MATHEMATICS

LOGARITHM

1. **Official Ans. by NTA (1)**

Sol. $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$

$\log_{(x+1)}(2x+5)(x+1)+2\log_{(2x+5)}(x+1)=4$

$\log_{(x+1)}(2x+5)+1+2\log_{(2x+5)}(x+1)=4$

Put $\log_{(x+1)}(2x+5)=t$

$t+\frac{2}{t}=3 \Rightarrow t^2-3t+2=0$

$t=1, 2$

$\log_{(x+1)}(2x+5)=1$ & $\log_{(x+1)}(2x+5)=2$

$x+1=2x+3$ & $2x+5=(x+1)^2$

$x=-4$ (rejected)

$x^2=4 \Rightarrow x=2, -2$ (rejected)

So, $x=2$

No. of solution = 1

2. **Official Ans. by NTA (2)**

Sol. $x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$

$\log_2(2^{x+1})-\log_2(3+2^x)^2+\log_2(10-2^{-x})=0$

$$\log_2\left(\frac{2^{x+1}.(10-2^{-x})}{(3+2^x)^2}\right)=0$$

$$\frac{2(10.2^x-1)}{(3+2^x)^2}=1$$

$\Rightarrow 20.2^x-2=9+2^{2x}+6.2^x$

$\therefore (2^x)^2-14(2^x)+11=0$

Roots are 2^{x_1} & 2^{x_2}

$\therefore 2^{x_1}.2^{x_2}=11$

$x_1+x_2=\log_2(11)$

COMPOUND ANGLE

1. **Official Ans. by NTA (2)**

Sol. $$\cot\theta=\frac{1+\cos 2\theta}{\sin 2\theta}=\frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)} \qquad \theta=\frac{\pi}{24}$$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right)=\frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$=\frac{\left(2\sqrt{2}+\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$=\frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$$

$$=\sqrt{6}+\sqrt{2}+\sqrt{3}+2$$

2. **Official Ans. by NTA (3)**

Sol. $$x=\frac{1}{2}\left(\tan\frac{\pi}{9}+\tan\frac{7\pi}{18}\right)$$

and $2y=\tan\frac{\pi}{9}+\tan\frac{5\pi}{18}$

so, $x-2y=\frac{1}{2}\left(\tan\frac{\pi}{9}+\tan\frac{7\pi}{18}\right)-\left(\tan\frac{\pi}{9}+\tan\frac{5\pi}{18}\right)$

$$\Rightarrow |x-2y|=\left|\frac{\cot\frac{\pi}{9}-\tan\frac{\pi}{9}}{2}-\tan\frac{5\pi}{18}\right|$$

$$=\left|\cot\frac{2\pi}{9}-\cot\frac{2\pi}{9}\right|=0$$

$$\left(\text{as }\tan\frac{5\pi}{18}=\cot\frac{2\pi}{9};\tan\frac{7\pi}{18}=\cot\frac{\pi}{9}\right)$$

3. Official Ans. by NTA (1)

Sol. $\alpha = \max\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$

$= \max\{2^{6\sin 3x} \cdot 2^{8\cos 3x}\}$

$= \max\{2^{6\sin 3x + 8\cos 3x}\}$

and $\beta = \min\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\} = \min\{2^{6\sin 3x + 8\cos 3x}\}$

Now range of 6 sin 3x + 8 cos 3x

$= \left[-\sqrt{6^2+8^2}, +\sqrt{6^2+8^2}\right] = [-10, 10]$

$\alpha = 2^{10}$ & $\beta = 2^{-10}$

So, $\alpha^{1/5} = 2^2 = 4$

$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$

quadratic $8x^2 + bx + c = 0$, $c - b$

$= 8 \times [(\text{product of roots}] + (\text{sum of roots})$

$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4}\right] = 8 \times \left[\frac{21}{4}\right] = 42$

4. Official Ans. by NTA (3)

Sol. $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$$

$$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{8}$$

5. Official Ans. by NTA (2)

Sol. $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$

As A,B,C are angles of triangle

$A + B + C = \pi$

$A = \pi - (B + C)$

So, $\sin A = \sin(B + C)$ …(1)

Similarly $\sin B = \sin(A + C)$ …(2)

From (1) and (2)

$$\frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$$

$\sin(C + B).\ \sin(C - B) = \sin(A - C)\sin(A + C)$

$$\sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$$

$$\{\because \sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y\}$$

$$2\sin^2 C = \sin^2 A + \sin^2 B$$

By sine rule

$2c^2 = a^2 + b^2$

$\Rightarrow b^2, c^2$ and a^2 are in A.P.

6. Official Ans. by NTA (1)

Sol. $2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right) = 1$

$$2\cos x\left(4\left(\sin^2\frac{\pi}{4} - \sin^2 x\right)-1\right) = 1$$

$$2\cos x\left(4\left(\frac{1}{2} - \sin^2 x\right)-1\right) = 1$$

$$2\cos x(2 - 4\sin^2 x - 1) = 1$$

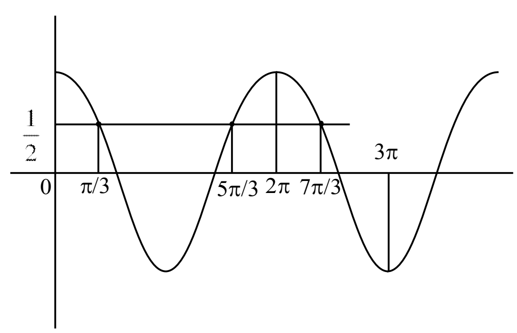
$$2\cos x(1 - 4\sin^2 x) = 1$$

$$2\cos x(4\cos^2 x - 3) = 1$$

$$4\cos^3 x - 3\cos x = \frac{1}{2}$$

$$\cos 3x = \frac{1}{2}$$

$x \in [0, \pi]$ $\therefore$ $3x \in [0, 3\pi]$



7. Official Ans. by NTA (1)

Sol. Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}=\theta$

$\sin 4\theta=\frac{\sqrt{63}}{8}$

$\cos 4\theta=\frac{1}{8}$

$2\cos^2 2\theta-1=\frac{1}{8}$

$\cos^2 2\theta=\frac{9}{16}$

$\cos 2\theta=\frac{3}{4}$

$2\cos^2\theta-1=\frac{3}{4}$

$\cos^2\theta=\frac{7}{8}$

$\cos\theta=\frac{\sqrt{7}}{2\sqrt{2}}$

$\tan\theta=\frac{1}{\sqrt{7}}$

8. Official Ans. by NTA (4)

Sol. $e^{(\cos^2\theta+\cos^4\theta+.....\infty)\ell n^2}=2^{\cos^2\theta+\cos^4\theta+....\infty}$

$=2^{\cot^2\theta}$

Now $t^2-9t+9=0 \Rightarrow t=1,8$

$\Rightarrow\ 2^{\cot^2\theta}=1,8\Rightarrow\cot^2\theta=0,3$

$0<\theta<\frac{\pi}{2}\Rightarrow\cot\theta=\sqrt{3}$

$\Rightarrow\ \frac{2\sin\theta}{\sin\theta+\sqrt{3}\sin\theta}=\frac{2}{1+\sqrt{3}\cot\theta}=\frac{2}{4}=\frac{1}{2}$

9. Official Ans. by NTA (4)

Sol. $15\sin^4\alpha+10\cos^4\alpha=6$

$15\sin^4\alpha+10\cos^4\alpha=6(\sin^2\alpha+\cos^2\alpha)^2$

$(3\sin^2\alpha-2\cos^2\alpha)^2=0$

$\tan^2\alpha=\frac{2}{3}\ .\ \cot^2\alpha=\frac{3}{2}$

$\Rightarrow 27\sec^6\alpha+8\text{cosec}^6\alpha$

$=27(\sec^6\alpha)^3+8(\text{cosec}^6\alpha)^3$

$=27(1+\tan^2\alpha)3+8(1+\cot^2\alpha)^3$

$=250$

QUADRATIC EQUATION

1. Official Ans. by NTA (3)

Sol. As, $\left(\alpha^2+\sqrt{3}\right)=-(3)^{1/4}\cdot\alpha$

$\Rightarrow\left(\alpha^4+2\sqrt{3}\alpha^2+3\right)=\sqrt{3}\alpha^2$ (On squaring)

$\therefore\ (\alpha^4+3)=(-)\sqrt{3}\alpha^2$

$\Rightarrow\ \alpha^8+6\alpha^4+9=3\alpha^4$ (Again squaring)

$\therefore\ \alpha^8+3\alpha^4+9=0$

$\Rightarrow\ \boxed{\alpha^8=-9-3\alpha^4}$

(Multiply by α^4)

So, $\alpha^{12}=-9\alpha^4-3\alpha^8$

$\therefore\ \alpha^{12}=-9\alpha^4-3(-9-3\alpha^4)$

$\Rightarrow\ \alpha^{12}=\cancel{-9\alpha^4}+27+\cancel{9\alpha^4}$

Hence, $\boxed{\alpha^{12}=(27)^2}$

$\Rightarrow\ (\alpha^{12})^8=(27)^8$

$\Rightarrow\ \alpha^{96}=(3)^{24}$

Similarly $\beta^{96}=(3)^{24}$

$\therefore\ \alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)=(3)^{24}\times 52$

$\Rightarrow$ Option (3) is correct.

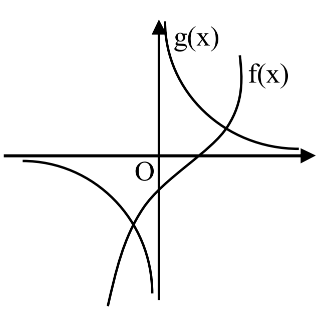
2. Official Ans. by NTA (1)

Sol. $e^{6x}-e^{4x}-2e^{3x}-12e^{2x}+e^x+1=0$

$\Rightarrow\left(e^{3x}-1\right)^2-e^x\left(e^{3x}-1\right)=12e^{2x}$

$\left(e^{3x}-1\right)^2\left(e^x-e^{-x}-e^{-2x}\right)=12$

$\Rightarrow\ \underbrace{e^x-e^{-x}-e^{-2x}}_{\text{increasing (let } f(x))}=\frac{12}{\underbrace{e^{3x}-1}_{\text{decreasing (let } g(x))}}$



$\Rightarrow$ No. of real roots = 2

3. Official Ans. by NTA (1)

Sol. $x^2+5\sqrt{2}\,x+10=0$

& $p_n = \alpha^n - \beta^n$ (Given)

Now $\dfrac{P_{17}P_{20}+5\sqrt{2}\,P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}=\dfrac{P_{17}(P_{20}+5\sqrt{2}\,P_{19})}{P_{18}(P_{19}+5\sqrt{2}P_{18})}$

$\dfrac{P_{17}(\alpha^{20}-\beta^{20}+5\sqrt{2}(\alpha^{19}-\beta^{19}))}{P_{18}(\alpha^{19}-\beta^{19}+5\sqrt{2}(\alpha^{18}-\beta^{18}))}$

$\dfrac{P_{17}(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2}))}{P_{18}(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2}))}$

Since $\alpha+5\sqrt{2}=-10/\alpha$ ....(1)

& $\beta+5\sqrt{2}=-10/\beta$ ....(2)

Now put there values in above expression

$=-\dfrac{10\,P_{17}\,P_{18}}{-10\,P_{18}\,P_{17}}=1$

4. Official Ans. by NTA (1)

Sol. $|x|^2 - |x| - 12 = 0$

$(|x| + 3)(|x| - 4) = 0$

$|x| = 4 \Rightarrow x = \pm 2$

5. Official Ans. by NTA (13)

Sol. $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\Sigma ab = -3$

$(ab + bc + ca)^2 = \Sigma(ab)^2 + 2abc\Sigma a$

$\Rightarrow \Sigma(ab)^2 = -2$

$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - 2\Sigma(ab)^2$

$= 9 - 2(-2) = 13$

6. Official Ans. by NTA (3)

Sol. $\left(x^2+\sqrt{5}\right)^2=\sqrt{20}\,x^2$

$x^4 = -5 \Rightarrow x^8 = 25$

$\alpha^8 + \beta^8 = 50$

7. Official Ans. by NTA (2)

Sol. $t^4 - t^3 - 4t^2 - t + 1 = 0,\ e^x = t > 0$

$\Rightarrow t^2 - t - 4 - \dfrac{1}{t}+\dfrac{1}{t^2}=0$

$\Rightarrow\ \alpha^2 - \alpha - 6 = 0,\ \alpha = t+\dfrac{1}{t}\ge 2$

$\Rightarrow \alpha = 3, -2$ (reject)

$\Rightarrow t+\dfrac{1}{t}=3$

$\Rightarrow$ The number of real roots = 2

8. Official Ans. by NTA (66)

Sol. $\dfrac{2}{x-1}-\dfrac{1}{x-2}=\dfrac{2}{k}$

$x \in R - \{1, 2\}$

$\Rightarrow\ k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$

$\Rightarrow\ k(x - 3) = 2(x^2 - 3x + 2)$

for $x \ne 3$, $\quad k=2\left(x-3+\dfrac{2}{x-3}+3\right)$

$x-3+\dfrac{2}{x-3}\ge 2\sqrt{2},\ \forall x>3$

& $x-3+\dfrac{2}{x-3}\le -2\sqrt{2},\ \forall x<-3$

$\Rightarrow 2\left(x-3+\dfrac{2}{x-3}+3\right)\in\left(-\infty,6-4\sqrt{2}\right]\cup\left[6+4\sqrt{2},\infty\right)$

for no real roots

$k\in(6-4\sqrt{2},6+4\sqrt{2})-\{0\}$

Integral $k \in \{1, 2….. 11\}$

Sum of k = 66

9. Official Ans. by NTA (18)

Sol. $3\alpha^2 - 10\alpha + 27\lambda = 0$ _____(1)

$\alpha^2 - \alpha + 2\lambda = 0$ _____(2)

(1) – 3(2) gives

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (1) we get

$9\lambda^2 - 3\lambda + 2\lambda - 0$

$9\lambda^2=\lambda \Rightarrow \lambda=\dfrac{1}{9}$ as $\lambda \ne 0$

Now $\alpha = 3\lambda \Rightarrow \lambda=\dfrac{1}{3}$

$\alpha + \beta = 1 \Rightarrow \beta = 2/3$

$\alpha+\gamma=\dfrac{10}{3}\ \Rightarrow\ \gamma=3$

$\dfrac{\beta\gamma}{\lambda}=\dfrac{\frac{2}{3}\times 3}{\frac{1}{9}}=18$

10. Official Ans. by NTA (4)

Sol. $x^2 + 9y^2 - 4x + 3 = 0$

$(x^2 - 4x) + (9y^2) + 3 = 0$

$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$

$(x - 2)^2 + (3y)^2 = 1$

$\frac{(x-2)^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$ (equation of an ellipse).

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x - 2 \in [-1, 1]$ and $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

$x \in [1, 3]$ $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

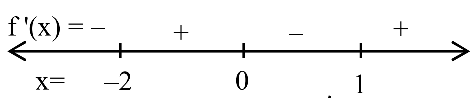
11. Official Ans. by NTA (4)

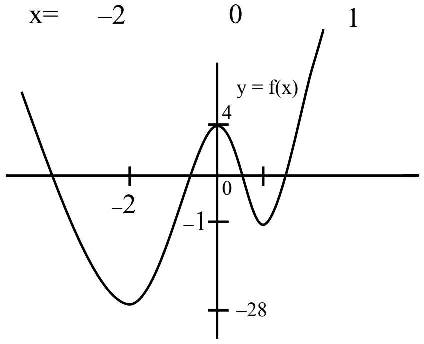
Sol. $3x^4 + 4x^3 - 12x^2 + 4 = 0$

So, Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$\therefore f'(x) = 12x(x^2 + x - 2)$

$= 12x(x + 2)(x - 1)$





12. Official Ans. by NTA (1)

Sol. $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2)$

$+ k(3x^2 + 4x + 2)^2 = 0$

Let $3x^2 + 4x + 3 = a$

and $3x^2 + 4x + 2 = b \Rightarrow b = a - 1$

Given equation becomes

$\Rightarrow a^2 - (k + 1)ab + kb^2 = 0$

$\Rightarrow a(a - kb) - b(a - kb) = 0$

$\Rightarrow (a - kb)(a - b) = 0 \Rightarrow a = kb$ or $a = b$ (reject)

$\because a = kb$

$\Rightarrow 3x^2 + 4x + 3 = k(3x^2 + 4x + 2)$

$\Rightarrow 3(k - 1)x^2 + 4(k - 1)x + (2k - 3) = 0$

for real roots $D \geq 0$

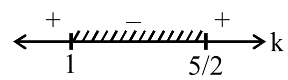
$\Rightarrow 16(k - 1)^2 - 4(3(k - 1))(2k - 3) \geq 0$

$\Rightarrow 4(k - 1)\{4(k - 1) - 3(2k - 3)\} \geq 0$

$\Rightarrow 4(k - 1)\{-2k + 5\} \geq 0$

$\Rightarrow -4(k - 1)\{2k - 5\} \geq 0$

$\Rightarrow (k - 1)(2k - 5) \leq 0$



$\therefore k \in \left[1, \frac{5}{2}\right]$

$\because k \neq 1$

$\therefore k \in \left(1, \frac{5}{2}\right]$ Ans.

