CONTINUITY

1. Let a function $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\sin x - e^x & \text{if } x \le 0\\ a+[-x] & \text{if } 0<x<1\\ 2x-b & \text{if } x \ge 1\end{cases}$$

Where [x] is the greatest integer less than or equal to x. If f is continuous on **R**, then (a + b) is equal to:

(1) 4 (2) 3 (3) 2 (4) 5

2. Let f : **R** → **R** be defined as

$$f(x)=\begin{cases}\dfrac{\lambda|x^2-5x+6|}{\mu(5x-x^2-6)}, & x<2\\ e^{\frac{\tan(x-2)}{x-[x]}} & ,x>2\\ \mu & ,x=2\end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then λ + μ is equal to :

(1) e(–e + 1) (2) e(e – 2)

(3) 1 (4) 2e – 1

3. Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}} & , -\frac{\pi}{4}<x<0\\ b & , x=0\\ e^{\cot 4x/\cot 2x} & , 0<x<\frac{\pi}{4}\end{cases}$$

If f is continuous at x = 0, then the value of $6a + b^2$ is equal to :

(1) 1 – e (2) e –1 (3) 1 + e (4) e

4. Let a, b ∈ **R**, b ≠ 0, Define a function

$$f(x)=\begin{cases}a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0\\ \dfrac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0.\end{cases}$$

If f is continuous at x = 0, then 10 – ab is equal to ________.

5. If the function $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , x<0\\ k & , x=0\\ \dfrac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , x>0\end{cases}$

is continuous at x = 0, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

(1) –5 (2) 5 (3) – 4 (4) 4

6. Let [t] denote the greatest integer ≤ t. The number of points where the function

$$f(x)=[x]|x^2-1|+\sin\left(\frac{\pi}{[x]+3}\right)-[x+1], x\in(-2,2)$$

is not continuous is ______.

7. If f : R → R is a function defined by $f(x)=[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is :

(1) discontinuous at all integral values of x except at x = 1

(2) continuous only at x = 1

(3) continuous for every real x

(4) discontinuous only at x = 1

8. Let f : R → R be defined as

$$f(x)=\begin{cases}2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1\\ |ax^2+x+b|, & \text{if } -1\le x\le 1\\ \sin(\pi x), & \text{if } x>1\end{cases}$$

If $f(x)$ is continuous on R, then a + b equals:

(1) –3 (2) –1 (3) 3 (4) 1

9. Let f : R → R and g : R → R be defined as

$$f(x)=\begin{cases} x+a, & x<0 \\ |x-1|, & x\geq 0 \end{cases} \text{ and}$$

$$g(x)=\begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x\geq 0 \end{cases}$$

where a, b are non-negative real numbers. If (gof)(x) is continuous for all x ∈ R, then a + b is equal to ___________.

10. If the function $f(x)=\dfrac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0)=\dfrac{1}{k}$, then k is _________ .

11. Let f : R → R be a function defined as

$$f(x)=\begin{cases} \dfrac{\sin(a+1)x+\sin 2x}{2x} & , \text{if } x<0 \\ b & , \text{if } x=0 \\ \dfrac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}} & , \text{if } x>0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to :

(1) $-\dfrac{5}{2}$ (2) –2 (3) –3 (4) $-\dfrac{3}{2}$

SOLUTION

1. Official Ans. by NTA (2)

Sol. Continuous at x = 0

$f(0^+) = f(0^-) \Rightarrow a - 1 = 0 - e^0$

$\Rightarrow a = 0$

Continuous at x = 1

$f(1^+) = f(1^-)$

$\Rightarrow 2(1) - b = a + (-1)$

$\Rightarrow b = 2 - a + 1 \Rightarrow b = 3$

$\therefore\ a + b = 3$

2. Official Ans. by NTA (1)

Sol. $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} e^{\frac{\tan(x-2)}{x-2}} = e^1$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \frac{-\lambda(x-2)(x-3)}{\mu(x-2)(x-3)} = -\frac{\lambda}{\mu}$

For continuity $\mu = e = -\frac{\lambda}{\mu} \Rightarrow \mu = e,\ \lambda = -e^2$

$\lambda + \mu = e(-e + 1)$

3. Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to 0} f(x) = b$

$\lim\limits_{x\to 0^+} xe^{\frac{\cot 4x}{\cot 2x}} = e^{\frac{1}{2}} = b$

$\lim\limits_{x\to 0^-} (1+|\sin x|)^{\frac{3a}{|\sin x|}} = e^{3a} = e^{\frac{1}{2}}$

$\lim\limits_{x\to 0^-} (1+|\sin x|)^{\frac{3a}{|\sin x|}} = e^{3a} = e^{\frac{1}{2}}$

$a = \frac{1}{6} \Rightarrow 6a = 1$

$(6a + b^2) = (1 + e)$

4. Official Ans. by NTA (14)

Sol. $f(x) = \begin{cases} a\sin\frac{\pi}{2}(x-1), & x \le 0 \\ \frac{\tan 2x - \sin 2x}{bx^3}, & x > 0 \end{cases}$

For continuity at '0'

$\lim\limits_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \lim\limits_{x\to 0^+} \frac{\tan 2x - \sin 2x}{bx^3} = -a$

