CIRCLE

1. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (– 4,1) and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then a + b is equal to :

(1) 3 (2) 11 (3) 5 (4) 7

2. Let the circle $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, x – 2y = 4 and 2x – y = 5 lies inside the circle S, then :

(1) $\frac{25}{9} < C < \frac{13}{3}$ (2) $100 < C < 165$

(3) $81 < C < 156$ (4) $100 < C < 156$

3. Two tangents are drawn from the point P(–1, 1) to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to:

(1) 2 (2) $(3\sqrt{2}+2)$

(3) 4 (4) $3(\sqrt{2}-1)$

4. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P, Q} is equal to

(1) $\{(4,0),(0,6)\}$

(2) $\{(2+2\sqrt{2}, 3-\sqrt{5}), (2-2\sqrt{2}, 3+\sqrt{5})\}$

(3) $\{(2+2\sqrt{2}, 3+\sqrt{5}), (2-2\sqrt{2}, 3-\sqrt{5})\}$

(4) $\{(-1,5),(5,1)\}$

5. Let

$A = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid 2x^2 + 2y^2 - 2x - 2y = 1\}$,

$B = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid 4x^2 + 4y^2 - 16y + 7 = 0\}$ and

$C = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\}$.

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to

(1) $\frac{3+\sqrt{10}}{2}$ (2) $\frac{2+\sqrt{10}}{2}$

(3) $\frac{3+2\sqrt{5}}{2}$ (4) $1+\sqrt{5}$

6. Consider a circle C which touches the y-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to :

(1) $\sqrt{53}$ (2) 9 (3) 8 (4) $\sqrt{82}$

7. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1) (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to ________.

8. A circle C touches the line x = 2y at the point (2,1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

(1) $7\sqrt{5}$ (2) 15

(3) $\sqrt{285}$ (4) $4\sqrt{15}$

9. Let the equation $x^2 + y^2 + px + (1 - p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer} is _________.

10. Let $\mathbb{Z}$ be the set of all integers,

$A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \le 4\}$,

$B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \le 4\}$ and

$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y-2)^2 \le 4\}$

If the total number of relation from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is :

(1) 16 (2) 25 (3) 49 (4) 9

11. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is 4x + 3y = 10, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \ne C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to ___________.

12. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is _________.

13. Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8.\left(\frac{\text{area } \Delta APQ}{\text{area } \Delta BPQ}\right)$ is equal to _____.

14. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to_____.

15. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is ______.

16. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to :

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

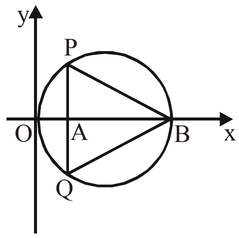
17. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x-1)^2 + (y-1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points P, A and B lie on :

(1) a straight line (2) a hyperbola

(3) an ellipse (4) a parabola

18. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $\left(4, -2\sqrt{2}\right)$, and given that $a - 2\sqrt{2}\,b = 3$, then $(a^2 + b^2 + ab)$ is equal to ______.

19. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

20. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is euqal to :

(1) $\sqrt{11}$ (2) $\sqrt{7}$ (3) $\sqrt{6}$ (4) $\sqrt{10}$

21. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to_______.

22. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

(1) 11 : 4 (2) 9 : 4 (3) 3 :1 (4) 2 : 1

23. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3, 4) meet x-axis and y-axis at point P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to

(1) $\frac{529}{64}$ (2) $\frac{125}{72}$ (3) $\frac{625}{72}$ (4) $\frac{585}{66}$

24. The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is:

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$ (3) $5\sqrt{4}$ (4) $4\sqrt{5}$

25. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles.

(2) Both circles' centres lie inside region of one another.

(3) Both circles pass through the centre of each other.

(4) Circles have two intersection points.

26. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 24x - 10y + 160 = 0$ is ______ .