13. Official Ans. by NTA (4)

Sol. $\text{cosec}18° = \frac{1}{\sin 18°} = \frac{4}{\sqrt{5} - 1} = \sqrt{5} + 1$

Let $\text{cosec}18° = x = \sqrt{5} + 1$

$\Rightarrow x - 1 = \sqrt{5}$

Squaring both sides, we get

$x^2 - 2x + 1 = 5$

$\Rightarrow x^2 - 2x - 4 = 0$

14. Official Ans. by NTA (1)

Sol. Consider the equation $x^2 + ax + b = 0$

If has two roots (not necessarily real α & β)

Either $\alpha = \beta$ or $\alpha \neq \beta$

Case (1) If $\alpha = \beta$, then it is repeated root. Given that $\alpha^2 - 2$ is also a root

So, $\alpha = \alpha^2 - 2 \Rightarrow (\alpha + 1)(\alpha - 2) = 0$

$\Rightarrow \alpha = -1$ or $\alpha = 2$

When $\alpha = -1$ then $(a, b) = (2, 1)$

$\alpha = 2$ then $(a, b) = (-4, 4)$

Case (2) If $\alpha \neq \beta$ Then

(I) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$

Here $(\alpha, \beta) = (2, -1)$ or $(-1, 2)$

Hence $(a, b) = (-(\alpha + \beta), \alpha\beta)$

$= (-1, -2)$

(II) $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

Then $\alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha)$

Since $\alpha \neq \beta$ we get $\alpha + \beta = \beta^2 + \alpha^2 - 4$

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$

Thus $-1 = 1 - 2\alpha\beta - 4$ which implies

$\alpha\beta = -1$ Therefore $(a, b) = (-(\alpha + \beta), \alpha\beta)$

$= (1, -1)$

(III) $\alpha = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$

$\Rightarrow \alpha = -\beta$

Thus $\alpha = 2, \beta = -2$

$\alpha = -1, \beta = 1$

Therefore $(a, b) = (0, -4)$ & $(0, -1)$

(IV) $\beta = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$ is same as (III)

Therefore we get 6 pairs of (a, b)

Which are (2, 1), (–4, 4), (–1, –2), (1, –1) (0, –4)

Option (1)

15. Official Ans. by NTA (2)

Sol. **Case-I**

$x \leq 5$

$$(x + 1)^2 - (x - 5) = \frac{27}{4}$$

$$(x + 1)^2 - (x + 1) - \frac{3}{4} = 0$$

$$x + 1 = \frac{3}{2}, -\frac{1}{2}$$

$$x = \frac{1}{2}, -\frac{3}{2}$$

Case-II

$x > 5$

$$(x + 1) + (x - 5) = \frac{27}{4}$$

$$(x + 1)^2 + (x + 1) - \frac{51}{4} = 0$$

$$x = \frac{-1 \pm \sqrt{52}}{2} \text{ (rejected as } x > 5)$$

So, the equation have two real root.

16. Official Ans. by NTA (4)

Sol. Consider $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 - 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$$(pq)^2 - 8pq - 128 = 0$$

$$pq = \frac{8 \pm 24}{2} = 16, -8$$

$\therefore$ $pq = 16$

$\therefore$ Required equation : $x^2 - (2)x + 16 = 0$

17. Official Ans. by NTA (1)

Sol. $x^2 - 2(3K - 1)x + 8K^2 - 7 > 0$

Now, $D < 0$

$\Rightarrow 4(3K - 1)^2 - 4 \times 1 \times (8K^2 - 7) < 0$

$\Rightarrow 9K^2 - 6K + 1 - 8K^2 + 7 < 0$

$\Rightarrow K^2 - 6K + 8 < 0$

$\Rightarrow (K - 4)(K - 2) < 0$

$\Rightarrow \boxed{K \in (2, 4)}$

18. Official Ans. by NTA (2)

Sol. $\because \alpha, \beta \in R \Rightarrow$ other root is $1 + 2i$

$\alpha = -(\text{sum of roots}) = -(1 - 2i + 1 + 2i) = -2$

$\beta = \text{product of roots} = (1 - 2i)(1 + 2i) = 5$

$\therefore \alpha - \beta = -7$

option (2)

19. Official Ans. by NTA (1)

Sol. $\alpha^2 - 6\alpha - 2 = 0$

$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$

Similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$

$(\alpha^{10} - \beta^{10}) - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$\Rightarrow a_{10} - 6a_9 - 2a_8 = 0$

$$\Rightarrow \frac{a_{10} - 2a_8}{3a_9} = 2$$

20. Official Ans. by NTA (324)

Sol. $x^2 - x - 1 = 0$ roots = α, β

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^{n+1} = \alpha^n + \alpha^{n-1}$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^{n+1} = \beta^n + \beta^{n-1}$

+

$P_{n+1} = P_n + P_{n-1}$

$29 = P_n + 11$

$P_n = 18$

$P_n^2 = 324$

21. Official Ans. by NTA (1)

Sol. $\log_4(x-1) = \log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1)^{1/2} = \log_2(x-3)$

$\Rightarrow (x-1)^{1/2} = x-3$

$\Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x-2)(x-5) = 0$

$\Rightarrow x = 2, 5$

But $x \neq 2$ because it is not satisfying the domain of given equation i.e $\log_2(x-3) \rightarrow$ its domain $x > 3$

finally x is 5

$\therefore$ No. of solutions = 1.

22. Official Ans. by NTA (5)

Sol. $f : [-1, 1] \rightarrow R$

$f(x) = ax^2 + bx + c$

$f(-1) = a - b + c = 2$ ...(1)

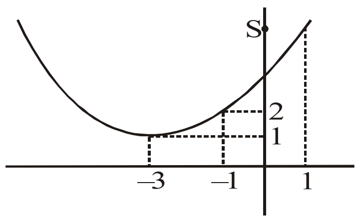
$f'(-1) = -2a + b = 1$ ...(2)

$f''(x) = 2a$

$\Rightarrow$ Max. value of $f''(x) = 2a = \frac{1}{2}$

$\Rightarrow a = \frac{1}{4};\ b = \frac{3}{2};\ c = \frac{13}{4}$

$\therefore f(x) = \frac{x^2}{4} + \frac{3}{2}x + \frac{13}{4}$



For, $x \in [-1, 1] \Rightarrow 2 \le f(x) \le 5$

$\therefore$ Least value of α is 5

23. Official Ans. by NTA (1)

Sol. Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$

So, $x = 3 + \cfrac{1}{4 + \cfrac{1}{x}} = 3 + \cfrac{1}{\cfrac{4x+1}{x}}$

$\Rightarrow (x-3) = \frac{x}{(4x+1)}$

$\Rightarrow (4x+1)(x-3) = x$

$\Rightarrow 4x^2 - 12x + x - 3 = x$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$x = \frac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \frac{12 \pm \sqrt{12(16)}}{8}$

$= \frac{12 \pm 4 \times 2\sqrt{3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$

$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$.

But only positive value is accepted

So, $x = 1.5 + \sqrt{3}$

SEQUENCE & PROGRESSION

1. Official Ans. by NTA (4)

Sol. $s = 2\log_9 x + 3\log_9 x + \ldots\ldots + 22\log_9 x$

$s = \log_9 x\,(2 + 3 + \ldots.. + 22)$

$s = \log_9 x \left\{\frac{21}{2}(2+22)\right\}$

Given $252\log_9 x = 504$

$\Rightarrow \log_9 x = 2 \Rightarrow x = 81$

2. Official Ans. by NTA (7)

Sol. $a_{n+2} = 2a_{n+1} + a_n$, let $\sum_{n=1}^{\infty}\frac{a_n}{8^n} = P$

Divide by 8^n we get

$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$

$\Rightarrow 64\frac{a_{n+2}}{8^{n+2}} = \frac{16a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$

$$64\sum_{n=1}^{\infty}\frac{a_{n+2}}{8^{n+2}}=16\sum_{n=1}^{\infty}\frac{a_{n+1}}{8^{n+1}}+\sum_{n=1}^{\infty}\frac{a_n}{8^n}$$

$$64\left(P-\frac{a_1}{8}-\frac{a_2}{8^2}\right)=16\left(P-\frac{a_1}{8}\right)+P$$

$$\Rightarrow 64\left(P-\frac{1}{8}-\frac{1}{64}\right)=16\left(P-\frac{1}{8}\right)+P$$

64P – 8 – 1 = 16 P – 2 + P

47P = 7

3. Official Ans. by NTA (1)

Sol. $S_{10}=530\Rightarrow \frac{10}{2}\{2a+9d\}=530$

$\Rightarrow$ 2a + 9d = 106 ….(1)

and S_5 = 140 $\Rightarrow \frac{5}{2}\{2a+4d\}=140$

$\Rightarrow$ 2a + 4d = 56 …..(2)

$\Rightarrow 5d=50\Rightarrow \boxed{d=10}\Rightarrow \boxed{a=8}$

Now, $S_{20}-S_6=\frac{20}{2}\{2a+19d\}-\frac{6}{2}\{2a+5d\}$

= 14a + 175d

= (14 × 8) + (175 × 10)

= 1862

4. Official Ans. by NTA (1251)

Sol. $2040=2^3\times 3\times 5\times 17$

n should not be multiple of 2, 3, 5 and 17.

Sum of all n = (1 + 3 + 5 … + 99) – (3 + 9 + 15 + 21 + … + 99) – (5 + 25 + 35 + 55 + 65 + 85 + 95) – (17)

$=2500-\frac{17}{2}(3+99)-365-17$

= 2500 – 867 – 365 – 17

= 1251

5. Official Ans. by NTA (1)

Sol. Let a be first term and d be common diff. of this A.P.

Given $S_{3n}=3S_{2n}$

$$\Rightarrow \frac{3n}{2}[2a+(3n-1)d]=3\frac{2n}{2}[2a+(2n-1)d]$$

$\Rightarrow$ 2a + (3n – 1)d = 4a + (4n – 2)d

$\Rightarrow$ 2a + (n – 1)d = 0

Now

$$\frac{S_{4n}}{S_{2n}}=\frac{\frac{4n}{2}[2a+(4n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]}=\frac{2\left[\underbrace{2a+(n-1)d}_{=0}+3nd\right]}{\left[\underbrace{2a+(n-1)d}_{=0}+nd\right]}$$

$$=\frac{6nd}{nd}=6$$

6. Official Ans. by NTA (3)

Sol. $$\ell=\left(\underbrace{1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}}_{S}+\ldots\right)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\ldots\right)}$$

$$S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\ldots.$$

$$\frac{S}{3}=\quad\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\ldots.$$

$$\frac{2S}{3}=1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\ldots.$$

$$\frac{2S}{3}=\frac{4}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\ldots.$$

$$S=\frac{3}{2}\left(\frac{4/3}{1-1/3}\right)=3$$

Now $\ell=(3)^{\log_{0.25}\left(\frac{1/3}{1-1/3}\right)}$

$$\ell=3^{\log_{((1/4))}\left(\frac{1}{2}\right)}=3^{1/2}=\sqrt{3}$$

$\Rightarrow \ell^2=3$

7. Official Ans. by NTA (3)

Sol. $2\log_3(2^x-5)=\log_3 2+\log_3\left(2^x-\frac{7}{2}\right)$

Let 2^x = t

$$\log_3(t-5)^2=\log_3 2\left(t-\frac{7}{2}\right)$$

$$(t-5)^2=2t-7$$

$$t^2-12t+32=0$$

$$(t-4)(t-8)=0$$

$$\Rightarrow 2^x=4 \text{ or } 2^x=8$$

X = 2 (Rejected)

Or x = 3

8. Official Ans. by NTA (832)

Sol. $B - C \equiv \{7,13,19,...97, ...\}$

Now, $n^2 - n \le 100 \times 100$

$\Rightarrow n(n-1) \le 100 \times 100$

$\Rightarrow A = \{1,2,...., 100\}$.

So, $A \cap (B - C) = \{7,13,19,...,97\}$

Hence, sum = $\frac{16}{2}(7+97) = 832$

9. Official Ans. by NTA (4)

Allen Ans. (BONUS)

Sol. $S = \frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + ... \frac{2^{100}}{x^{2^{100}}+1}$

$S + \frac{1}{1-x} = \frac{1}{1-x} + \frac{1}{x+1} + ...$

$= \frac{2}{1-x^2} + \frac{2}{1+x^2} + ...$

$S + \frac{1}{1-x} = \frac{2^{101}}{1-x^{2^{101}}}$

Put x = 2

$S = 1 - \frac{2^{101}}{2^{2^{101}} - 1}$

Not in option (BONUS)

10. Official Ans. by NTA (2)

Sol. Sum of infinite terms :

$\frac{a}{1-r} = 15$ .... (i)

Series formed by square of terms:

$a^2, a^2r^2, a^2r^4, a^2r^6$ .....

Sum = $\frac{a^2}{1-r^2} = 150$

$\Rightarrow \frac{a}{1-r}.\frac{a}{1+r} = 150 \Rightarrow 15.\frac{a}{1+r} = 150$

$\Rightarrow \frac{a}{1+r} = 10$ ...... (ii)

by (i) and (ii) a = 12 ; $r = \frac{1}{5}$

Now series : ar^2, ar^4, ar^6

Sum = $\frac{ar^2}{1-r^2} = \frac{12.\left(1/25\right)}{1-1/25} = \frac{1}{2}$

11. Official Ans. by NTA (2021)

Sol. $c_2 = a_2 + b_2 = a_1 - 3 + 2b_1 = 12$

$a_1 + 2b_1 = 15$ ____(1)

$c_3 = a_3 + b_3 = a_1 - 6 + 4b_1 = 13$

$a_1 + 4b_1 = 19$ ____(2)

from (1) & (2) $b_1 = 2$, $a_1 = 11$

$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10}(a_k + b_k) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$

$= \frac{10}{2}(2\times 11 + 9\times(-3)) + \frac{2(2^{10}-1)}{2-1}$

$= 5(22 - 27) + 2(1023)$

$= 2046 - 25 = 2021$

12. Official Ans. by NTA (4)

Sol. $\frac{2(1+2+3+...+y)}{3(1+2+3+...+y)} = \frac{4}{\log_{10} x}$

$\Rightarrow \log_{10}x = 6 \Rightarrow x = 10^6$

Now,

$y = (\log_{10} x) + \left(\log_{10} x^{\frac{1}{3}}\right) + \left(\log_{10} x^{\frac{1}{9}}\right) + ..\infty$

$= \left(1 + \frac{1}{3} + \frac{1}{9} + ...\infty\right)\log_{10} x$

$= \left(\frac{1}{1-\frac{1}{3}}\right)\log_{10} x = 9$

So, $(x,y) = (10^6, 9)$

13. Official Ans. by NTA (2)

Sol. $S = \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + ...$

$= \left[\frac{1}{1^2} - \frac{1}{2^2}\right] + \left[\frac{1}{2^2} - \frac{1}{3^2}\right] + \left[\frac{1}{3^2} - \frac{1}{4^2}\right] + ... + \left[\frac{1}{10^2} - \frac{1}{11^2}\right]$

$= 1 - \frac{1}{121}$

$= \frac{120}{121}$

14. Official Ans. by NTA (2)

Sol. Let numbers be $\frac{a}{r}$, a, ar $\rightarrow$ G.P

$\frac{a}{r}$, 2a, ar $\rightarrow$ A.P $\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

$r = 2 \pm \sqrt{3}$

4^{th} form of G.P $= 3r^2 \Rightarrow ar^2 = 3r^2 \Rightarrow a = 3$

$r = 2 + \sqrt{3}$, a = 3, $d = 2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = \left(2+\sqrt{3}\right)^2 - 3\sqrt{3}$

$= 7 + 4\sqrt{3} - 3\sqrt{3}$

$= 7 + \sqrt{3}$

15. Official Ans. by NTA (398)

Sol. 7 × 8, 10 × 10, 13 × 12, 16 × 14 ……

$T_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3)$

$= 2(3n^2 + 13n + 12) = 6n^2 + 26n + 24$

$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$

$= \frac{6(10\times11\times21)}{6} + 26\times\frac{10\times11}{2} + 24\times10$

$= 10 \times 11 (21 + 13) + 240$

$= 3980$

Mean $= \frac{S_{10}}{10} = \frac{3980}{10} = 398$

16. Official Ans. by NTA (3)

Sol. $\frac{\frac{10}{2}(2a_1 + 9d)}{\frac{p}{2}(2a_1 + (p-1)d)} = \frac{100}{p^2}$

$(2a_1 + 9d)p = 10(2a_1 + (p - 1)d)$

$9dp = 20a_1 - 2pa_1 + 10d(p - 1)$

$9p = (20 - 2p)\frac{a_1}{d} + 10(p - 1)$

$\frac{a_1}{d} = \frac{(10-p)}{2(10-p)} = \frac{1}{2}$

$\therefore \frac{a_{11}}{a_{10}} = \frac{a_1 + 10d}{a_1 + 9d} = \frac{\frac{1}{2}+10}{\frac{1}{2}+9} = \frac{21}{19}$

17. Official Ans. by NTA (5143)

Sol. A = 4 – digit numbers divisible by 3

A = 1002, 1005, …, 9999.