$\Rightarrow \lim\limits_{x\to 0^+} \frac{\frac{8x^3}{3} + \frac{8x^3}{3!}}{bx^3} = -a$

$\Rightarrow 8\left(\frac{1}{3} + \frac{1}{3!}\right) = -ab$

$\Rightarrow 4 = -ab$

$\Rightarrow 10 - ab = 14$

5. Official Ans. by NTA (1)

Sol. If $f(x)$ is continuous at x = 0, RHL = LHL = $f(0)$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1}\cdot\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$

(Rationalisation)

$\lim\limits_{x\to 0^+} -\frac{2\sin^2 x}{x^2}.\left(\sqrt{x^2+1}+1\right) = -4$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{1}{x}\ \ell n\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$

$\lim\limits_{x\to 0^-} \frac{\ell n\left(1+\frac{x}{a}\right)}{\left(\frac{x}{a}\right).a} + \frac{\ell n\left(1-\frac{x}{b}\right)}{\left(-\frac{x}{b}\right).b}$

$= \frac{1}{a} + \frac{1}{b}$

So $\frac{1}{a} + \frac{1}{b} = -4 = k$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{4}{k} = -4 - 1 = -5$

6. Official Ans. by NTA (2)

Sol. $f(x) = [x]\,|x^2 - 1| + \sin \dfrac{\pi}{[x+3]} - [x + 1]$

$$f(x)=\begin{cases} 3-2x^2, & -2<x<-1 \\ x^2, & -1\le x<0 \\ \dfrac{\sqrt{3}}{2}+1 & 0\le x<1 \\ x^2+1+\dfrac{1}{\sqrt{2}}, & 1\le x<2 \end{cases}$$

discontinuous at x = 0 , 1

7. Official Ans. by NTA (3)

Sol. For x = n, $n \in Z$

$$\text{LHL} = \lim_{x\to n^-} f(x) = \lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$$

$= 0$

$$\text{RHL} = \lim_{x\to n^+} f(x) = \lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$$

$= 0$

$f(n) = 0$

$\Rightarrow$ LHL = RHL = $f(n)$

$\Rightarrow$ $f(x)$ is continuous for every real x.

8. Official Ans. by NTA (2)

Sol. $f(x)$ is continuous on R

$\Rightarrow f(1^-) = f(1) = f(1^+)$

$|a + 1 + b| = \lim_{x\to 1} \sin(\pi x)$

$|a + 1 + b| = 0 \Rightarrow a + b = -1$ ...(1)

$\Rightarrow$ Also $f(-1^-) = f(-1) = f(-1^+)$

$$\lim_{x\to -1} 2\sin\left(\frac{-\pi x}{2}\right) = |a-1+b|$$

$|a - 1 + b| = 2$

Either a– 1 + b = 2 or a – 1 + b = –2

a + b = 3 ...(2) or a + b = –1 ...(3)

from (1) and (2) $\Rightarrow$ a + b = 3 = – 1(reject)

from (1) and (3) $\Rightarrow$ a + b = –1

9. Official Ans by NTA (1)

Sol. $$g[f(x)] = \begin{cases} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x)\ge 0 \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x+a<0\ \&\ x<0 \\ |x-1|+1 & |x-1|<0\ \&\ x\ge 0 \\ (x+a-1)^2+b & x+a\ge 0\ \&\ x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0\ \&\ x\ge 0 \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a)\ \&\ x\in(-\infty,0) \\ |x-1|+1 & x\in\phi \\ (x+a-1)^2+b & x\in[-a,\infty)\ \&\ x\in(-\infty,0) \\ (|x-1|-1)^2+b & x\in R\ \&\ x\in[0,\infty) \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a) \\ (x+a-1)^2+b & x\in[-a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty) \end{cases}$$

g(f(x)) is continuous

at x = –a & at x = 0

1 = b + 1 & $(a – 1)^2 + b = b$

b = 0 & a = 1

$\Rightarrow$ a + b = 1

10. Official Ans. by NTA (6)

Sol. $$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4} = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin\left(\dfrac{\sin x+x}{2}\right)\sin\left(\dfrac{x-\sin x}{2}\right)}{x^4} = \frac{1}{K}$$

$$\Rightarrow \lim_{x\to 0} 2\left(\frac{\sin x+x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right) = \frac{1}{K}$$

$$\Rightarrow 2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6} = \frac{1}{K}$$

$\Rightarrow$ K = 6

11. Official Ans. by NTA (4)

Sol. $f(x)$ is continuous at x = 0

$$\lim_{x\to0^+} f(x) = f(0) = \lim_{x\to0^-} f(x) \quad ...(1)$$

$$f(0) = b \quad ...(2)$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+}\left(\frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x}\right)$$

$$= \frac{a+1}{2} + 1 \quad ...(3)$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x\to0^+} \frac{(x+bx^3-x)}{bx^{5/2}\left(\sqrt{x+bx^3}+\sqrt{x}\right)}$$

$$= \lim_{x\to0^+} \frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+bx^2}+1\right)} = \frac{1}{2} \quad ...(4)$$

Use (2), (3) & (4) in (1)

$$\frac{1}{2} = b = \frac{a+1}{2} + 1$$

$$\Rightarrow b = \frac{1}{2},\ a = -2$$

$$a + b = \frac{-3}{2}$$