27. Choose the correct statement about two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) circles have same centre

(2) circles have no meeting point

(3) circles have only one meeting point

(4) circles have two meeting points

28. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

(1) Rhombus (2) Square

(3) Rectangle (4) Parallelogram

29. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

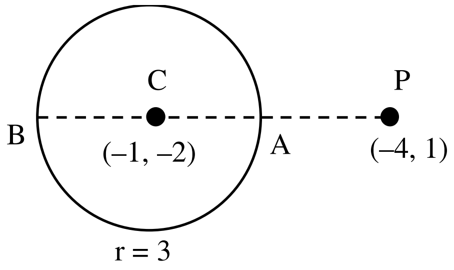
(1) $\left(0, \pm\sqrt{3}\right)$ (2) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

(3) $\left(2, \pm\frac{3}{2}\right)$ (4) $\left(1, \pm 2\right)$

SOLUTION

1. Official Ans. by NTA (3)

Sol.



Centre of smallest circle is A

Centre of largest circle is B

$r_2 = |CP - CA| = 3\sqrt{2} - 3$

$r_1 = CP + CB = 3\sqrt{2} + 3$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = \frac{(3\sqrt{2}+3)^2}{9} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$$

a = 3, b = 2

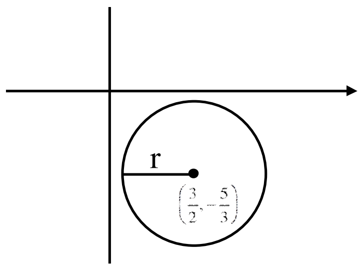
2. Official Ans. by NTA (4)

Sol. $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$

$$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$$

$$\text{Centre} \equiv (-g, -f) \equiv \left(\frac{3}{2}, \frac{-10}{6}\right)$$

$$\text{radius} = r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$$



Now,

$$\Rightarrow r < \frac{3}{2}$$

$$\Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$$

$\Rightarrow C > 100$ .....(1)

Now point of intersection of x – 2y = 4 and 2x – y = 5 is (2, –1), which lies inside the circle S.

$\therefore$ S(2, –1) < 0

$$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$$

$$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$$

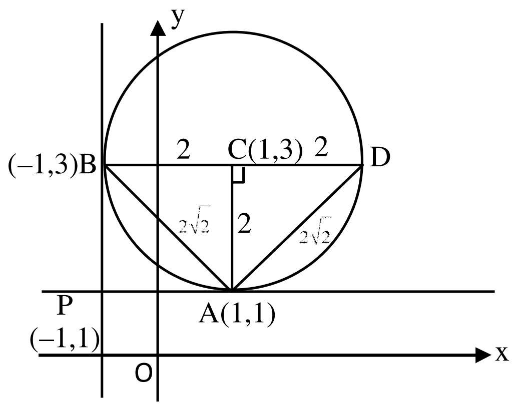
$\boxed{C < 156}$ .....(2)

From (1) & (2)

$\boxed{100 < C < 156}$ Ans.

3. Official Ans. by NTA (3)

Sol.

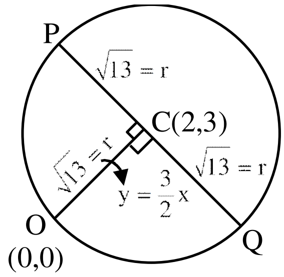


$$\Delta ABD = \frac{1}{2} \times 2 \times 4$$

= 4

4. Official Ans. by NTA (4)

Sol.



$$\tan\theta = -\frac{2}{3}$$

Using symmetric from of line

$P, Q : \left(2 \pm \sqrt{13}\cos\theta, 3 \pm \sqrt{13}\sin\theta\right)$

$$\left(2 \pm \sqrt{13}.\left(-\frac{3}{\sqrt{13}}\right), 3 \pm \sqrt{13}\left(\frac{2}{\sqrt{13}}\right)\right)$$

(–1, 5) & (5, 1)

5. Official Ans. by NTA (3)

Sol. $S_1 : x^2 + y^2 - x - y - \frac{1}{2} = 0$; $C_1\left(\frac{1}{2}, \frac{1}{2}\right)$