9999 = 1002 + (n – 1)3

$\Rightarrow$ (n – 1)3 = 8997 $\Rightarrow$ n = 3000

B = 4 – digit numbers divisible by 7

B = 1001, 1008, …, 9996

$\Rightarrow$ 9996 = 1001 + (n – 1)7

$\Rightarrow$ n = 1286

$A \cap B$ = 1008, 1029, …, 9996

9996 = 1008 + (n – 1)21

$\Rightarrow$ n = 429

So, no divisible by either 3 or 7

= 3000 + 1286 – 429 = 3857

total 4-digits numbers = 9000

required numbers = 9000 – 3857 = 5143

18. Official Ans. by NTA (305)

Sol. $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + ...$

$\frac{1}{5}S = \quad \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + ...$

On subtracting

$\frac{4}{5}S = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + ...$

$S = \frac{7}{4} + \frac{1}{10}\left(1 + \frac{2}{5} + \frac{3}{5^2} + ...\right)$

$S = \frac{7}{4} + \frac{1}{10}\left(1 - \frac{1}{5}\right)^{-2}$

$= \frac{7}{4} + \frac{1}{10}\times\frac{25}{16} = \frac{61}{32}$

$\Rightarrow$ 160S = 5 × 61 = 305

19. Official Ans. by NTA (1)

Sol. Let $T_r = r(n-r)$

$T_r = nr - r^2$

$$\Rightarrow S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n}(nr - r^2)$$

$$S_n = \frac{n.(n)(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n(n-1)(n+1)}{6}$$

Now $\sum_{r=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$

$$= \sum_{r=4}^{\infty}\left(2.\frac{n(n-1)(n+1)}{6.n(n-1)(n-2)!} - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty}\left(\frac{1}{3}\left(\frac{n-2+3}{(n-2)!}\right) - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty}\frac{1}{3}\cdot\frac{1}{(n-3)!} = \frac{1}{3}(e-1)$$

Option (1)

20. Official Ans. by NTA (2)

Sol. $\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}} = \sum_{n=1}^{20}\frac{1}{a_n(a_n+d)}$

$$= \frac{1}{d}\sum_{n=1}^{20}\left(\frac{1}{a_n} - \frac{1}{a_n+d}\right)$$

$$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{21}}\right) = \frac{4}{9} \text{ (Given)}$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_{21}-a_1}{a_1 a_{21}}\right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_1+20d-a_1}{a_1 a_2}\right) = \frac{4}{9} \Rightarrow a_1 a_2 = 45 \text{ .... (1)}$$

Now sum of first 21 terms $= \frac{21}{2}(2a_1 + 20d) = 189$

$\Rightarrow a_1 + 10d = 9$ ... (2)

For equation (1) & (2) we get

$a_1 = 3$ & $d = \frac{3}{5}$

OR

$a_1 = 15$ & $d = -\frac{3}{5}$

So, $a_6.a_{16} = (a_1 + 5d)(a_1 + 15d)$

$\Rightarrow a_6 a_{16} = 72$

Option (2)

21. Official Ans. by NTA (4)

Sol. $\frac{a+2+a}{3} = \frac{10}{3}$

$a = 4$

and $\frac{c+b+b}{3} = \frac{7}{3}$

$c + 2b = 7$

also $2b = a + c$

$2b - a + 2b = 7$

$b = \frac{11}{4}$

now $4x^2 + \frac{11}{4}x + 1 = 0 <^{\alpha}_{\beta}$

$\alpha^2 + \beta^2 - \alpha\beta = (\alpha+\beta)^2 - 3\alpha\beta$

$$= \left(\frac{-11}{16}\right)^2 - 3\left(\frac{1}{4}\right)$$

$$= \frac{121}{256} - \frac{3}{4} = \frac{-71}{256}$$

22. Official Ans. by NTA (3)

Sol. Let number are a, ar, ar^2, ar^3

$$a\frac{(r^4-1)}{r-1} = \frac{65}{12} \quad ...(1)$$

$$\frac{1}{a}\frac{\left(\frac{1}{r^4}-1\right)}{\frac{1}{r}-1} = \frac{65}{18}$$

$$\frac{1}{ar^3}\left(\frac{1-r^3}{1-r}\right) = \frac{65}{18} \quad ....(2)$$

$\frac{(1)}{(2)} \Rightarrow a^2r^3 = \frac{3}{2}$

and $a^3.r^3 = 1$

$ar = 1$

$(ar)^2.r = \frac{3}{2}$

$r = \frac{3}{2}, a = \frac{2}{3}$

So, third term $= ar^2 = \frac{2}{3} \times \frac{9}{4}$

$\alpha = \frac{3}{2}$

$2\alpha = 3$

23. Official Ans. by NTA (4)

Sol. $x = \frac{1}{1-\cos^2\theta} \Rightarrow \sin^2\theta = \frac{1}{x}$

Also, $\cos^2\theta = \frac{1}{y}$ & $1-\sin^2\theta\cos^2\theta = \frac{1}{z}$

So, $1-\frac{1}{x}\times\frac{1}{y} = \frac{1}{z} \Rightarrow z(xy-1) = xy$ ....(1)

Also, $\frac{1}{x}+\frac{1}{y} = 1 \quad \Rightarrow x+y = xy$ .....(2)

From (i) and (ii)

$xy + z = xyz = (x + y) z$

24. Official Ans. by NTA (9)

Sol. Let a_n be the side length of A_n.

So, $a_n = \sqrt{2}a_{n+1}$, $a_1 = 12$

$\Rightarrow a_n = 12\times\left(\frac{1}{\sqrt{2}}\right)^{n-1}$

Now, $\Rightarrow (a_n)^2 < 1 \qquad \Rightarrow \frac{144}{2^{(n-1)}} < 1$

$\Rightarrow 2^{(n-1)} > 144$

$\Rightarrow n - 1 \geq 8$

$\Rightarrow n \geq 9$

25. Official Ans. by NTA (2)

Sol. $T_n = \frac{n^2+6n+10}{(2n+1)!} = \frac{4n^2+24n+40}{4.(2n+1)!}$

$= \frac{(2n+1)^2+20n+39}{4.(2n+1)!}$

$= \frac{(2n+1)^2+(2n+1).10+29}{4(2n+1)!}$

$= \frac{1}{4}\left[\frac{(2n+1)^2}{(2n+1)(2n)!}+\frac{(2n+1)10}{(2n+1)(2n)!}+\frac{29}{(2n+1)!}\right]$

$= \frac{1}{4}\left[\frac{2n+1}{(2n)!}+\frac{10}{(2n)!}+\frac{29}{(2n+1)!}\right]$

$= \frac{1}{4}\left[\frac{1}{(2n-1)!}+\frac{11}{(2n)!}+\frac{29}{(2n+1)!}\right]$

$S_1 = \frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+... = \frac{e-\frac{1}{e}}{2}$

$S_2 = 11\left[\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+...\right] = 11\left[\frac{e+\frac{1}{e}-2}{2}\right]$

$S_3 = 29\left[\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+...\right] = 29\left[\frac{e-\frac{1}{e}-2}{2}\right]$

Now, $S = \frac{1}{4}[S_1+S_2+S_3]$

$= \frac{1}{4}\left[\frac{e}{2}-\frac{1}{2e}+\frac{11e}{2}+\frac{11}{2e}+\frac{29e}{2}-\frac{29}{2e}-4\right]$

$= \frac{41e}{8}-\frac{19}{8e}-10$

26. Official Ans. by NTA (10)

Sol. $4x^2 - 9x + 5 = 0 \Rightarrow x = 1, \frac{5}{4}$

Now given $\frac{5}{4} = \frac{t_p+t_q}{2}, t = t_pt_q$ where

$t_r = -16\left(-\frac{1}{2}\right)^{r-1}$

so $\frac{5}{4} = -8\left[\left(-\frac{1}{2}\right)^{p-1}+\left(-\frac{1}{2}\right)^{q-1}\right]$

$1 = 256\left(-\frac{1}{2}\right)^{p+q-2} \Rightarrow 2^{p+q-2} = (-1)^{p+q-2}2^8$

hence $p + q = 10$

27. Official Ans. by NTA (3)

Sol. $a, ar, ar^2, ...$

$T_2 + T_6 = \frac{25}{2} \Rightarrow ar(1+r^4) = \frac{25}{2}$

$a^2r^2(1+r^4)^2 = \frac{625}{4}$ .... (1)

$T_3 . T_5 = 25 \Rightarrow (ar^2)(ar^4) = 25$

$a^2r^6 = 25$ .... (2)

On dividing (1) by (2)

$\frac{(1+r^4)^2}{r^4} = \frac{25}{4}$

$4r^8 - 17r^4 + 4 = 0$

$(4r^4 - 1)(r^4 - 4) = 0$

$r^4 = \frac{1}{4}, 4 \Rightarrow r^4 = 4$

(an increasing geometric series)

$a^2r^6 = 25 \Rightarrow (ar^3)^2 = 25$

$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 5(1 + 2 + 4) = 35$

28. Official Ans. by NTA (1)

Sol. $S=1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+...$

$\frac{S}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+\frac{12}{3^4}+...$

$\frac{2S}{3}=1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+\frac{5}{3^4}+...+$ up to infinite terms

$\Rightarrow S=\frac{13}{4}$

29. Official Ans by NTA (14)

Sol. $a^2=\frac{b}{16}\Rightarrow\frac{1}{b}=\frac{1}{16a^2}$

$\frac{2}{b}=\frac{1}{a}+6$

$\frac{1}{8a^2}=\frac{1}{a}+6$

$\frac{1}{a^2}-\frac{8}{a}-48=0$

$\frac{1}{a}=12,-4 \quad\Rightarrow a=\frac{1}{12},-\frac{1}{4}$

$a=\frac{1}{12},\ a>0$

$b=16a^2=\frac{1}{9}$

$\Rightarrow 72(a+b)=6+8=14$

30. Official Ans by NTA (16)

Sol. $S_n(x)=(2+3+6+11+18+27+.......+\text{n-terms})\log_a x$

Let $S_1=2+3+6+11+18+27+....+T_n$

$S_1=2+3+6+..............................+T_n$

$T_n=2+1+3+5+......+\text{n terms}$

$T_n=2+(n-1)^2$

$S_1=\Sigma T_n=2n+\frac{(n-1)n(2n-1)}{6}$

$\Rightarrow S_n(x)=\left(2n+\frac{n(n-1)(2n-1)}{6}\right)\log_a x$

$S_{24}(x)=1093$ (Given)

$\log_a x\left(48+\frac{23.24.47}{6}\right)=1093$

$\log_a x=\frac{1}{4}$ .... (1)

$S_{12}(2x)=265$

$S_{12}(2x)=265$

$\log_a(2x)\left(24+\frac{11.12.23}{6}\right)=265$

$\log_a 2x=\frac{1}{2}$ .... (2)

(2) – (1)

$\log_a 2x-\log_a x=\frac{1}{4}$

$\log_a 2=\frac{1}{4}\ \Rightarrow\ a=16$

31. Official Ans. by NTA (3)

Sol. **GP :** 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

AP : 11, 16, 21, 26, 31, 36

Common terms : 16, 256, 4096 only

32. Official Ans. by NTA (2)

Sol. $2\log_{10}(4^x-2)=1+\log_{10}\left(4^x+\frac{18}{5}\right)$

$(4^x-2)^2=10\left(4^x+\frac{18}{5}\right)$

$(4^x)^2+4-4(4^x)-32=0$

$(4^x-16)(4^x+2)=0$

$4^x=16$

$x=2$

$\begin{vmatrix}3&1&4\\1&0&2\\2&1&0\end{vmatrix}=3(-2)-1(0-4)+4(1)$

$=-6+4+4=2$

33. Official Ans. by NTA (2)

Sol. $S=(100)(100)+(99)(101)+(98)(102)........$

$(2)(198)+(1)(199)$

$S=\sum_{x=0}^{99}(100-x)(100+x)=\sum 100^2-x^2$

$=100^3-\frac{99\times100\times199}{6}$

$\alpha=3 \qquad \beta=1650$

$\text{slope}=\frac{1650}{3}=550$

34. Official Ans. by NTA (2)

Sol. $T_n = \frac{1}{(2n+1)^2 - 1}\frac{1}{(2n+2)2n} = \frac{1}{4(n)(n+1)}$

$= \frac{(n+1)-n}{4n(n+1)} = \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$S = \frac{1}{4}\left(1 - \frac{1}{101}\right) = \frac{1}{4}\left(\frac{100}{101}\right) = \frac{25}{101}$

35. Official Ans. by NTA (4)

Sol. $S_{2n} = \frac{2n}{2}[2a + (2n-1)d],\ S_{4n} = \frac{4n}{2}[2a + (4n-1)d]$

$\Rightarrow S_2 - S_1 = \frac{4n}{2}[2a + (4n-1)d] - \frac{2n}{2}[2a + (2n-1)d]$

$= 4an + (4n-1)2nd - 2na - (2n-1)dn$

$= 2na + nd[8n - 2 - 2n + 1]$

$\Rightarrow 2na + nd[6n - 1] = 1000$

$2a + (6n-1)d = \frac{1000}{n}$

Now, $S_{6n} = \frac{6n}{2}[2a + (6n-1)d]$

$= 3n.\frac{1000}{n} = 3000$

36. Official Ans. by NTA (210)

Sol. $\left((x^{1/3}+1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r(x^{1/3})^{10-r}(-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$

$\Rightarrow r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

TRIGONOMETRIC EQUATION

1. Official Ans. by NTA (3)

Sol. $\sin^7 x \le \sin^2 x \le 1$ ...(1)

and $\cos^7 x \le \cos^2 x \le 1$ ...(2)

also $\sin^2 x + \cos^2 x = 1$

$\Rightarrow$ equality must hold for (1) & (2)

$\Rightarrow \sin^7 x = \sin^2 x$ & $\cos^7 = \cos^2 x$

$\Rightarrow \sin x = 0$ & $\cos x = 1$

or

$\cos x = 0$ & $\sin x = 1$

$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow$ 5 solutions

2. Official Ans. by NTA (4)

Sol. $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{\cos\frac{3x}{2} + \cos\frac{x}{2}\right\} = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{2\cos x\cos\frac{x}{2}\right\} = 0$

$2\sin\frac{5x}{2} = 0 \Rightarrow \frac{5X}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$

$\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

So sum $= 6\pi + \pi + 2\pi = 9\pi$

3. Official Ans. by NTA (3)

Sol. $\sin\theta + \cos\theta = \frac{1}{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$

$\sin 2\theta = -\frac{3}{4}$

Now :

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$= 1 - 2\left(-\frac{3}{4}\right)^2$

$= 1 - 2\times\frac{9}{16} = -\frac{1}{8}$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$= (3 - 4\sin^2 2\theta).\sin 2\theta$

$= \left[3 - 4\left(\frac{9}{16}\right)\right].\left(-\frac{3}{4}\right)$

$\Rightarrow \left[\frac{3}{4}\right] \times \left(-\frac{3}{4}\right) = -\frac{9}{16}$

$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$

$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$

4. Official Ans. by NTA (1)

Sol. $\frac{\cos x}{1 + \sin x} = |\tan 2x|$

$\Rightarrow \frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)} = |\tan 2x|$

$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$

$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$

$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$

or sum $= \frac{-11\pi}{6}$.

