \dfrac{\mu - 10}{\lambda - 2}$

$\therefore$ For no solution $\lambda = 2$ and $\mu \neq 10$.

4. $D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$

$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$

If $a = 3$, $b \neq 13$, no solution.

5. $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, \ \dfrac{-\pi}{4} \le x \le \dfrac{\pi}{4}$

Apply : $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\therefore x = \dfrac{\pi}{4}$

6. $\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix} \begin{pmatrix} R_1 \to R_1 - R_2 \\ \& \ R_2 \to R_2 - R_3 \end{pmatrix}$

$-2(\cos^2 x) + 2(2 + 2\cos 2x + \sin^2 x)$

$4 + 4\cos 2x - 2(\cos^2 x - \sin^2 x)$

$f(x) = 4 + \underbrace{2\cos 2x}_{\max = 1}$

$f(x)_{\max} = 4 + 2 = 6$

7. **Case–I**

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\sin^3\theta \end{vmatrix} = 0$$

or $4\sin 3\theta = -2$

$\sin 3\theta = -\dfrac{1}{2}$

$\theta = \dfrac{7\pi}{18}$

8. $2 \times$ (i) – (ii) – (iii) gives :

$-(1+\beta)z = 3 - \alpha$

For infinitely many solution

$\beta + 1 = 0 = 3 - \alpha \Rightarrow (\alpha,\beta) = (3,-1)$

Hence, $\alpha + \beta - \alpha\beta = 5$

9. $$\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$$

$R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$$

$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$

10. $$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28+[\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = -[\lambda] - 9$$

if $[\lambda] + 9 \neq 0$ then unique solution

if $[\lambda] + 9 = 0$ then $D_1 = D_2 = D_3 = 0$

so infinite solutions

Hence λ can be any red number.

11. Here $$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} \begin{matrix} = 2(-a-1) - 1(a-1) + 1 + 1 \\ = 1 - 3a \end{matrix}$$

$$D_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} \begin{matrix} = 2(-b-3) - 1(b-3) + 5(1+1) \\ = 7 - 3b \end{matrix}$$

for $a = \dfrac{1}{3},\ b \neq \dfrac{7}{3}$, system has no solutions

12. $a_r = e^{\frac{i2\pi r}{9}}$, $r = 1, 2, 3, \ldots$ $a_1, a_2, a_3, \ldots$ are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_n & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix} = a_1.a_1^4.a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$$

Now $a_1a_9 - a_3a_7 = a_1^{10} - a_1^{10} = 0$

13. $\alpha + \beta + \gamma = 2\pi$

$$\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$= 1 + 2\cos\alpha.\cos\beta.\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha+\beta) + \cos(\alpha-\beta))\cos\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha-\beta)\cos\gamma$

$= \sin^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0$

14. $\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$

$= -1(a^2 + 10) -1(-3a-10) + 2(-6 + 2a)$

$= -a^2 - 10 + 3a + 10 - 12 + 4a$

$\Delta = -a^2 + 7a - 12$

$\Delta = -[a^2 - 7a + 12]$

$\Delta = -[(a-3)(a-4)]$

$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$

$= 0 - 1(-a - 35) + 2(-2 + 7a)$

$\Rightarrow a + 35 - 4 + 14a$

$15a + 31$

Now $\Delta_1 = 15a + 31$

For inconsistent $\Delta = 0 \therefore a = 3, a = 4$

and for a = 3 and 4 $\Delta_1 \neq 0$

$n(S_1) = 2$

For infinite solution : $\Delta = 0$

and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

Not possible

$\therefore n(S_2) = 0$

15. $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$\Rightarrow 24 - 2(0) - k(8) = 0 \Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$

$= 10(8) - 2(-10m + 6) - 3(12 + 20m)$

$= 8(4 - 5m)$

$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$

$= 3(-6 + 10m) + 10(0) - 3(10m - 6)$

$= 0$

$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix}$

$= 3(-20m - 12) - 2(6 - 10m) + 10(8)$

$= 40m - 32 = 8(5m - 4)$

for inconsistent

$k = 3 \;\&\; m \neq \frac{4}{5}$

16. We observe $5P_2 - P_1 = 3P_3$

So, $15 - K = -6$

$\Rightarrow K = 21$

17. $2x + 3y + 2z = 9$ ...(1)

$3x + 2y + 2z = 9$ ...(2)

$x - y + 4z = 8$ ...(3)

$(1) - (2) \Rightarrow -x + y = 0 \Rightarrow x - y = 0$

from (3) $4z = 8 \Rightarrow z = 2$

from (1) $2x + 3y = 5$

$\Rightarrow x = y = 1$

$\therefore$ system has unique solution

18. $P_1 : x + 2y - 3z = a$

$P_2 : 2x + 6y - 11z = b$

$P_3 : x - 2y + 7z = c$

Clearly

$5P_1 = 2P_2 + P_3$ if $5a = 2b + c$

$\Rightarrow$ All the planes sharing a line of intersection

$\Rightarrow$ infinite solutions

19. $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$

so, A is correct and B, C, E are incorrect.

If k = 2

$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = -48 \neq 0$

So no solution

D is correct.

20. $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ a^2+7a+12-a^2-3a-2 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

$= 4(a + 2) - 4a - 10$

$= 4a + 8 - 4a - 10 = -2$

21. $C_1 + C_2 \to C_1$

$$\begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 - R_2 \to R_1$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

Open w.r.t. R_1

$-(2 \sin 2x - \cos 2x)$

$\cos 2x - 2 \sin 2x = f(x)$

$f(x)|_{max} = \sqrt{1+4} = \sqrt{5}$

22. $kx + y + z = 1$

$x + ky + z = k$

$x + y + zk = k^2$

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K(K^2-1) - 1(K-1) + 1(1-K)$$

$= K^3 - K - K + 1 + 1 - K$

$= K^3 - 3K + 2$

$= (K - 1)^2 (K + 2)$

For K = 1

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for $K = -2$, at least one out of Δ_1, Δ_2, Δ_3 are not zero

Hence for no sol^n, $K = -2$

23. $$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

use $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow (2+4\sin 2x) \begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$$\Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$x = \frac{\pi}{2} + \frac{\pi}{12}, \pi - \frac{\pi}{12}$$

24. $$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$\Rightarrow -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \Sigma\alpha\beta) = 0$

$\Rightarrow -(-a)(a^2 - 2b - b) = 0$

$\Rightarrow a(a^2 - 3b) = 0$

$\Rightarrow a^2 = 3b \Rightarrow \dfrac{a^2}{b} = 3$

25. For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$

when $\mu = 6$, $12 - 6\lambda + 6\lambda = 12$

which is satisfied by all λ