$r_1 = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}} = 1$

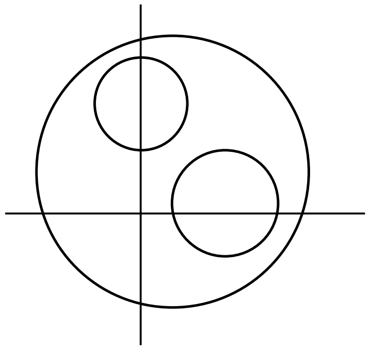
$S_2 : x^2 + y^2 - 4y + \frac{7}{4} = 0$; $C_2 : (0,2)$

$r_2 = \sqrt{4 - \frac{7}{4}} = \frac{3}{2}$

$S_3 : x^2 + y^2 - 4x - 2y + 5 - r^2 = 0$

$C_3 : (2,1)$

$r_3 = \sqrt{4 + 1 - 5 + r^2} = |r|$



$C_1C_3 = \sqrt{\frac{5}{2}}$

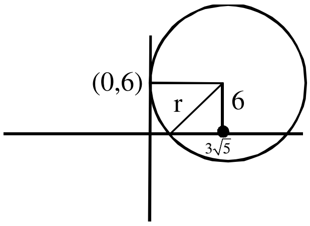
$\sqrt{\frac{5}{2}} \le |r - 1| \Rightarrow \left.\begin{matrix} r \le 1 + \sqrt{\frac{5}{2}} \\ r \ge \frac{3}{2} + \sqrt{5} \end{matrix}\right\}$

$C_2C_3 = \sqrt{5} \le \left|r - \frac{3}{2}\right|$

$\left.\begin{matrix} r - \frac{3}{2} \ge \sqrt{5} \\ r - \frac{3}{2} \le -\sqrt{5} \end{matrix}\right\}$

6. Official Ans. by NTA (2)

Sol.



$r = \sqrt{6^2 + (3\sqrt{5})^2}$

$= \sqrt{36 + 45} = 9$

7. Official Ans. by NTA (16)

Sol. Let P(x, y)

$x^2+y^2+x^2+(y-1)^2+(x-1)^2+y^2+(x-1)^2+(y-1)^2;$

$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$

$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$

$d = 2\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{7}{2}}$

$\Rightarrow d^2 = 16$

8. Official Ans. by NTA (1)

Sol. $(x - 2)^2 + (y - 1)^2 + \lambda(x - 2y) = 0$

$C : x^2 + y^2 + x(\lambda - 4) + y(-2 - 2\lambda) + 5 = 0$

$C_1 : x^2 + y^2 + 2y - 5 = 0$

$S_1 - S_2 = 0$ (Equation of PQ)

$(\lambda - 4)x - (2\lambda + 4)y + 10 = 0$ Passes through (0,–1)

$\Rightarrow \lambda = -7$

$C : x^2 + y^2 - 11x + 12y + 5 = 0$

$= \frac{\sqrt{245}}{4}$

Diometer = $7\sqrt{5}$

9. Official Ans. by NTA (61)

Sol. $r = \sqrt{\frac{p^2}{4} + \frac{(1-p)^2}{4} - 5} = \frac{\sqrt{2p^2 - 2p - 19}}{2}$

Since, $r \in (0, 5]$

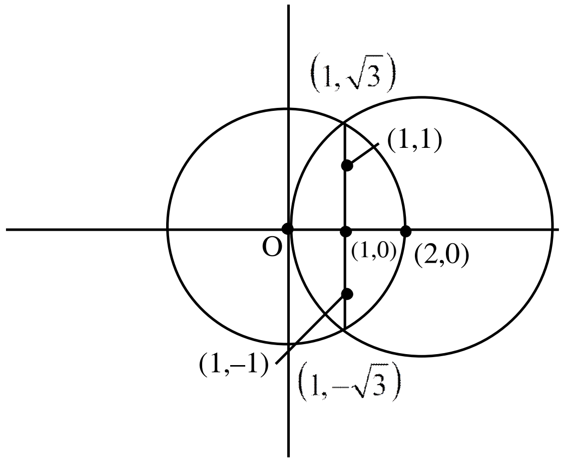
So, $0 < 2p^2 - 2p - 19 \le 100$

$\Rightarrow p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$ so,

number of integral values of p^2 is 61

10. Official Ans. by NTA (2)

Sol.