5. Official Ans. by NTA (56)

Sol. Given equation

$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 1 - \sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 2 - (\sin 2\theta)^2 - \sin 2\theta = 0$

$\Rightarrow (\sin 2\theta)^2 + (\sin 2\theta) - 2 = 0$

$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$

$\Rightarrow \sin 2\theta = 1$ or $\boxed{\sin 2\theta = -2}$ (not possible)

$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$\Rightarrow S = \frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4} = 7\pi$

$\Rightarrow \frac{8S}{\pi} = \frac{8 \times 7\pi}{\pi} = 56.00$

6. Official Ans. by NTA (2)

Sol. $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$

In interval $\left[0, \frac{\pi}{4}\right]$ only one solution

7. Official Ans. by NTA (4)

Sol. $\sin 2\theta + \tan 2\theta > 0$

$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$

$\Rightarrow \sin 2\theta \frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0 \Rightarrow \tan 2\theta(2\cos^2\theta) > 0$

Note : $\cos 2\theta \neq 0$

$\Rightarrow 1 - 2\sin^2\theta \neq 0 \Rightarrow \sin\theta \neq \pm\frac{1}{\sqrt{2}}$

Now, $\tan 2\theta (1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ (as $\cos 2\theta + 1 > 0$)

$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

As $\sin\theta \neq \pm\frac{1}{\sqrt{2}}$; which has been already considered

8. Official Ans. by NTA (2)

Sol. $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$

$\cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)$

$+ \frac{1}{4}.\cos^2\left(\frac{x-y}{2}\right) + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2}\cos\left(\frac{x-y}{2}\right)\right)^2 + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0$ and $\cos\left(\frac{x+y}{2}\right) = \frac{1}{2}\cos\left(\frac{x-y}{2}\right)$

$\Rightarrow x = y$ and $\cos x = \frac{1}{2} = \cos y$

$\therefore \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin x + \cos y = \frac{1+\sqrt{3}}{2}$

option (2)

9. Official Ans. by NTA (11)

Sol. 3 sinx + 4 cosx = k + 1

$\Rightarrow k+1 \in \left[-\sqrt{3^2+4^2}, \sqrt{3^2+4^2}\right]$

$\Rightarrow k+1 \in [-5,5]$

$\Rightarrow k \in [-6,4]$

No. of integral values of k = 11

10. Official Ans. by NTA (1)

Sol. $\sqrt{3}(\cos x)^2 - \sqrt{3}\cos x + \cos x - 1 = 0$

$\Rightarrow \left(\sqrt{3}\cos x + 1\right)(\cos x - 1) = 0$

$\Rightarrow \cos x = 1$ or $\cos x = -\frac{1}{\sqrt{3}}$ (reject)

$\Rightarrow x = 0$ only

11. Official Ans. by NTA (2)

Sol. $x \in \left(0, \frac{\pi}{2}\right)$

$\log_{10}\sin x + \log_{10}\cos x = -1$

$\Rightarrow \quad \log_{10}\sin x.\cos x = -1$

$\Rightarrow \quad \sin x.\cos x = \frac{1}{10}$ ......(1)

$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \quad \sin x + \cos x = 10^{\left(\log_{10}\sqrt{n} - \frac{1}{2}\right)} = \sqrt{\frac{n}{10}}$

by squaring

$1 + 2\sin x.\cos x = \frac{n}{10}$

$\Rightarrow \quad 1 + \frac{1}{5} = \frac{n}{10} \quad \Rightarrow \quad n = 12$

12. Official Ans. by NTA (2)

Sol. $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$

$(81)^{\sin^2 x} = t$

$t + \frac{81}{t} = 30$

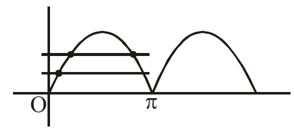
$t^2 - 30t + 81 = 0$

$(t-3)(t-27) = 0$

$(81)^{\sin^2 x} = 3^1$ or $(81)^{\sin^2 x} = 3^3$

$3^{4\sin^2 x} = 3^1$ or $3^{4\sin^2 x} = 3^3$

$\sin^2 x = \frac{1}{4}$ or $\sin^2 x = \frac{3}{4}$



Total sol. = 4

13. Official Ans. by NTA (1)

Sol. If cotx > 0 $\Rightarrow \frac{1}{\sin x} = 0$ (Not possible)

If cotx < 0 $\Rightarrow$ 2cotx + $\frac{1}{\sin x} = 0$

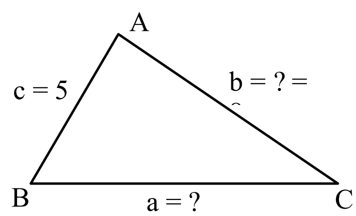
$\Rightarrow$ 2cosx = –1

$\Rightarrow x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ (reject)

SOLUTION OF TRIANGLE

1. Official Ans. by NTA (3)

Sol.



As, cosB = $\frac{3}{5} \Rightarrow \boxed{B = 53°}$

As, R = 5 $\Rightarrow \frac{c}{\sin c} = 2R$

$\Rightarrow \frac{5}{10} = \sin c \Rightarrow \boxed{C = 30°}$

Now, $\frac{b}{\sin B} = 2R \Rightarrow \boxed{b = 2(5)\left(\frac{4}{5}\right) = 8}$

Now, by cosine formula

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$

$\Rightarrow a^2 - 6a - 3g = 0$

$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$

$\Rightarrow \boxed{3 + 4\sqrt{3}}$ (Reject a = 3 $- 4\sqrt{3}$)

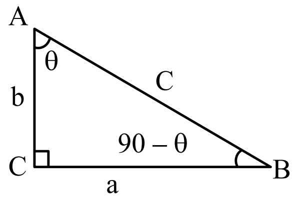
Now, $\Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(3+4\sqrt{3})$

$\Rightarrow \Delta = (6 + 8\sqrt{3})$

$\Rightarrow$ Option (3) is correct.

2. Official Ans. by NTA (2)

Sol.



$\angle A = \theta$

$\angle B = 90 - \theta$

a = smallest side

$c^2 = a^2 + b^2$

$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$

$\frac{b^2c^2}{a^2} = b^2 + c^2$

Use a = 2R sinA = 2R sinθ

b = 2R sin B = 2R sin (90 – θ) = 2R cosθ

c = 2R sinC = 2 sin90° = 2R

$\frac{4R^2\cos^2\theta}{4R^2\sin^2\theta} = 4R^2\cos^2\theta + 4R^2$

$\cos^2\theta = \sin^2\theta\cos^2\theta + \sin^2\theta$

$1 - \sin^2\theta = \sin^2\theta(1-\sin^2\theta) + \sin^2\theta$

$\sin^2\theta = \frac{3-\sqrt{5}}{2}$

$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

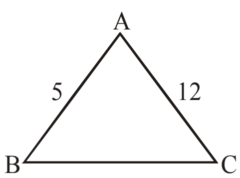
3. Official Ans by NTA (15)

Sol. $\Delta = \frac{1}{2}.5.12.\sin A = 30$

sinA = 1

A = 90° $\Rightarrow$ BC = 13

BC = 2R = 13



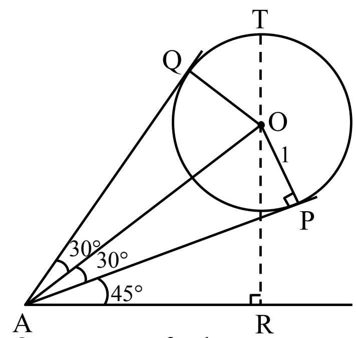
$r = \frac{\Delta}{S} = \frac{30}{15} = 2$

2R + r = 15

HEIGHT & DISTANCE

1. Official Ans. by NTA (2)

Sol.



O $\rightarrow$ centre of sphere

P,Q $\rightarrow$ point of contact of tangents from A

Let T be top most point of balloon & R be foot of perpendicular from O to ground.

From triangle OAP, OA = 16cosec30° = 32

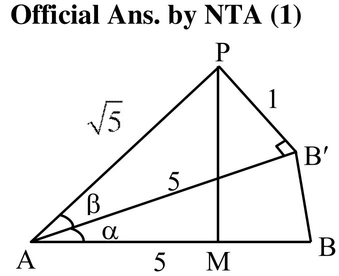
From triangle ABO, OR = OA sin75° = $32\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

So level of top most point = OR + OT

$= 8(\sqrt{6}+\sqrt{2}+2)$

2. Official Ans. by NTA (1)

Sol.



From figure.

$\sin\beta = \frac{1}{\sqrt{5}}$

$\therefore \tan\beta = \frac{1}{2}$

$\tan(\alpha+\beta) = 2$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta} = 2$

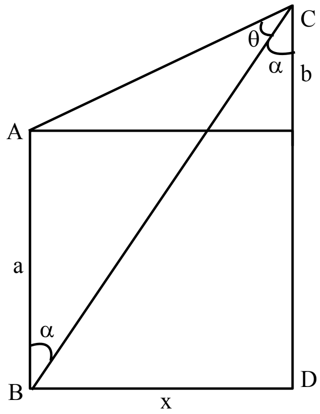
$\frac{\tan\alpha + \frac{1}{2}}{1 - \tan\alpha\left(\frac{1}{2}\right)} = 2$

$\tan\alpha = \frac{3}{4}$

$\boxed{\alpha = \tan^{1}\left(\frac{3}{4}\right)}$

3. Official Ans. by NTA (3)

Sol.



$\tan\theta = \frac{1}{2}$

$\tan(\theta+\alpha) = \frac{x}{b}$, $\tan\alpha = \frac{x}{a+b}$

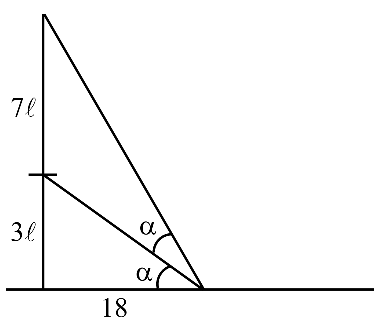
$\Rightarrow \frac{1}{2} + \frac{x}{a+b}$

$\Rightarrow \frac{\frac{1}{2}+\frac{x}{a+b}}{1-\frac{1}{2}\times\frac{x}{a+b}} = \frac{x}{b}$.

$\Rightarrow x^2 - 2ax + ab + b^2 = 0$

4. Official Ans. by NTA (2)

Sol.



Let height of pole = 10ℓ

$\tan\alpha = \frac{3\ell}{18} = \frac{\ell}{6}$

$\tan 2\alpha = \frac{10\ell}{18}$

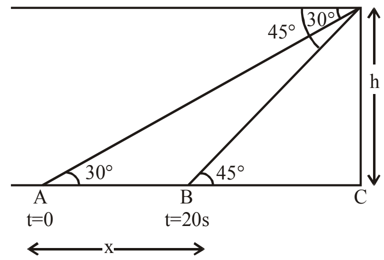
$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{10\ell}{18}$

use $\tan\alpha = \frac{\ell}{6} \Rightarrow \ell = \sqrt{\frac{72}{5}}$

height of pole = $10\ell = 12\sqrt{10}$

5. Official Ans. by NTA (3)

Sol.



Let speed of boat is u m/s and height of tower is h meter & distance AB = x metre

$\therefore\ x = h\cot 30° - h\cot 45°$

$\Rightarrow x = h(\sqrt{3}-1)$

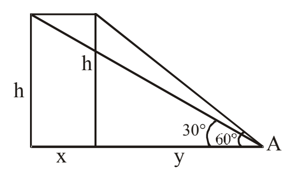
$\therefore\ u = \frac{x}{20} = \frac{h(\sqrt{3}-1)}{20}\text{ m/s}$

$\therefore$ Time taken to travel from B to C (Distance = h meter)

$= \frac{h}{u} = \frac{h}{h\frac{(\sqrt{3}-1)}{20}} = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3}+1)\text{ sec}$.

6. Official Ans. by NTA (4)

Sol.



$\tan 60° = \frac{h}{y}$

$\sqrt{3} = \frac{h}{y} \Rightarrow h = \sqrt{3}y$ …(1)

$\tan 30° = \frac{h}{x+y}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+y} \Rightarrow \sqrt{3}h = x+y$ …(2)

Speed 432 km/h $\Rightarrow \frac{432\times 20}{60\times 60} \Rightarrow \frac{12}{5}$ km

$\sqrt{3}h = \frac{12}{5} + y$

$\sqrt{3}h - \frac{12}{5} = y$

from (1)

$h = \sqrt{3}\left[\sqrt{3}h - \frac{12}{5}\right]$

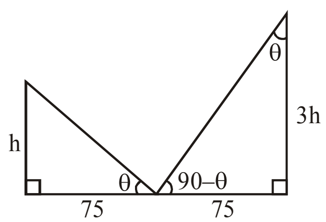
$h = 3h - \frac{12\sqrt{3}}{5}$

$h = \frac{6\sqrt{3}}{5}$ km

$h = 1200\sqrt{3}$ m

7. Official Ans. by NTA (2)

Sol.



$\tan\theta = \frac{h}{75} = \frac{75}{3h}$

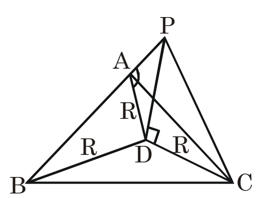
$\Rightarrow\ h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}$ m

8. Official Ans. by NTA (2)

Sol. Let PD = h, R = 2

As angle of elevation of top of pole from A, B, C are equal So D must be circumcentre of ΔABC



$\tan\left(\frac{\pi}{3}\right) = \frac{PD}{R} = \frac{h}{R}$

$h = R\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$

DETERMINANT

1. Official Ans. by NTA (1)

Sol. $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$

$C_2 \to C_2 - C_3$

$\Rightarrow \begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix} = 2$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

$\Rightarrow \lambda \begin{vmatrix} x-2\lambda & 1 & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix} = 2$

$\Rightarrow 1\,(4\lambda^2 - 4\lambda^2 + 2\lambda) = 2$

$\Rightarrow \boxed{\lambda^2 = 1}$

2. Official Ans. by NTA (2)

Sol. $\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & K \end{vmatrix} = 0$

$\Rightarrow 3(2K + 15) + K + 18 - 28 = 0$

$\Rightarrow 7K + 35 = 0 \Rightarrow K = -5$

3. Official Ans. by NTA (4)

Sol. $x + y + z = 6$ …(i)

$3x + 5y + 5z = 26$ …(ii)

$x + 2y + \lambda z = \mu$ …(iii)

$5 \times$ (i) – (ii) $\Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore$ from (i) and (iii)

$y + z = 4$ …(iv)

$2y + \lambda z = \mu - 2$ …(v)

(v) – 2 × (iv)

$\Rightarrow (\lambda - 2)z = \mu - 10$

$\Rightarrow z = \frac{\mu-10}{\lambda-2}$ & $y = 4 - \frac{\mu-10}{\lambda-2}$

$\therefore$ For no solution $\lambda = 2$ and $\mu \neq 10$.

4. Official Ans. by NTA (1)

Sol. $D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$

$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$

If a = 3, b ≠ 13, no solution.

5. Official Ans. by NTA (2)

Sol. $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, \ \frac{-\pi}{4} \le x \le \frac{\pi}{4}$

Apply : $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\therefore x = \frac{\pi}{4}$

6. Official Ans. by NTA (6)

Sol. $\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix} \begin{pmatrix} R_1 \to R_1 - R_2 \\ \& \ R_2 \to R_2 - R_3 \end{pmatrix}$

$-2(\cos^2 x) + 2(2 + 2\cos 2x + \sin^2 x)$

$4 + 4\cos 2x - 2(\cos^2 x - \sin^2 x)$

$f(x) = 4 + \underbrace{2\cos 2x}_{\max = 1}$

$f(x)_{\max} = 4 + 2 = 6$

7. Official Ans. by NTA (2)

Sol. Case–I

$\begin{vmatrix} 1 + \cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1 + \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4\sin 3\theta \end{vmatrix} = 0$

$C_1 \to C_1 + C_2$

$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1 + \sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1 + 4\sin 3\theta \end{vmatrix} = 0$

$R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1 + 4\sin^3\theta \end{vmatrix} = 0$

or 4 sin 3θ = – 2

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

8. Official Ans. by NTA (5)

Sol. 2 × (i) – (ii) – (iii) gives :

$-(1 + \beta)z = 3 - \alpha$

For infinitely many solution

$\beta + 1 = 0 = 3 - \alpha \Rightarrow (\alpha, \beta) = (3, -1)$

Hence, $\alpha + \beta - \alpha\beta = 5$

9. Official Ans. by NTA (2)

Sol. $\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$

$R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$

$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$

10. Official Ans. by NTA (1)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28 + [\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = - [\lambda] - 9$

if $[\lambda] + 9 \ne 0$ then unique solution

if $[\lambda] + 9 = 0$ then $D_1 = D_2 = D_3 = 0$

so infinite solutions

Hence λ can be any red number.

