COMPOUND ANGLE

1. The value of $\cot\frac{\pi}{24}$ is :
(1) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$
(2) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$
(3) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$
(4) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

2. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x-2y|$ is equal to :
(1) 4 (2) 3 (3) 0 (4) 1

3. Let $\alpha = \max_{x\in\mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$ and $\beta = \min_{x\in\mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$. If $8x^2+bx+c=0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c-b$ is equal to :
(1) 42 (2) 47 (3) 43 (4) 50

4. The value of
$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$
is :
(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{1}{8\sqrt{2}}$

5. Let $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :
(1) $b^2-a^2=a^2+c^2$
(2) b^2, c^2, a^2 are in A.P.
(3) c^2, a^2, b^2 are in A.P.
(4) a^2, b^2, c^2 are in A.P.

6. If n is the number of solutions of the equation
$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1,$$
$x\in[0,\pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is :
(1) $(3, 13\pi/9)$
(2) $(2, 2\pi/3)$
(3) $(2, 8\pi/9)$
(4) $(3, 5\pi/3)$

7. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:
(1) $\frac{1}{\sqrt{7}}$ (2) $2\sqrt{2}-1$
(3) $\sqrt{7}-1$ (4) $\frac{1}{2\sqrt{2}}$

8. If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+.....\infty)\log_e 2}$ satisfies the equation $t^2-9t+8=0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is
(1) $2\sqrt{3}$ (2) $\frac{3}{2}$
(3) $\sqrt{3}$ (4) $\frac{1}{2}$

9. If $15\sin^4\alpha+10\cos^4\alpha=6$, for some $\alpha\in R$, then the value of $27\sec^6\alpha+8\csc^6\alpha$ is equal to :
(1) 350 (2) 500
(3) 400 (4) 250

SOLUTION

1. $\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$ $\quad \theta = \frac{\pi}{24}$

$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$= \frac{\left(2\sqrt{2}+\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$

$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$

$= \sqrt{6}+\sqrt{2}+\sqrt{3}+2$

2. $x = \frac{1}{2}\left(\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right)$

and $2y = \tan\frac{\pi}{9} + \tan\frac{5\pi}{18}$

so, $x - 2y = \frac{1}{2}\left(\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right) - \left(\tan\frac{\pi}{9} + \tan\frac{5\pi}{18}\right)$

$\Rightarrow |x - 2y| = \left|\frac{\cot\frac{\pi}{9} - \tan\frac{\pi}{9}}{2} - \tan\frac{5\pi}{18}\right|$

$= \left|\cot\frac{2\pi}{9} - \cot\frac{2\pi}{9}\right| = 0$

$\left(\text{as } \tan\frac{5\pi}{18} = \cot\frac{2\pi}{9}; \tan\frac{7\pi}{18} = \cot\frac{\pi}{9}\right)$

3. $\alpha = \max\left\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\right\}$

$= \max\left\{2^{6\sin 3x} \cdot 2^{8\cos 3x}\right\}$

$= \max\left\{2^{6\sin 3x + 8\cos 3x}\right\}$

and $\beta = \min\left\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\right\} = \min\left\{2^{6\sin 3x + 8\cos 3x}\right\}$

Now range of 6 sin 3x + 8 cos 3x

$= \left[-\sqrt{6^2+8^2}, +\sqrt{6^2+8^2}\right] = [-10, 10]$

$\alpha = 2^{10}$ & $\beta = 2^{-10}$

So, $\alpha^{1/5} = 2^2 = 4$

$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$

quadratic $8x^2 + bx + c = 0$, c – b

$= 8 \times [(\text{product of roots}] + (\text{sum of roots})$

$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4}\right] = 8 \times \left[\frac{21}{4}\right] = 42$

4. $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$

$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{8}$

5. $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$

As A,B,C are angles of triangle

$A + B + C = \pi$

$A = \pi - (B + C)$

So, $\sin A = \sin(B + C)$ …(1)

Similarly $\sin B = \sin(A + C)$ …(2)

From (1) and (2)

$\frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$

$\sin(C + B).\sin(C - B) = \sin(A - C)\sin(A + C)$

$\sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$

$\{\because \sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y\}$

$2\sin^2 C = \sin^2 A + \sin^2 B$

By sine rule

$2c^2 = a^2 + b^2$

$\Rightarrow b^2, c^2$ and a^2 are in A.P.

6. $2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$

$2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1$

$2\cos x\left(4\left(\frac{1}{2}-\sin^2 x\right)-1\right)=1$

$2\cos x\left(2-4\sin^2 x-1\right)=1$

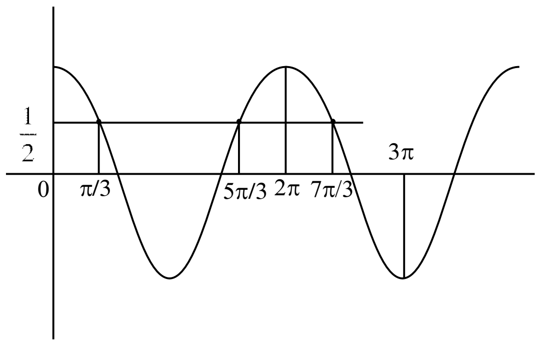
$2\cos x\left(1-4\sin^2 x\right)=1$

$2\cos x\left(4\cos^2 x-3\right)=1$

$4\cos^3 x-3\cos x=\frac{1}{2}$

$\cos 3x=\frac{1}{2}$

$x \in [0, \pi] \therefore 3x \in [0, 3\pi]$



7. Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}=\theta$

$\sin 4\theta=\frac{\sqrt{63}}{8}$

$\cos 4\theta=\frac{1}{8}$

$2\cos^2 2\theta-1=\frac{1}{8}$

$\cos^2 2\theta=\frac{9}{16}$

$\cos 2\theta=\frac{3}{4}$

$2\cos^2\theta-1=\frac{3}{4}$

$\cos^2\theta=\frac{7}{8}$

$\cos\theta=\frac{\sqrt{7}}{2\sqrt{2}}$

$\tan\theta=\frac{1}{\sqrt{7}}$

8. $e^{(\cos^2\theta+\cos^4\theta+.....\infty)\ell n^2}=2^{\cos^2\theta+\cos^4\theta+....\infty}$

$=2^{\cot^2\theta}$

Now $t^2-9t+9=0 \Rightarrow t=1, 8$

$\Rightarrow 2^{\cot^2\theta}=1,8 \Rightarrow \cot^2\theta=0,3$

$0<\theta<\frac{\pi}{2} \Rightarrow \cot\theta=\sqrt{3}$

$\Rightarrow \frac{2\sin\theta}{\sin\theta+\sqrt{3}\sin\theta}=\frac{2}{1+\sqrt{3}\cot\theta}=\frac{2}{4}=\frac{1}{2}$

9. $15\sin^4\alpha+10\cos^4\alpha=6$

$15\sin^4\alpha+10\cos^4\alpha=6(\sin^2\alpha+\cos^2\alpha)^2$

$(3\sin^2\alpha-2\cos^2\alpha)^2=0$

$\tan^2\alpha=\frac{2}{3}$. $\cot^2\alpha=\frac{3}{2}$

$\Rightarrow 27\sec^6\alpha+8\text{cosec}^6\alpha$

$=27(\sec^6\alpha)^3+8(\text{cosec}^6\alpha)^3$

$=27(1+\tan^2\alpha)3+8(1+\cot^2\alpha)^3$

$=250$