$(x-2)^2 + y^2 \le 4$

$x^2 + y^2 \le 4$

No. of points common in C_1 & C_2 is 5.

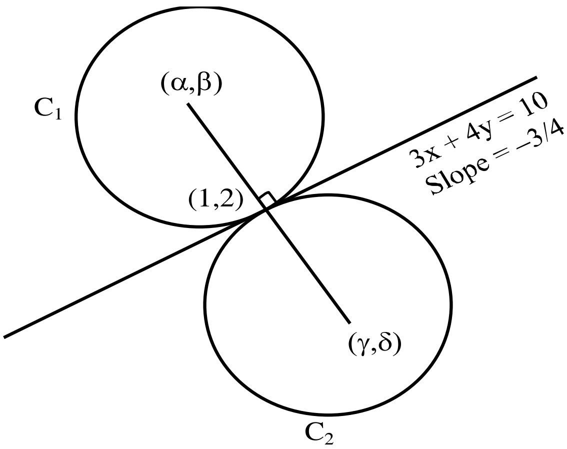
(0, 0) , (1, 0) , (2, 0) , (1, 1), (1, –1)

Similarly in C_2 & C_3 is 5.

No. of relations = $2^{5\times5} = 2^{25}$.

11. Official Ans. by NTA (40)

Sol. Slope of line joining centres of circles $= \frac{4}{3} = \tan\theta$



$\Rightarrow \cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$

Now using parametric form

$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm5$

$\oplus\ (x, y) = (1+5\cos\theta, 2+5\sin\theta)$

$(\alpha,\beta) = (4,6)$

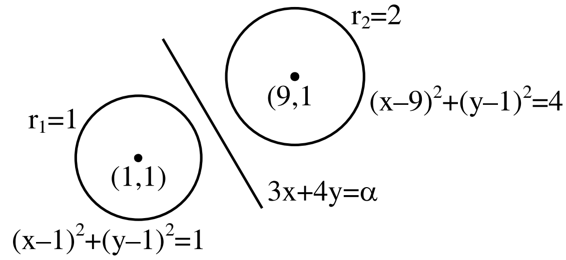
$\Theta\ (x, y) = (\gamma,\delta) = (1-5\cos\theta, 2-5\sin\theta)$

$(\gamma,s) = (-2,-2)$

$\Rightarrow |(\alpha+\beta)(\gamma+\delta)| = |10x-4| = 40$

12. Official Ans. by NTA (165)

Sol.



Both centres should lie on either side of the line as well as line can be tangent to circle.

$(3 + 4 - \alpha) . (27 + 4 - \alpha) < 0$

$(7 - \alpha) . (31 - \alpha) < 0 \Rightarrow \alpha \in (7, 31)$ …(1)

d_1 = distance of (1, 1) from line

d_2 = distance of (9, 1) from line

$d_1 \ge r_1 \Rightarrow \frac{|7-\alpha|}{5}\ 1 \Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty)$ …(2)

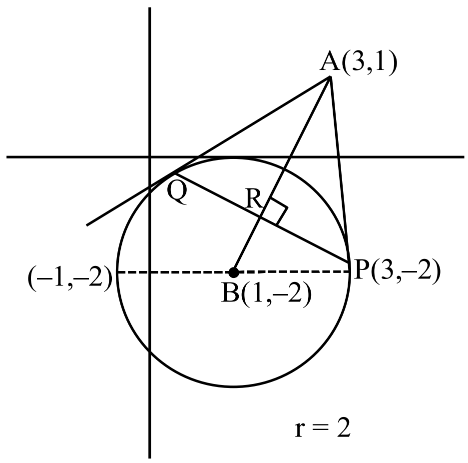
$d_2 \ge r_2 \Rightarrow \frac{|31-\alpha|}{5} \ge 2 \Rightarrow \alpha \in (-\infty, 21] \cup [41, \infty)$ …(3)

$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12, 21]$

Sum of integers = 165

13. Official Ans. by NTA (18)

Sol.