11. Official Ans. by NTA (4)

Sol. Here $D=\begin{vmatrix}2&1&1\\1&-1&1\\1&1&a\end{vmatrix}=2(-a-1)-1(a-1)+1+1=1-3a$

$D_3=\begin{vmatrix}2&1&5\\1&-1&3\\1&1&b\end{vmatrix}=2(-b-3)-1(b-3)+5(1+1)=7-3b$

for $a=\frac{1}{3},\ b\neq\frac{7}{3}$, system has no solutions

12. Official Ans. by NTA (3)

Sol. $a_r = e^{\frac{i2\pi r}{9}}$, r = 1, 2, 3, … a_1, a_2, a_3, … are in G.P. $\begin{vmatrix}a_1&a_2&a_3\\a_n&a_5&a_6\\a_7&a_8&a_9\end{vmatrix}=\begin{vmatrix}a_1&a_2^2&a_1^3\\a_1^4&a_1^5&a_1^6\\a_1^7&a_1^8&a_1^9\end{vmatrix}=a_1.a_1^4.a_1^7\begin{vmatrix}1&a_1&a_1^2\\1&a_1&a_1^2\\1&a_1&a_1^2\end{vmatrix}=0$

Now $a_1a_9 - a_3a_7 = a_1^{10} - a_1^{10} = 0$

13. Official Ans. by NTA (2)

Sol. $\alpha+\beta+\gamma=2\pi$

$$\begin{vmatrix}1&\cos\gamma&\cos\beta\\\cos\gamma&1&\cos\alpha\\\cos\beta&\cos\alpha&1\end{vmatrix}$$

$= 1 + 2\cos\alpha.\cos\beta.\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha+\beta)+\cos(\alpha-\beta))\cos\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha-\beta)\cos\gamma$

$= \sin^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0$

14. Official Ans. by NTA (3)

Sol. $\Delta=\begin{vmatrix}-1&1&2\\3&-a&5\\2&-2&-a\end{vmatrix}$

$= -1(a^2+10) - 1(-3a-10) + 2(-6+2a)$

$= -a^2 - 10 + 3a + 10 - 12 + 4a$

$\Delta = -a^2 + 7a - 12$

$\Delta = -[a^2 - 7a + 12]$

$\Delta = -[(a-3)(a-4)]$

$\Delta_1=\begin{vmatrix}0&1&2\\1&-a&5\\7&-2&-a\end{vmatrix}$

$= 0 - 1(-a-35) + 2(-2+7a)$

$\Rightarrow a + 35 - 4 + 14a$

$15a + 31$

Now $\Delta_1 = 15a + 31$

For inconsistent $\Delta = 0\ \therefore\ a = 3, a = 4$

and for a = 3 and 4 $\quad \Delta_1 \neq 0$

$n(S_1) = 2$

For infinite solution : $\Delta = 0$

and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

Not possible

$\therefore\ n(S_2) = 0$

15. Official Ans. by NTA (4)

Sol. $\Delta=\begin{vmatrix}3&-2&-k\\2&-4&-2\\1&2&-1\end{vmatrix}=0$

$\Rightarrow\ 24 - 2(0) - k(8) = 0\ \Rightarrow k = 3$

$\Delta_x=\begin{vmatrix}10&-2&-3\\6&-4&-2\\5m&2&-1\end{vmatrix}$

$= 10(8) - 2(-10m+6) - 3(12+20m)$

$= 8(4-5m)$

$\Delta_y=\begin{vmatrix}3&10&-3\\2&6&-2\\1&5m&-1\end{vmatrix}$

$= 3(-6+10m) + 10(0) - 3(10m-6)$

$= 0$

$\Delta_z=\begin{vmatrix}3&-2&10\\2&-4&6\\1&2&5m\end{vmatrix}$

$= 3(-20m-12) - 2(6-10m) + 10(8)$

$= 40m - 32 = 8(5m-4)$

for inconsistent

$k = 3\ \&\ m \neq \frac{4}{5}$

16. Official Ans. by NTA (21)

Sol. We observe $5P_2 - P_1 = 3P_3$

So, $15 - K = -6$

$\Rightarrow K = 21$

17. Official Ans. by NTA (4)

Sol. $2x + 3y + 2z = 9$ ...(1)

$3x + 2y + 2z = 9$ ...(2)

$x - y + 4z = 8$ ...(3)

$(1) - (2) \Rightarrow -x + y = 0 \Rightarrow x - y = 0$

from (3) $4z = 8 \Rightarrow z = 2$

from (1) $2x + 3y = 5$

$\Rightarrow x = y = 1$

$\therefore$ system has unique solution

18. Official Ans. by NTA (2)

Sol. $P_1 : x + 2y - 3z = a$

$P_2 : 2x + 6y - 11z = b$

$P_3 : x - 2y + 7z = c$

Clearly

$5P_1 = 2P_2 + P_3$ if $5a = 2b + c$

$\Rightarrow$ All the planes sharing a line of intersection

$\Rightarrow$ infinite solutions

19. Official Ans. by NTA (4)

Sol. $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$

so, A is correct and B, C, E are incorrect.

If $k = 2$

$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = -48 \neq 0$

So no solution

D is correct.

20. Official Ans. by NTA (2)

Sol. $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ a^2 + 7a + 12 - a^2 - 3a - 2 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 + 3a + 2 & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a + 10 & 2 & 0 \end{vmatrix}$$

$= 4(a + 2) - 4a - 10$

$= 4a + 8 - 4a - 10 = -2$

21. Official Ans by NTA (3)

Sol. $C_1 + C_2 \to C_1$

$$\begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 - R_2 \to R_1$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

Open w.r.t. R_1

$-(2 \sin 2x - \cos 2x)$

$\cos 2x - 2 \sin 2x = f(x)$

$f(x)|_{max} = \sqrt{1+4} = \sqrt{5}$

22. Official Ans. by NTA (4)

Sol. $kx + y + z = 1$

$x + ky + z = k$

$x + y + zk = k^2$

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K(K^2 - 1) - 1(K - 1) + 1(1 - K)$$

$= K^3 - K - K + 1 + 1 - K$

$= K^3 - 3K + 2$

$= (K - 1)^2 (K + 2)$

For $K = 1$

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for $K = -2$, at least one out of $\Delta_1, \Delta_2, \Delta_3$ are not zero

Hence for no sol^n, $K = -2$

23. Official Ans. by NTA (4)

Sol. $$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

use $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow (2+4\sin 2x)\begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$$\Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$x = \frac{\pi}{2} + \frac{\pi}{12}, \pi - \frac{\pi}{12}$$

24. Official Ans. by NTA (2)

Sol. $$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow -(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\Sigma\alpha\beta) = 0$$

$$\Rightarrow -(-a)(a^2-2b-b) = 0$$

$$\Rightarrow a(a^2-3b) = 0$$

$$\Rightarrow a^2 = 3b \Rightarrow \frac{a^2}{b} = 3$$

25. Official Ans. by NTA (1)

Sol. For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$$

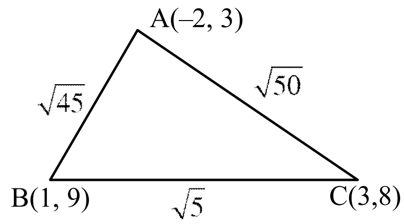
when $\mu = 6$, $12 - 6\lambda + 6\lambda = 12$

which is satisfied by all λ

STRAIGHT LINE

1. Official Ans. by NTA (9)

Sol.



$$\left(\sqrt{50}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{5}\right)^2$$

$\angle B = 90°$

Circum-center = $\left(\frac{1}{2}, \frac{11}{2}\right)$

Mid point of BC = $\left(2, \frac{17}{2}\right)$

Line : $\left(y - \frac{11}{2}\right) = 2\left(x - \frac{1}{2}\right) \Rightarrow y = 2x + \frac{9}{2}$

Passing though $\left(0, \frac{\alpha}{2}\right)$

$$\frac{\alpha}{2} = \frac{9}{2} \Rightarrow \alpha = 9$$

2. Official Ans. by NTA (3)

Sol. $$\frac{x^2-y^2}{1-(-5)} = \frac{xy}{-2}$$

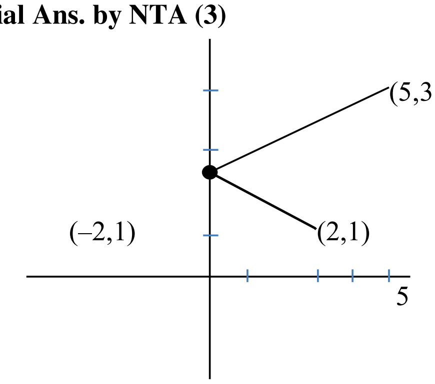
$$\frac{x^2-y^2}{6} = \frac{xy}{-2}$$

$$\Rightarrow x^2 - y^2 = -3xy$$

$$\Rightarrow x^2 + 3xy - y^2 = 0$$

3. Official Ans. by NTA (3)

Sol.



Equation of reflected Ray

$$y - 1 = \frac{2}{7}(x+2)$$

$$7y - 7 = 2x + 4$$

$$2x - 7y + 11 = 0$$

Let the equation of other directrix is

$2x - 7y + \lambda$

Distance of directrix from Focub

$$\frac{a}{e} - ae = \frac{8}{\sqrt{53}}$$

$$3a - \frac{a}{3} = \frac{8}{\sqrt{53}} \text{ or } a = \frac{3}{\sqrt{53}}$$

Distance from other focus $\frac{a}{e} + ae$

$$3a + \frac{a}{3} = \frac{10a}{3} = \frac{10}{3} \times \frac{3}{\sqrt{53}} = \frac{10}{\sqrt{53}}$$

Distance between two directrix $= \frac{2a}{e}$

$$= 2 \times 3 \times \frac{3}{\sqrt{53}} = \frac{18}{\sqrt{53}}$$

$$\left|\frac{\lambda - 11}{\sqrt{53}}\right| = \frac{18}{\sqrt{53}}$$

$\lambda - 11 = 18$ or -18

$\lambda = 29$ or -7

$2x - 7y - 7 = 0$ or $2x - 7y + 29 = 0$

4. Official Ans. by NTA (2)

Sol. Image of A(a,b) along y = x is B(b,a). Translating it 2 units it becomes C(b + 2, a).

Now, applying rotation theorem

$$-\frac{1}{2} + \frac{7}{\sqrt{2}} i = ((b+2) + ai)\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$\frac{-1}{\sqrt{2}} + \frac{7}{\sqrt{2}} i = \left(\frac{b+2}{\sqrt{2}} - \frac{a}{\sqrt{2}}\right) + i\left(\frac{b+2}{\sqrt{2}} + \frac{a}{\sqrt{2}}\right)$$

$\Rightarrow b - a + 2 = -1$ ....(i)

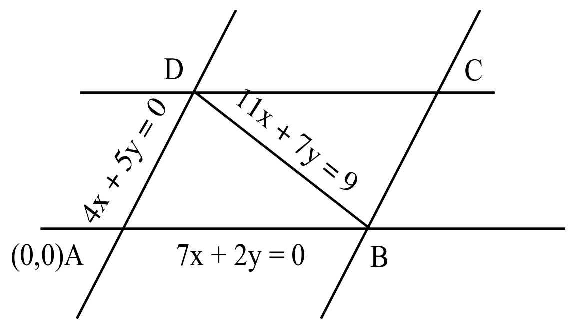
and $b + 2 + a = 7$ ....(ii)

$\Rightarrow a = 4; b = 1$

$\Rightarrow 2a + b = 9$

5. Official Ans. by NTA (2)

Sol. Both the lines pass through origin.



point D is equal of intersection of 4x + 5y = 0 &

11x + 7y = 9

So, coordinates of point $D = \left(\frac{5}{3}, -\frac{4}{3}\right)$

Also, point B is point of intersection of 7x + 2y = 0 & 11x + 7y = 9

So, coordinates of point $B = \left(-\frac{2}{3}, \frac{7}{3}\right)$

diagonals of parallelogram intersect at middle

let middle point of B,D

$$\Rightarrow \left(\frac{\frac{5}{3} - \frac{2}{3}}{2}, \frac{\frac{-4}{3} + \frac{7}{3}}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

equation of diagonal AC

$$\Rightarrow (y - 0) = \frac{\frac{1}{\alpha} - 0}{\frac{1}{\alpha} - 0}(\pi - 0)$$

y = x

diagonal AC passes through (2, 2).

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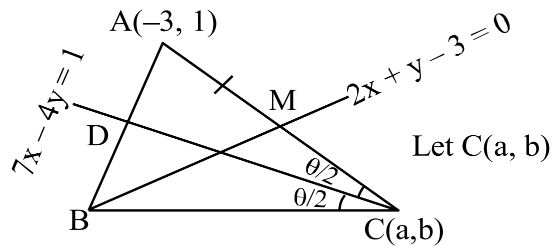
6. Official Ans. by NTA (3)

Sol.



$\therefore\ M\left(\frac{a-3}{2},\frac{b+1}{2}\right)$ lies on $2x + y - 3 = 0$

$\Rightarrow 2a + b = 11$ .... (i)

$\because$ C lies on $7x - 4y = 1$

$\Rightarrow 7a - 4b = 1$ ......(ii)

$\therefore$ by (i) and (ii) : $a = 3, b = 5$

$\Rightarrow C(3, 5)$

$\therefore\ m_{AC} = 2/3$

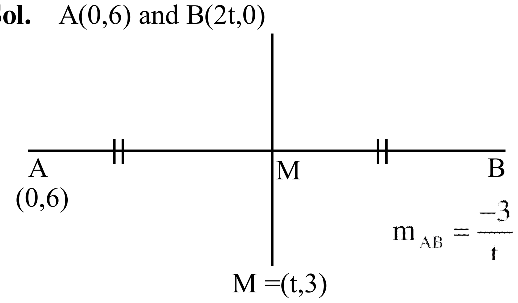
Also, $m_{CD} = 7/4$

$$\Rightarrow \tan\frac{\theta}{2}=\left|\frac{\frac{2}{3}-\frac{4}{4}}{1+\frac{14}{12}}\right| \Rightarrow \tan\frac{\theta}{2}=\frac{1}{2}$$

$$\Rightarrow \tan\theta=\frac{2.\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$$

7. Official Ans. by NTA (3)

Sol. A(0,6) and B(2t,0)



Perpendicular bisector of AB is

$$(y-3)=\frac{t}{3}(x-t)$$

$$\text{So, } C=\left(0,3-\frac{t^2}{3}\right)$$

Let P be (h,k)

$$h=\frac{t}{2};k=\left(3-\frac{t^2}{6}\right)$$

$$\Rightarrow k=3-\frac{4h^2}{6}\Rightarrow 2x^2+3y-9=0 \text{ option (3)}$$

8. Official Ans. by NTA (4)

Sol. $$\left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right|=1$$

$$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}=\pm 2$$

$\Rightarrow a\ (2b + 1 - b) - 0 + 1\ (b^2 - 0) = \pm 2$

$$\Rightarrow a=\frac{\pm 2-b^2}{b+1}$$

$$\therefore\ a=\frac{2-b^2}{b+1} \text{ and } a=\frac{-2-b^2}{b+1}$$

sum of possible values of 'a' is

$$=\frac{-2b^2}{a+1} \text{ Ans.}$$

9. Official Ans. by NTA (1)

Sol. First line is $\frac{x}{\sin\alpha}-\frac{y}{\cos\alpha}=\frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p=\left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$ …(i)

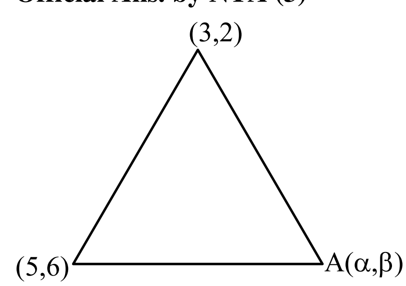
second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$ …(ii)

Hence $4p^2 + q^2 = k^2$ (From (i) & (ii))

10. Official Ans. by NTA (3)

Sol.



$$\left|\frac{1}{2}\begin{vmatrix} 5 & 6 & 1 \\ 3 & 2 & 1 \\ \alpha & \beta & 1 \end{vmatrix}\right|=12$$

$4\alpha - 2\beta = \pm 24 + 8$

$\Rightarrow 4\alpha - 2\beta = +24 + 8 \Rightarrow 2\alpha - \beta = 16$

$2x - y - 16 = 0$ …(1)

$\Rightarrow 4\alpha - 2\beta = -24 + 8 \Rightarrow 2\alpha - \beta = -8$

$2x - y + 8 = 0$ …(2)

perpendicular distance of (1) from (0, 0)

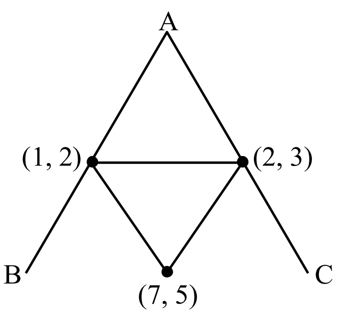
$$\left|\frac{0-0-16}{\sqrt{5}}\right| = \frac{16}{\sqrt{5}}$$

perpendicular distance of (2) from (0, 0) is

$$\left|\frac{0-0+8}{\sqrt{5}}\right| = \frac{8}{\sqrt{5}}$$

11. Official Ans. by NTA (6)

Sol. intersection point of give lines are (1, 2), (7, 5), (2,3)



$$\Delta = \frac{1}{2}\begin{vmatrix}1 & 2 & 1\\ 7 & 5 & 1\\ 2 & 3 & 1\end{vmatrix}$$

$$=\frac{1}{2}[1(5-3)-2(7-2)+1(21-10)]$$

$$=\frac{1}{2}[2-10+11]$$

$$\Delta DEF = \frac{1}{2}(3) = \frac{3}{2}$$

$$\Delta ABC = 4\,\Delta DEF = 4\left(\frac{3}{2}\right) = 6$$

12. Official Ans. by NTA (4)

Sol. $m = -\frac{1}{\sqrt{3}}, c = 2$

(1) $c = a\sqrt{1+m^2}$

$c = \sqrt{7}\,\frac{2}{\sqrt{3}}$ (incorrect)

(2) $c = \frac{a}{m} = \frac{\frac{1}{24\sqrt{3}}}{\frac{-1}{\sqrt{3}}} = -\frac{1}{24}$ (incorrect)

(3) $c = \sqrt{a^2m^2 - b^2}$

$c = \sqrt{\frac{9}{2}\cdot\frac{1}{3}-\frac{1}{2}} = 1$ (incorrect)

(4) $c = \sqrt{a^2m^2 + b^2}$

$c = \sqrt{9\cdot\frac{1}{3}+1} = 2$ (correct)

13. Official Ans. by NTA (56)

Sol. Let point is (h, k)

So, $\sqrt{(h-5)^2+k^2} = 3\sqrt{(h+5)^2+k^2}$

$8x^2 + 8y^2 + 100x + 200 = 0$

$x^2 + y^2 + \frac{25}{2}x + 25 = 0$

$r^2 = \frac{(25)^2}{4^2} - 25$

$4r^2 = \frac{25^2}{4} - 100$

$4r^2 = 156.25 - 100$

$4r^2 = 56.25$

After round of $4r^2 = 56$

14. Official Ans. by NTA (4)

Sol. Let the line be $y = mx + c$

x-intercept : $-\frac{c}{m}$

y-intercept : c

A.M of reciprocals of the intercepts :

$$\frac{-\frac{m}{c}+\frac{1}{c}}{2} = \frac{1}{4} \Rightarrow 2(1-m) = c$$

line : $y = mx + 2(1-m) = c$

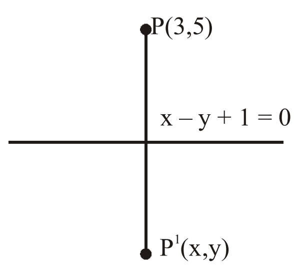
$\Rightarrow (y-2) - m(x-2) = 0$

$\Rightarrow$ line always passes through (2, 2)

Ans. 4

15. Official Ans. by NTA (4)

Sol.