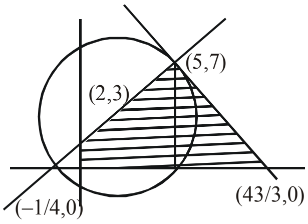
$\tan\theta = \frac{3}{2}$

$\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$

$8\left(\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ}\right) = 18$

14. Official Ans. by NTA (BONUS)

Sol.



Equation of normal

$4x - 3y + 1 = 0$

and equation of tangents

$3x + 4y - 43 = 0$

Area of triangle = $\frac{1}{2}\left(\frac{43}{3}+\frac{1}{4}\right)\times(7)$

$= \frac{1}{2}\left(\frac{172+3}{12}\right)\times 7$

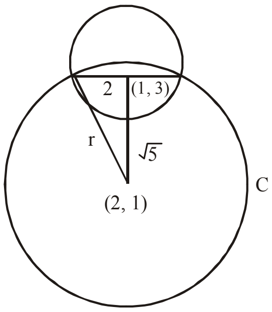
$A = \frac{1225}{24}$

$24A = 1225$

* as positive x-axis is given in the question so question should be bonus.

15. Official Ans. by NTA (3)

Sol.



$x^2 + y^2 + 2x - 6y + 6 = 0$

center (1, 3)

radius = 2

distance between (1, 3) and (2, 1) is $\sqrt{5}$

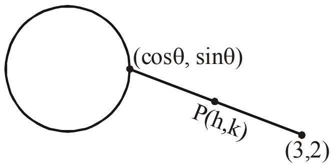
$\because \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

16. Official Ans. by NTA (2)

Sol. $h = \frac{\cos\theta + 3}{2}$

$k = \frac{\sin\theta + 2}{2}$



$\Rightarrow \left(h-\frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4}$

$\Rightarrow r = \frac{1}{2}$

17. Official Ans. by NTA (1)

Sol. P be a point on $(x - 1)^2 + (y - 1)^2 = 1$

so $P(1 + \cos\theta, 1 + \sin\theta)$

A(1, 4) B(1, –5)

$(PA)^2 + (PB)^2$

$= (\cos\theta)^2 + (\sin\theta - 3)^2 + (\cos\theta)^2 + (\sin\theta + 6)^2$

$= 47 + 6\sin\theta$

is maximum if $\sin\theta = 1$

$\Rightarrow \sin\theta = 1, \cos\theta = 0$

P(1, 1) A(1, 4) B(1, –5)

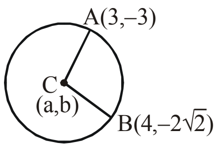
P, A, B are collinear points.

18. Official Ans. by NTA (9)

Sol. All normals of circle passes through centre

Radius = CA = CB

$CA^2 = CB^2$



$(a - 3)^2 + (b + 3)^2$

$= (a - 4)^2 + \left(b-2\sqrt{2}\right)^2$

$a+\left(3-2\sqrt{2}\right)b = 3$

$a - 2\sqrt{2}\,b + 3b = 3$ ...(1)

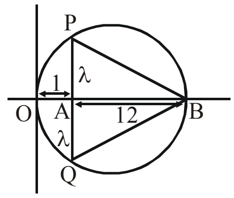
given that $a - 2\sqrt{2}\,b = 3$ ...(2)

from (1) & (2) $\Rightarrow a = 3, b = 0$

$a^2 + b^2 + ab = 9$

19. Official Ans. by NTA (2)

Sol. $PA = AQ = \lambda$



$OA \cdot AB$

$= AP \cdot AQ$

$\Rightarrow 1.12 = \lambda \cdot \lambda$

$\Rightarrow \lambda = 2\sqrt{3}$

Area $\Delta PQB = \dfrac{1}{2} \times 2\lambda \times AB$

$\Delta = \dfrac{1}{2}.4\sqrt{3} \times 12$

$= 24\sqrt{3}$

20. Official Ans by NTA (3)

Sol. $x^2 + y^2 + ax + 2ay + c = 0$

$2\sqrt{g^2 - c} = 2\sqrt{\dfrac{a^2}{4} - c} = 2\sqrt{2}$

$\Rightarrow \dfrac{a^2}{4} - c = 2$ ...(1)