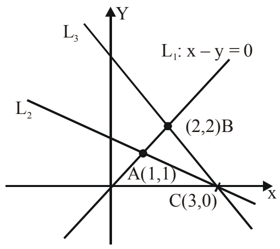
$$\frac{x-3}{1} = \frac{y-5}{-1} = -2\left(\frac{3-5+1}{1+1}\right)$$

So, $x = 4, y = 4$

Hence, $(x-2)^2 + (y-4)^2 = 4$

16. Official Ans. by NTA (3)

Sol.



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : x + y = 6$

on solving L_1 and L_2 :

y = L and x = 1

L_1 and L_3 :

x = 2

y = 2

L_2 and L_3 :

x + y = 3

2x + y = 6

x = 3

y = 0

$AC = \sqrt{4+1} = \sqrt{5}$

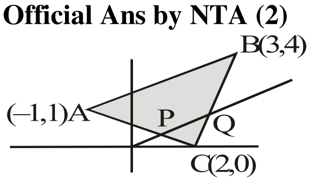
$BC = \sqrt{4+1} = \sqrt{5}$

$AB = \sqrt{1+1} = \sqrt{2}$

so its an isosceles triangle

17. Official Ans by NTA (2)

Sol.



$P \equiv (x_1, mx_1)$

$Q \equiv (x_2, mx_2)$

$$A_1 = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{13}{2}$$

$$A_2 = \frac{1}{2}\begin{vmatrix} x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$A_2 = \frac{1}{2}\left|2(mx_1 - mx_2)\right| = m\left|x_1 - x_2\right|$$

$$A_1 = 3A_2 \Rightarrow \frac{13}{2} = 3m\left|x_1 - x_2\right|$$

$$\Rightarrow \left|x_1 - x_2\right| = \frac{16}{6m}$$

AC : x + 3y = 2

BC : y = 4x – 8

$P : x + 3y = 2 \ \& \ y = mx \Rightarrow x_1 = \frac{2}{1+3m}$

$Q : y = 4x - 8 \ \& \ y = mx \Rightarrow x_2 = \frac{8}{4-m}$

$$\left|x_1 - x_2\right| = \left|\frac{2}{1+3m} - \frac{8}{4-m}\right|$$

$$= \left|\frac{-26m}{(1+3m)(4-m)}\right| = \frac{26m}{(3m+1)\,|m-4|}$$

$$= \frac{26m}{(3m+1)(4-m)}$$

$$\left|x_1 - x_2\right| = \frac{13}{6m}$$

$$\frac{26m}{(3m+1)(4-m)} = \frac{13}{6m}$$

$\Rightarrow 12m^2 = -(3m + 1)(m - 4)$

$\Rightarrow 12m^2 = -(3m^2 - 11m - 4)$

$\Rightarrow 15m^2 - 11m - 4 = 0$

$\Rightarrow 15m^2 - 15m + 4m - 4 = 0$

$\Rightarrow (15m + 4)(m - 1) = 0$

$\Rightarrow m = 1$

18. Official Ans. by NTA (144)

Sol. Since orthocentre and circumcentre both lies on y-axis

$\Rightarrow$ Centroid also lies on y-axis

$\Rightarrow \Sigma\cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

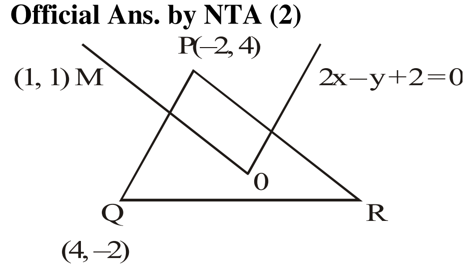
$$\therefore \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}$$

$$= \frac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\cos\gamma}$$

= 12

19. Official Ans. by NTA (2)

Sol.



Equation of perpendicular bisector of PR is

y = x

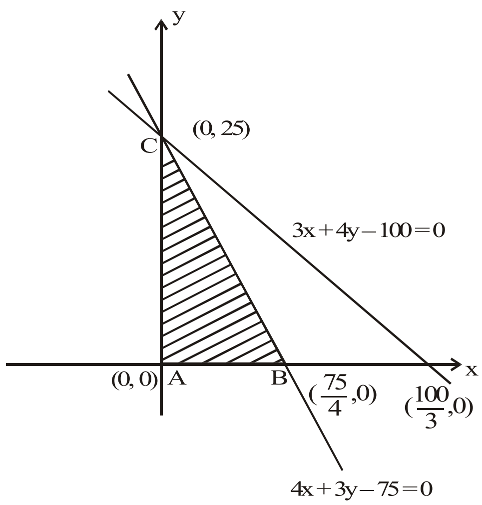
Solving with 2x – y + 2 = 0 will give

(–2, 2)

20. Official Ans. by NTA (904)

Allen Answer (904 or 904.01 or 904.02)

Sol.



$z = 6xy + y^2 = y(6x + y)$

$3x + 4y \leq 100$ .....(i)

$4x + 3y \leq 75$ ......(ii)

$x \geq 0$

$y \geq 0$

$x \leq \frac{75-3y}{4}$

$Z = y(6x + y)$

$Z \leq y\left(6.\left(\frac{75-3y}{4}\right)+y\right)$

$Z \leq \frac{1}{2}(225y - 7y^2) \leq \frac{(225)^2}{2\times4\times7}$

$= \frac{50625}{56}$

≈ 904.0178

≈ 904.02

It will be attained at $y = \frac{225}{14}$

21. Official Ans. by NTA (2)

Sol. $3x + 4y = 9$

$y = mx + 1$

$\Rightarrow 3x + 4mx + 4 = 9$

$\Rightarrow (3 + 4m)x = 5$

$\Rightarrow$ x will be an integer when

$3 + 4m = 5, -5, 1, -1$

$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$

so, number of integral values of m is 2

22. Official Ans. by NTA (1)

Sol. $y = mx + c$

$3 = m + c$

$\sqrt{2} = \left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right|$

$= 6m + \sqrt{2} = m - 3\sqrt{2}$

$= \sin = -4\sqrt{2} \rightarrow m = \frac{-4\sqrt{2}}{5}$

$= = 6m - \sqrt{2} = m - 3\sqrt{2}$

$= 7m - 2\sqrt{2} \rightarrow m = \frac{2\sqrt{2}}{7}$

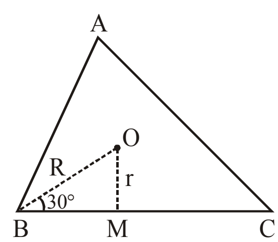
According to options take $m = \frac{-4\sqrt{2}}{5}$

So $y = \frac{-4\sqrt{2}x}{5} + \frac{3+4\sqrt{2}}{5}$

$4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

23. Official Ans. by NTA (1)

Sol.



$r = OM = \frac{3}{\sqrt{2}}$

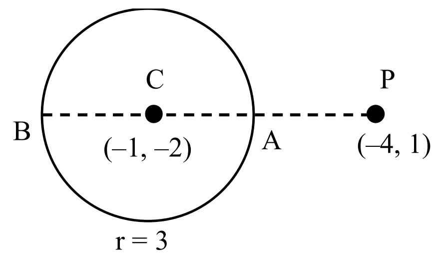
$\& \sin 30° = \frac{1}{2} = \frac{r}{R} \Rightarrow R = \frac{6}{\sqrt{2}}$

$\therefore r + R = \frac{9}{\sqrt{2}}$

CIRCLE

1. Official Ans. by NTA (3)

Sol.



Centre of smallest circle is A

Centre of largest circle is B

$r_2 = |CP - CA| = 3\sqrt{2} - 3$

$r_1 = CP + CB = 3\sqrt{2} + 3$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = \frac{(3\sqrt{2}+3)^2}{9} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$$

a = 3, b = 2

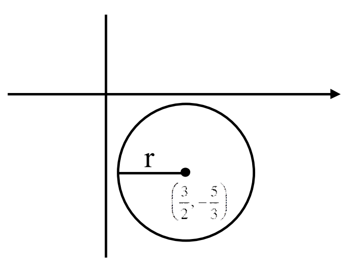
2. Official Ans. by NTA (4)

Sol. $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$

$$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$$

$$\text{Centre} \equiv (-g, -f) \equiv \left(\frac{3}{2}, \frac{-10}{6}\right)$$

$$\text{radius} = r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$$



Now,

$$\Rightarrow r < \frac{3}{2}$$

$$\Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$$

$\Rightarrow C > 100$ .....(1)

Now point of intersection of x – 2y = 4 and 2x – y = 5 is (2, –1), which lies inside the circle S.

$\therefore$ S(2, –1) < 0

$$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$$

$$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$$

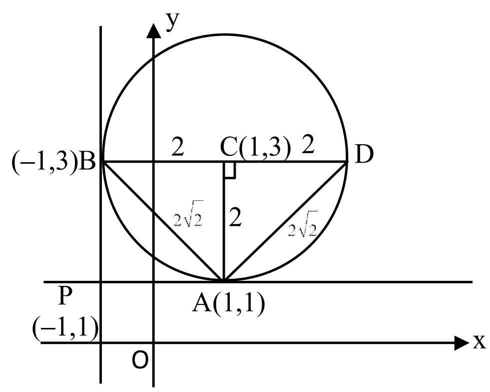
$\boxed{C < 156}$ .....(2)

From (1) & (2)

$\boxed{100 < C < 156}$ Ans.

3. Official Ans. by NTA (3)

Sol.

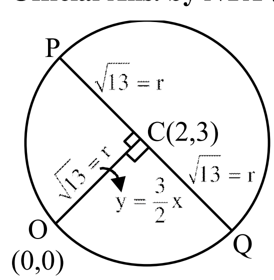


$$\Delta ABD = \frac{1}{2} \times 2 \times 4$$

= 4

4. Official Ans. by NTA (4)

Sol.



$$\tan\theta = -\frac{2}{3}$$

Using symmetric from of line

$$P,Q : \left(2 \pm \sqrt{13}\cos\theta, 3 \pm \sqrt{13}\sin\theta\right)$$

$$\left(2 \pm \sqrt{13}.\left(-\frac{3}{\sqrt{13}}\right), 3 \pm \sqrt{13}\left(\frac{2}{\sqrt{13}}\right)\right)$$

(–1, 5) & (5, 1)

5. Official Ans. by NTA (3)

Sol. $S_1 : x^2 + y^2 - x - y - \frac{1}{2} = 0$; $C_1\left(\frac{1}{2}, \frac{1}{2}\right)$

$$r_1 = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}} = 1$$

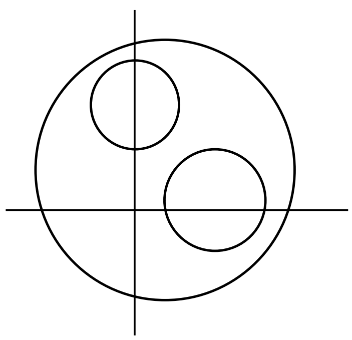
$S_2 : x^2 + y^2 - 4y + \frac{7}{4} = 0$; $C_2 : (0,2)$

$$r_2 = \sqrt{4 - \frac{7}{4}} = \frac{3}{2}$$

$S_3 : x^2 + y^2 - 4x - 2y + 5 - r^2 = 0$

$C_3 : (2,1)$

$$r_3 = \sqrt{4 + 1 - 5 + r^2} = |r|$$



$$C_1C_3 = \sqrt{\frac{5}{2}}$$

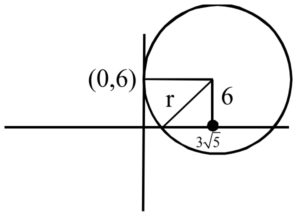
$$\sqrt{\frac{5}{2}} \le |r - 1| \Rightarrow \left.\begin{matrix} r \le 1 + \sqrt{\frac{5}{2}} \\ r \ge \frac{3}{2} + \sqrt{5} \end{matrix}\right\}$$

$$C_2C_3 = \sqrt{5} \le \left|r - \frac{3}{2}\right|$$

$$\left.\begin{matrix} r - \frac{3}{2} \ge \sqrt{5} \\ r - \frac{3}{2} \le -\sqrt{5} \end{matrix}\right\}$$

6. Official Ans. by NTA (2)

Sol.



$$r = \sqrt{6^2 + (3\sqrt{5})^2}$$

$$= \sqrt{36 + 45} = 9$$

7. Official Ans. by NTA (16)

Sol. Let P(x, y)

$x^2+y^2+x^2+(y-1)^2+(x-1)^2+y^2+(x-1)^2+(y-1)^2$;

$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$

$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$

$$d = 2\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{7}{2}}$$

$\Rightarrow d^2 = 16$

8. Official Ans. by NTA (1)

Sol. $(x - 2)^2 + (y - 1)^2 + \lambda(x - 2y) = 0$

$C : x^2 + y^2 + x(\lambda - 4) + y(-2 - 2\lambda) + 5 = 0$

$C_1 : x^2 + y^2 + 2y - 5 = 0$

$S_1 - S_2 = 0$ (Equation of PQ)

$(\lambda - 4)x - (2\lambda + 4)y + 10 = 0$ Passes through $(0,-1)$

$\Rightarrow \lambda = -7$

$C : x^2 + y^2 - 11x + 12y + 5 = 0$

$$= \frac{\sqrt{245}}{4}$$

Diometer = $7\sqrt{5}$

9. Official Ans. by NTA (61)

Sol. $r = \sqrt{\frac{p^2}{4} + \frac{(1-p)^2}{4} - 5} = \frac{\sqrt{2p^2 - 2p - 19}}{2}$

Since, $r \in (0, 5]$

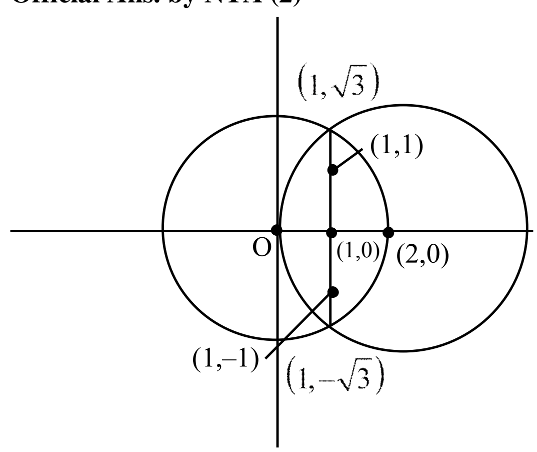
So, $0 < 2p^2 - 2p - 19 \le 100$

$\Rightarrow p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$ so,

number of integral values of p^2 is 61

10. Official Ans. by NTA (2)

Sol.



$(x-2)^2 + y^2 \le 4$

$x^2 + y^2 \le 4$

No. of points common in C_1 & C_2 is 5.

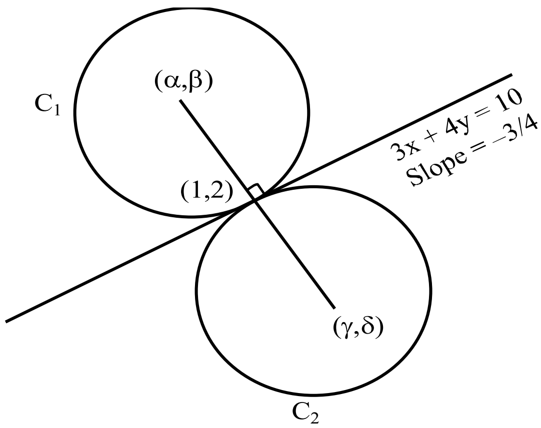
(0, 0) , (1, 0) , (2, 0) , (1, 1), (1, –1)

Similarly in C_2 & C_3 is 5.