$2\sqrt{f^2 - c} = 2\sqrt{a^2 - c} = 2\sqrt{5}$

$\Rightarrow a^2 - c = 5$ ...(2)

(1) & (2)

$\dfrac{3a^2}{4} = 3 \Rightarrow a = -2 \ (a < 0)$

$\therefore c = -1$

Circle $\Rightarrow x^2 + y^2 - 2x - 4y - 1 = 0$

$\Rightarrow (x - 1)^2 + (y - 2)^2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\dfrac{1}{2}$

$m_{tangent} = 2$

Equation of tangent

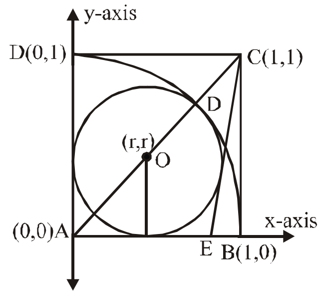
$\Rightarrow (y - 2) = 2(x - 1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Perpendicular distance from (0, 0) = $\left|\dfrac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

21. Official Ans. by NTA (1)

Sol.



Here AO + OD = 1 or $(\sqrt{2} + 1)r = 1$

$\Rightarrow \quad r = \sqrt{2-1}$

equation of circle $(x - r)^2 + (y - r)^2 = r^2$

Equation of CE

$y - 1 = m(x - 1)$

$mx - y + 1 - M = 0$

It is tangent to circle

$\therefore \quad \left|\dfrac{mr - r + 1 - m}{\sqrt{m^2+1}}\right| = r$

$\left|\dfrac{(m-1)r + 1 - m}{\sqrt{m^2+1}}\right| = r$

$\dfrac{(m-1)^2(r-1)^2}{m^2+1} = r^2$

Put $r = \sqrt{2} - 1$

On solving $m = 2 - \sqrt{3}, 2 + \sqrt{3}$

Taking greater slope of CE as

$2 + \sqrt{3}$

$y - 1 = (2 + \sqrt{3})(x - 1)$

Put $\quad y = 0$

$-1 = (2 + \sqrt{3})(x - 1)$

$\dfrac{-1}{2+\sqrt{3}} \times \left(\dfrac{2-\sqrt{3}}{2-\sqrt{3}}\right) = x - 1$

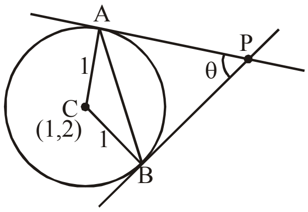
$x - 1 = \sqrt{3} - 1$

$EB = 1 - x = 1 - (\sqrt{3} - 1)$

$EB = 2 - \sqrt{3}$

22. Official Ans. by NTA (2)

Sol.



$\tan\theta = \frac{12}{5}$

$PA = \cot\frac{\theta}{2}$

$\therefore$ area of $\Delta PAB = \frac{1}{2}(PA)^2\sin\theta = \frac{1}{2}\cot^2\frac{\theta}{2}\sin\theta$

$= \frac{1}{2}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta$

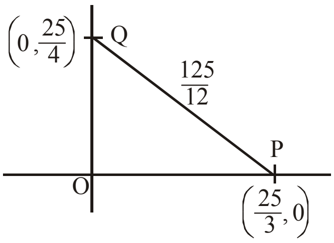
$= \frac{1}{2}\left(\frac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)\left(\frac{12}{13}\right) = \frac{1}{2}\frac{18}{18}\times\frac{2}{13} = \frac{27}{26}$

area of $\Delta CAB = \frac{1}{2}\sin\theta = \frac{1}{2}\left(\frac{12}{13}\right) = \frac{6}{13}$