No. of relations = $2^{5\times5} = 2^{25}$.

11. Official Ans. by NTA (40)

Sol. Slope of line joining centres of circles $= \frac{4}{3} = \tan\theta$



$\Rightarrow \cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$

Now using parametric form

$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm 5$

$\oplus$ $(x, y) = (1+5\cos\theta, 2+5\sin\theta)$

$(\alpha,\beta) = (4, 6)$

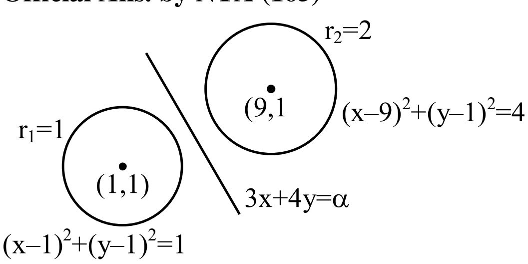
Θ $(x, y) = (\gamma,\delta) = (1-5\cos\theta, 2-5\sin\theta)$

$(\gamma, s) = (-2, -2)$

$\Rightarrow |(\alpha+\beta)(\gamma+\delta)| = |10x - 4| = 40$

12. Official Ans. by NTA (165)

Sol.



Both centres should lie on either side of the line as well as line can be tangent to circle.

$(3 + 4 - \alpha) . (27 + 4 - \alpha) < 0$

$(7 - \alpha) . (31 - \alpha) < 0 \Rightarrow \alpha \in (7, 31)$ …(1)

d_1 = distance of (1, 1) from line

d_2 = distance of (9, 1) from line

$d_1 \ge r_1 \Rightarrow \frac{|7-\alpha|}{5}$ $1 \Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty)$ …(2)

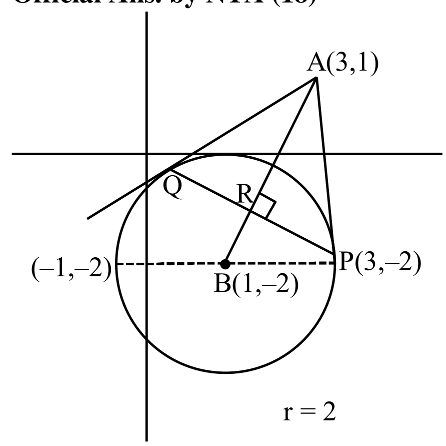
$d_2 \ge r_2 \Rightarrow \frac{|31-\alpha|}{5} \ge 2 \Rightarrow \alpha \in (-\infty, 21] \cup [41, \infty)$ …(3)

$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12, 21]$

Sum of integers = 165

13. Official Ans. by NTA (18)

Sol.



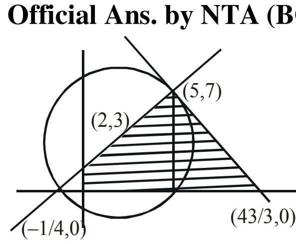
$\tan\theta = \frac{3}{2}$

$$\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$$

$$8\left(\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ}\right) = 18$$

14. Official Ans. by NTA (BONUS)

Sol.



Equation of normal

$4x - 3y + 1 = 0$

and equation of tangents

$3x + 4y - 43 = 0$

$$\text{Area of triangle} = \frac{1}{2}\left(\frac{43}{3} + \frac{1}{4}\right) \times (7)$$

$$= \frac{1}{2}\left(\frac{172+3}{12}\right) \times 7$$

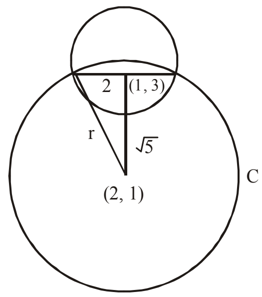
$$A = \frac{1225}{24}$$

$24A = 1225$

* as positive x-axis is given in the question so question should be bonus.

15. Official Ans. by NTA (3)

Sol.



$x^2 + y^2 + 2x - 6y + 6 = 0$

center (1, 3)

radius = 2

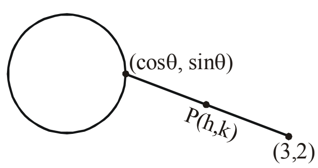
distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\because \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

16. Official Ans. by NTA (2)

Sol. $h = \dfrac{\cos\theta + 3}{2}$



$k = \dfrac{\sin\theta + 2}{2}$

$\Rightarrow \left(h - \dfrac{3}{2}\right)^2 + (k-1)^2 = \dfrac{1}{4}$

$\Rightarrow r = \dfrac{1}{2}$

17. Official Ans. by NTA (1)

Sol. P be a point on $(x - 1)^2 + (y - 1)^2 = 1$

so $P(1 + \cos\theta, 1 + \sin\theta)$

A(1, 4) B(1, –5)

$(PA)^2 + (PB)^2$

$= (\cos\theta)^2 + (\sin\theta - 3)^2 + (\cos\theta)^2 + (\sin\theta + 6)^2$

$= 47 + 6\sin\theta$

is maximum if $\sin\theta = 1$

$\Rightarrow \sin\theta = 1, \cos\theta = 0$

P(1, 1) A(1, 4) B(1, –5)

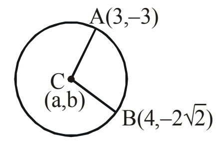
P, A, B are collinear points.

18. Official Ans. by NTA (9)

Sol. All normals of circle passes through centre

Radius = CA = CB

$CA^2 = CB^2$



$(a - 3)^2 + (b + 3)^2$

$= (a - 4)^2 + \left(b - 2\sqrt{2}\right)^2$

$a + \left(3 - 2\sqrt{2}\right)b = 3$

$a - 2\sqrt{2}\,b + 3b = 3$ ...(1)

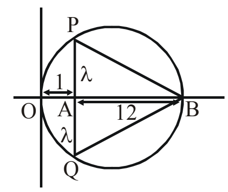
given that $a - 2\sqrt{2}\,b = 3$ ...(2)

from (1) & (2) $\Rightarrow a = 3, b = 0$

$a^2 + b^2 + ab = 9$

19. Official Ans. by NTA (2)

Sol. $PA = AQ = \lambda$



$OA \cdot AB$

$= AP \cdot AQ$

$\Rightarrow 1.12 = \lambda \cdot \lambda$

$\Rightarrow \lambda = 2\sqrt{3}$

Area $\Delta PQB = \dfrac{1}{2} \times 2\lambda \times AB$

$\Delta = \dfrac{1}{2}.4\sqrt{3} \times 12$

$= 24\sqrt{3}$

20. Official Ans by NTA (3)

Sol. $x^2 + y^2 + ax + 2ay + c = 0$

$2\sqrt{g^2 - c} = 2\sqrt{\dfrac{a^2}{4} - c} = 2\sqrt{2}$

$\Rightarrow \dfrac{a^2}{4} - c = 2$ ...(1)

$2\sqrt{f^2 - c} = 2\sqrt{a^2 - c} = 2\sqrt{5}$

$\Rightarrow a^2 - c = 5$ ...(2)

(1) & (2)

$\dfrac{3a^2}{4} = 3 \Rightarrow a = -2 \ (a < 0)$

$\therefore c = -1$

Circle $\Rightarrow x^2 + y^2 - 2x - 4y - 1 = 0$

$\Rightarrow (x - 1)^2 + (y - 2)^2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\dfrac{1}{2}$

$m_{tangent} = 2$

Equation of tangent

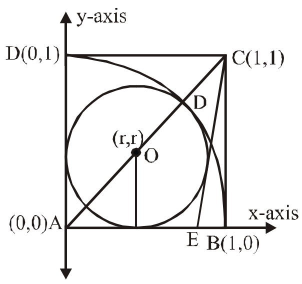
$\Rightarrow (y - 2) = 2(x - 1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Perpendicular distance from (0, 0) $= \left|\dfrac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

21. Official Ans. by NTA (1)

Sol.



Here AO + OD = 1 or $(\sqrt{2}+1)r = 1$

$\Rightarrow \quad r = \sqrt{2-1}$

equation of circle $(x-r)^2 + (y-r)^2 = r^2$

Equation of CE

$y - 1 = m(x-1)$

$mx - y + 1 - M = 0$

It is tangent to circle

$\therefore \quad \left|\frac{mr - r + 1 - m}{\sqrt{m^2+1}}\right| = r$

$\left|\frac{(m-1)r + 1 - m}{\sqrt{m^2+1}}\right| = r$

$\frac{(m-1)^2(r-1)^2}{m^2+1} = r^2$

Put $r = \sqrt{2}-1$

On solving $m = 2-\sqrt{3},\ 2+\sqrt{3}$

Taking greater slope of CE as

$2+\sqrt{3}$

$y - 1 = (2+\sqrt{3})(x-1)$

Put $\quad y = 0$

$-1 = (2+\sqrt{3})(x-1)$

$\frac{-1}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = x - 1$

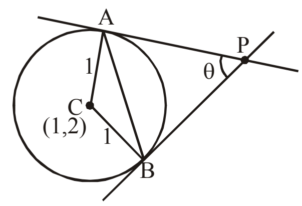
$x - 1 = \sqrt{3} - 1$

$EB = 1 - x = 1 - (\sqrt{3}-1)$

$EB = 2 - \sqrt{3}$

22. Official Ans. by NTA (2)

Sol.



$\tan\theta = \frac{12}{5}$

$PA = \cot\frac{\theta}{2}$

$\therefore$ area of $\Delta PAB = \frac{1}{2}(PA)^2\sin\theta = \frac{1}{2}\cot^2\frac{\theta}{2}\sin\theta$

$= \frac{1}{2}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta$

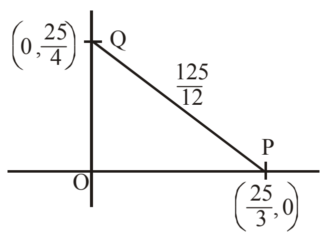
$= \frac{1}{2}\left(\frac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)\left(\frac{12}{13}\right) = \frac{1}{2}\frac{18}{18}\times\frac{2}{13} = \frac{27}{26}$

area of $\Delta CAB = \frac{1}{2}\sin\theta = \frac{1}{2}\left(\frac{12}{13}\right) = \frac{6}{13}$

$\therefore \frac{\text{area of }\Delta PAB}{\text{area of }\Delta CAB} = \frac{9}{4}$ **Option (2)**

23. Official Ans. by NTA (3)

Sol. Tangent to circle $3x + 4y = 25$



OP + OQ + OR = 25

Incentre $= \left(\frac{\frac{25}{4}\times\frac{25}{3}}{25}, \frac{\frac{25}{4}\times\frac{25}{3}}{25}\right)$

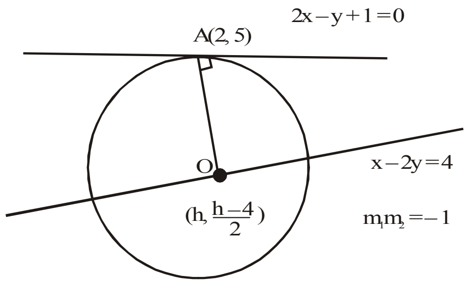
$= \left(\frac{25}{12}, \frac{25}{12}\right)$

$\therefore r^2 = 2\left(\frac{25}{12}\right)^2 = 2\times\frac{625}{144} = \frac{625}{72}$

Option (3)

24. Official Ans. by NTA (1)

Sol.



$$\left(\frac{h-\frac{(h-4)}{2}}{2-h}\right)(2)=-1$$

h = 8

center (8, 2)

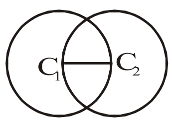
$\text{Radius} = \sqrt{(8-2)^2+(2-5)^2} = 3\sqrt{5}$)

25. Official Ans. by NTA (2)

Sol. $r_1 = 3$, c_1 (5, 5)

$r_2 = 3$, c_2 (8, 5)

$C_1C_2 = 3$, $r_1 = 3$, $r_2 = 3$

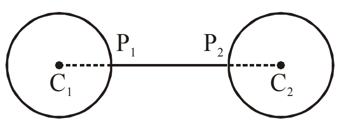


26. Official Ans. by NTA (1)

Sol. Given $C_1(5, 5)$, $r_1 = 3$ and C_2 (12, 5), $r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{min} = 7 - 6 = 1$



27. Official Ans. by NTA (3)

Sol. $x^2 + y^2 - 10x - 10y + 41 = 0$

$A(5,5)$, $R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

$B(11,5)$, $R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

⇒ circles have only one meeting point.

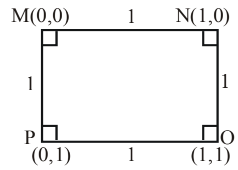
28. Official Ans. by NTA (2)

Sol. $M : x^2 + y^2 = 1$ (0,0)

$N : x^2 + y^2 - 2x = 0$ (1,0)

$O : x^2 + y^2 - 2x - 2y + 1 = 0$ (1,1)

$P : x^2 + y^2 - 2y = 0$ (0,1)

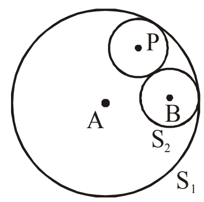


29. Official Ans. by NTA (3)

Sol. $S_1 : x^2 + y^2 = 9$ — $r_1 = 3$, A(0, 0)

$S_2 : (x - 2)^2 + y^2 = 1$ — $r_2 = 1$, B(2, 0)

$\because\ c_1c_2 = r_1 - r_2$



∴ given circle are touching internally

Let a veriable circle with centre P and radius r

$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2$

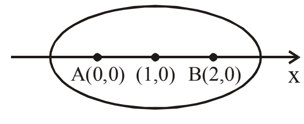
$\Rightarrow PA + PB = 4$ (> AB)

⇒ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is 2a = 4,

$e = \frac{1}{2}$

⇒ centre is at (1, 0) and $b^2 = a^2(1 - e^2) = 3$

if x-ellipse



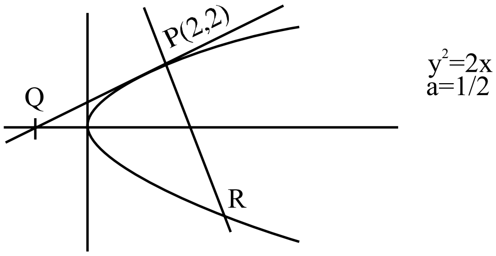
$$\Rightarrow E : \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$

PARABOLA

1. Official Ans. by NTA (1)

Sol.



Tangent at P : $y(2) = 2\,(1/2)\,(x + 2)$

$\Rightarrow 2y = x + 2$

$\therefore\ Q = (-2, 0)$

Normal at P : $y - 2 = -\dfrac{(2)}{2.\frac{1}{2}}(x - 2)$

$\Rightarrow y - 2 = -2(x - 2)$

$\Rightarrow y = 6 - 2x$

$\therefore$ Solving with $y^2 = 2x \Rightarrow R\left(\dfrac{9}{2} - 3\right)$

$$\therefore \text{Ar}(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 1 & 1 \\ \frac{9}{2} & 3- & 1 \end{vmatrix}$$

$= \dfrac{25}{2}$ sq.units

2. Official Ans. by NTA (34)

Sol. $y^2 = -64x$

focus : $(-16, 0)$

$y = mx + c$ is focal chord

$\Rightarrow c = 16m$ ....(1)

$y = mx + c$ is tangent to $(x + 10)^2 + y^2 = 4$

$\Rightarrow y = m(x+10) \pm 2\sqrt{1+m^2}$

$\Rightarrow c = 10m \pm 2\sqrt{1+m^2}$

$\Rightarrow 16m = 10m \pm 2\sqrt{1+m^2}$

$\Rightarrow 6m = 2\sqrt{1+m^2}$ $\quad (m > 0)$

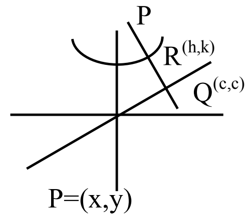
$\Rightarrow 9m^2 = 1 + m^2$

$\Rightarrow m = \dfrac{1}{2\sqrt{2}}$ & $c = \dfrac{8}{\sqrt{2}}$

$4\sqrt{2}(m+c) = 4\sqrt{2}\left(\dfrac{17}{2\sqrt{2}}\right) = \boxed{34}$

3. Official Ans. by NTA (2)

Sol.



$\dfrac{K - C}{h - C} = -1$

$C = \dfrac{h + K}{2}$ $\quad$ P(x,y)

$R = \left(\dfrac{x + C}{2}, \dfrac{y + C}{2}\right)$

$R = \left(\dfrac{x}{2} + \dfrac{h}{4} + \dfrac{K}{4}, \dfrac{y}{2} + \dfrac{h}{4} + \dfrac{k}{4}\right)$

$h = \dfrac{x}{2} + \dfrac{h}{4} + \dfrac{K}{4}$

$K = \dfrac{y}{2} + \dfrac{h}{4} + \dfrac{K}{4}$

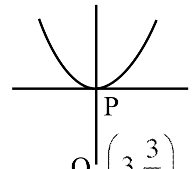
$\Rightarrow x = \dfrac{3h}{2} - \dfrac{K}{2}, y = \dfrac{3K}{2} - \dfrac{h}{2}$

$Y = 4x^2 + 1$

$\left(\dfrac{3k - h}{2}\right) = 4\left(\dfrac{3h - k}{2}\right)^2 + 1$

4. Official Ans. by NTA (9)

Sol.