$\therefore \frac{\text{area of }\Delta PAB}{\text{area of }\Delta CAB} = \frac{9}{4}$ **Option (2)**

23. Official Ans. by NTA (3)

Sol. Tangent to circle $3x + 4y = 25$



$OP + OQ + OR = 25$

Incentre $= \left(\frac{\frac{25}{4}\times\frac{25}{3}}{25}, \frac{\frac{25}{4}\times\frac{25}{3}}{25}\right)$

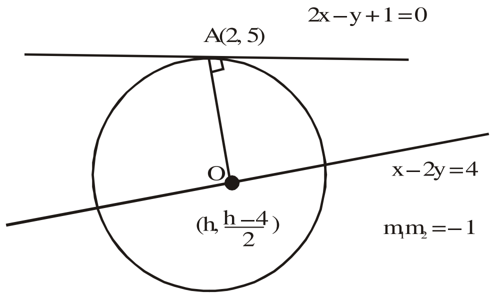
$= \left(\frac{25}{12}, \frac{25}{12}\right)$

$\therefore r^2 = 2\left(\frac{25}{12}\right)^2 = 2\times\frac{625}{144} = \frac{625}{72}$

Option (3)

24. Official Ans. by NTA (1)

Sol.



$\left(\frac{h-\frac{(h-4)}{2}}{2-h}\right)(2) = -1$

$h = 8$

center $(8, 2)$

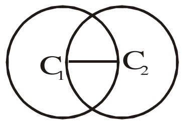
$\text{Radius} = \sqrt{(8-2)^2+(2-5)^2} = 3\sqrt{5}$)

25. Official Ans. by NTA (2)

Sol. $r_1 = 3$, c_1 (5, 5)

$r_2 = 3$, c_2 (8, 5)

$C_1C_2 = 3$, $r_1 = 3$, $r_2 = 3$

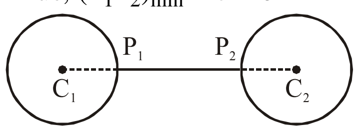


26. Official Ans. by NTA (1)

Sol. Given $C_1(5, 5)$, $r_1 = 3$ and C_2 (12, 5), $r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{min} = 7 - 6 = 1$



27. Official Ans. by NTA (3)

Sol. $x^2 + y^2 - 10x - 10y + 41 = 0$

$A(5,5)$, $R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

$B(11,5)$, $R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

$\Rightarrow$ circles have only one meeting point.

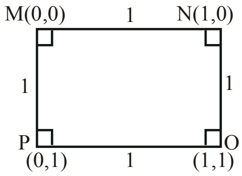
28. Official Ans. by NTA (2)

Sol. $M : x^2 + y^2 = 1 \quad (0,0)$

$N : x^2 + y^2 - 2x = 0 \quad (1,0)$

$O : x^2 + y^2 - 2x - 2y + 1 = 0 \quad (1,1)$

$P : x^2 + y^2 - 2y = 0 \quad (0,1)$

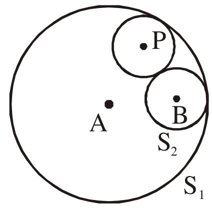


29. Official Ans. by NTA (3)

Sol. $S_1 : x^2 + y^2 = 9$ ⟨ $r_1 = 3$, A(0, 0)

$S_2 : (x-2)^2 + y^2 = 1$ ⟨ $r_2 = 1$, B(2, 0)

$\because\ c_1c_2 = r_1 - r_2$



$\therefore$ given circle are touching internally

Let a veriable circle with centre P and radius r

$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

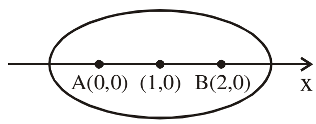
$\Rightarrow PA + PB = r_1 + r_2$

$\Rightarrow PA + PB = 4 \quad (> AB)$

$\Rightarrow$ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is $2a = 4$, $e = \frac{1}{2}$

$\Rightarrow$ centre is at (1, 0) and $b^2 = a^2(1 - e^2) = 3$

if x-ellipse



$\Rightarrow E : \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$