$\left(3, \dfrac{3}{2}\right)$

$P \equiv \left(\dfrac{3}{2}t^2, 3t\right)$

Normal at point P

$tx + y = 3t + \dfrac{3}{2}t^3$

Passes through $\left(3, \dfrac{3}{2}\right)$

$\Rightarrow 3t + \dfrac{3}{2} = 3t + \dfrac{3}{2}t^3$

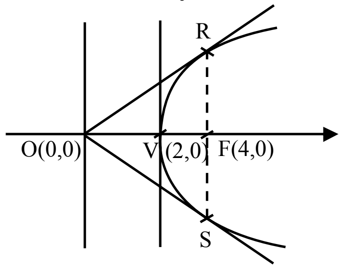
$P \equiv \left(\dfrac{3}{2}, 3\right) = (\alpha, \beta)$

$\Rightarrow t^3 = 1 \Rightarrow t = 1$

$2(\alpha + \beta) = 2\left(\dfrac{3}{2} + 3\right) = 9$

5. Official Ans. by NTA (2)

Sol.



Clearly RS is latus-rectum

$\because$ VF = 2 = a

$\therefore$ RS = 4a = 8

Now OF = 2a = 4

$\Rightarrow$ Area of triangle ORS = 16

6. Official Ans. by NTA (4)

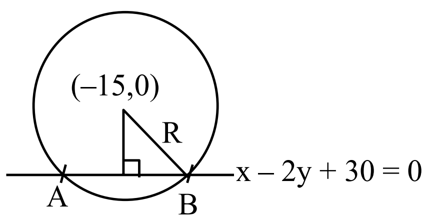
Sol. Equation of tangent to $y^2 = 30x$

$$y = mx + \frac{30}{4m}$$

Pass thru (–30, 0) : $a = -30m + \frac{30}{4m} \Rightarrow m^2 = 1/4$

$$\Rightarrow m = \frac{1}{2} \text{ or } m = -\frac{1}{2}$$

At $m = \frac{1}{2}$: $y = \frac{x}{2} + 15 \Rightarrow x - 2y + 30 = 0$



$$P = \frac{15}{\sqrt{5}}$$

$$\ell_{AB} = 2\sqrt{R^2 - P^2} = 2\sqrt{\frac{225}{4} - \frac{225}{5}}$$

$$\Rightarrow \ell_{AB} = 30.\sqrt{\frac{1}{20}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

7. Official Ans. by NTA (3)

Sol. $T = S_1$

$xh - yk = h^2 - k^2$

$$y = \frac{xh}{k} - \frac{(h^2 - k^2)}{k}$$

this touches $y^2 = 8x$ then $c = \frac{a}{m}$

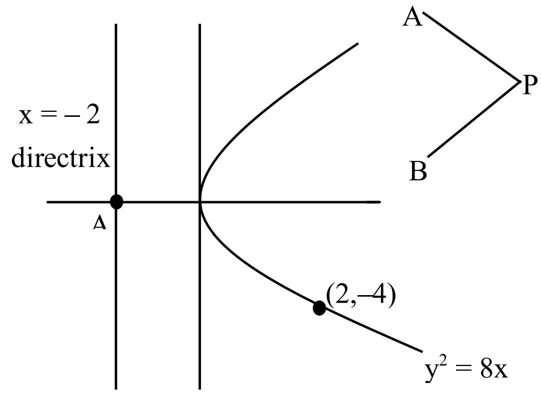
$$\left(\frac{k^2 - h^2}{k}\right) = \frac{2k}{h}$$

$2y^2 = x(y^2 - x^2)$

$y^2(x - 2) = x^3$

8. Official Ans. by NTA (1)

Sol.



Equation of tangent at (2,–4) (T = 0)

$-4y = 4(x + 2)$

$x + y + 2 = 0$ …(1)

equation of normal

$x - y + \lambda = 0$

$\downarrow$(2,–4)

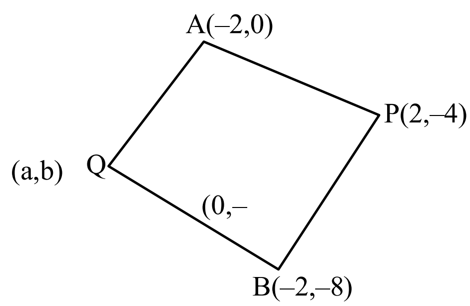
$\lambda = -6$

thus $x - y = 6$ …(2) equation of normal

POI of (1) & x = – 2 is A(–2,0)

POI of (2) & x = – 2 is A(–2,8)

Given AQBP is a sq.



$$\Rightarrow m_{AQ}.m_{AP} = -1$$

$$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a + 2 = b \text{ …(1)}$$

Also PQ must be parallel to x-axis thus

$\Rightarrow b = -4$

$\therefore$ a = –6

Thus 2a + b = – 16

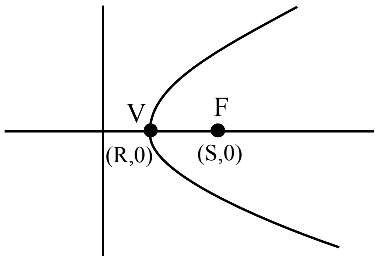
9. Official Ans. by NTA (2)

Sol. Locus is directrix of parabola

$x - 3 + 4 = 0 \Rightarrow x + 1 = 0.$

10. Official Ans. by NTA (3)

Sol.



$V \rightarrow$ Vertex

$F \rightarrow$ focus

VF = S – R

So latus rectum = 4(S – R)

11. Official Ans. by NTA (2)

Sol. tangent of $y^2 = 8x$ is $y = mx + \frac{2}{m}$

$P(2, -4) \Rightarrow -4 = 2m + \frac{2}{m}$

$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$

$\therefore$ tangent is $y = -x - 2$

$\Rightarrow x + y + 2 = 0$ …(1)

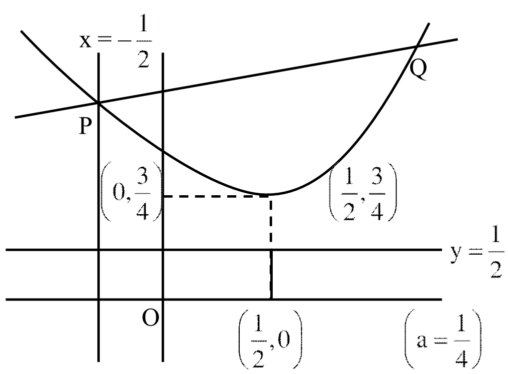
(1) is also tangent to $x^2 + y^2 = a$

So $\frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$

$\Rightarrow a = 2$

12. Official Ans. by NTA (2)

Sol.



$\left(y - \frac{3}{4}\right) = \left(x - \frac{1}{2}\right)^2$ ... (1)

For $x = -\frac{1}{2}$

$y - \frac{3}{4} = 1 \Rightarrow y = \frac{7}{4} \Rightarrow P\left(-\frac{1}{2}, \frac{7}{4}\right)$

Now $y' = 2\left(x - \frac{1}{2}\right)$ At $x = -\frac{1}{2}$

$\Rightarrow m_T = -2, m_N = \frac{1}{2}$

Equation of Normal is

$y - \frac{7}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right)$

$y = \frac{x}{2} + 2$

Now put y in equation (1)

$\frac{x}{2} + 2 - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$

$\Rightarrow x = 2 \ \& -\frac{1}{2}$

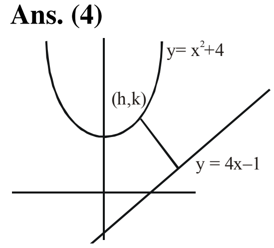
$\Rightarrow Q(2, 3)$

Now $(PQ)^2 = \frac{125}{16}$

Option (2)

13. Official Ans. by NTA (4)

Sol. Ans. (4)



P : $y = x^2 + 4$

$k = h^2 + 4$

L : $y = 4x - 1$

$y - 4x + 1 = 0$

$d = AB = \left|\frac{k - 4h + 1}{\sqrt{5}}\right| = \left|\frac{h^2 - 4 - 4h + 1}{\sqrt{5}}\right|$

$\frac{d(d)}{dh} = \frac{2h - 4}{\sqrt{5}} = 0$

$h = 2$

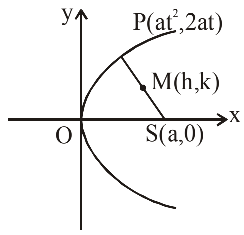
$\frac{d^2(d)}{dh^2} = \frac{2}{\sqrt{5}} > 0$

$\therefore$ $k = 4 + 4 = 8$

$\therefore$ Point (2, 8)

14. Official Ans. by NTA (3)

Sol.



$$h=\frac{at^2+a}{2},\ k=\frac{2at+0}{2}$$

$$\Rightarrow\ t^2=\frac{2h-a}{a}\ \text{and}\ t=\frac{k}{a}$$

$$\Rightarrow\ \frac{k^2}{a^2}=\frac{2h-a}{a}$$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$$\Rightarrow\ y^2=2a\left(x-\frac{a}{2}\right)$$

Its directrix is $x-\frac{a}{2}=-\frac{a}{2}\ \Rightarrow\ x=0$

15. Official Ans. by NTA (3)

Sol. Slope of tangent = $m_T = m$

So, $m(-2) = -1 \Rightarrow m=\frac{1}{2}$

Equation : $y = mx + \frac{a}{m}$

$$\Rightarrow\ y=\frac{1}{2}x+\frac{3}{2\times\frac{1}{2}}\left(a=\frac{6}{4}=\frac{3}{2}\right)$$

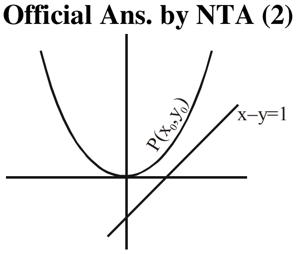
$$\Rightarrow\ y=\frac{x}{2}+3$$

$\Rightarrow 2y = x + 6$

Point (5, 4) will not lie on it

16. Official Ans. by NTA (2)

Sol.



Shortest distance between curves is always along common normal.

$$\left.\frac{dy}{dx}\right|_P=\text{slope of line}=1$$

$$\Rightarrow x_0=1 \qquad \therefore\ y_0=\frac{1}{2}$$

$$\Rightarrow P\left(1,\frac{1}{2}\right)$$

$$\therefore\ \text{Shortest distance}=\left|\frac{1-\frac{1}{2}-1}{\sqrt{1^2+1^2}}\right|=\frac{1}{2\sqrt{2}}$$

option (2)

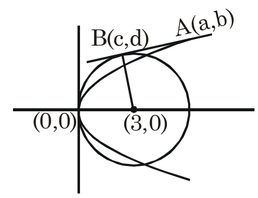
17. Official Ans. by NTA (9)

Sol. Let coordinate of point $A(t^2, 2t)$ $(\because a = 1)$

equation of tangent at point A

$yt = x + t^2$

$x - ty + t^2 = 0$



centre of circle (3, 0)

Now PD = radius

$$\left|\frac{3-0+t^2}{\sqrt{1+t^2}}\right|=3$$

$$(3 + t^2)^2 = 9(1 + t^2)$$

$$9 + t^4 + 6t^2 = 9 + 9t^2$$

$$t=0,-\sqrt{3},\sqrt{3}$$

So point $A\left(3,2\sqrt{3}\right)$

$\Rightarrow a = 3,\ b=2\sqrt{3}$

(Since it lies in first quadrant)

For point B which is foot of perpendicular from centre (3, 0) to the tangent $x-\sqrt{3}y+3=0$

$$\frac{c-3}{1}=\frac{d-0}{-\sqrt{3}}=\frac{-(3-0+3)}{4}$$

$$\Rightarrow c=\frac{3}{2} \qquad d=\frac{3\sqrt{3}}{2}$$

$$\Rightarrow 2\left(\frac{3}{2}+3\right)=9$$

18. Official Ans by NTA (1)

Sol. Given $y^2 = 4x$

Mirror image on $y = x \Rightarrow C : x^2 = 4y$

$$2x = 4\cdot\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

$$\left.\frac{dy}{dx}\right|_{P(2,1)} = \frac{2}{2} = 1$$

Equation of tangent at (2, 1)

$\Rightarrow y - 1 = 1(x - 2)$

$\Rightarrow x - y = 1$

19. Official Ans. by NTA (4)

Sol. For standard parabola

For more than 3 normals (on axis)

$x > \frac{L}{2}$ (where L is length of L.R.)

For $y^2 = 2x$

L.R. = 2

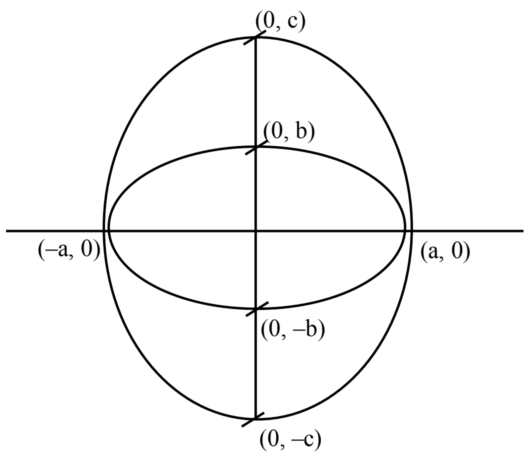
for (a, 0)

$$a > \frac{L.R.}{2} \Rightarrow a > 1$$

ELLIPSE

1. Official Ans. by NTA (1)

Sol.



$$e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{a^2}{c^2}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{a^2}{c^2}$$

$$\Rightarrow \mathbf{c^2} = \frac{a^4}{b^2} \Rightarrow c = \frac{a^2}{b}$$

Also b = ce

$$\Rightarrow c = \frac{b}{e}$$

$$\frac{b}{e} = \frac{a^2}{b}$$

$$\Rightarrow e = \frac{b^2}{a^2} = 1 - e^2$$

$$\Rightarrow e^2 + e - 1 = 0$$

$$\Rightarrow e = \frac{-1+\sqrt{5}}{2}$$

2. Official Ans. by NTA (3)

Sol. $\frac{3}{2a^2} + \frac{1}{b^2} = 1$ and $1 - \frac{b^2}{a^2} = \frac{1}{3}$

$\Rightarrow a^2 = 3\ b^2 = 3$

$$\Rightarrow \frac{x^2}{3} + \frac{y^2}{2} = 1 \quad .....(i)$$

Its focus is (1,0)

Now, eqn of circle is

$$(x-1)^2 + y^2 = \frac{4}{3} \quad .....(ii)$$

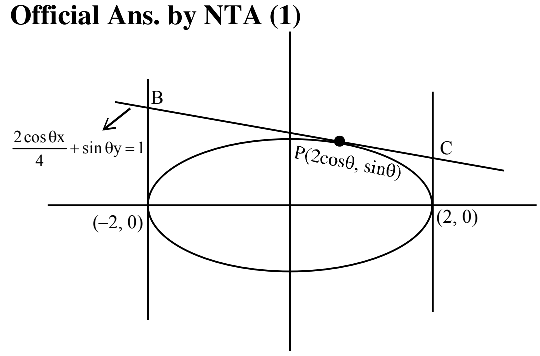
Solving (i) and (ii) we get

$$y = \pm\frac{2}{\sqrt{3}}, x = 1$$

$$\Rightarrow PQ^2 = \left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16}{3}$$

3. Official Ans. by NTA (1)

Sol.



$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Equation of tangent is $(\cos\theta)x + 2\sin\theta y = 2$

$B\left(-2, \frac{1+\cos\theta}{\sin\theta}\right)$, $C\left(2, \frac{1-\cos\theta}{\sin\theta}\right)$

$B\left(-2, \cot\frac{\theta}{2}\right)$ $C\left(2, \tan\frac{\theta}{2}\right)$

Equation of circle is

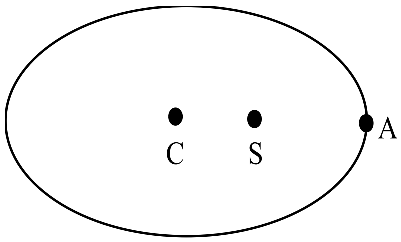
$$(x+2)(x-2) + \left(y - \cot\frac{\theta}{2}\right)\left(y - \tan\frac{\theta}{2}\right) = 0$$

$$x^2 - 4 + y^2 - \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)y + 1 = 0$$

so, $\left(\sqrt{3}, 0\right)$ satisfying option (1)

4. Official Ans. by NTA (3)

Sol. Given C(3,–4), S(4,–4)



and A(5,–4)

Hence, a = 2 & ae = 1

$\Rightarrow e = \frac{1}{2}$

$\Rightarrow b^2 = 3.$

So, $E : \frac{(x-3)^2}{4} + \frac{(y+4)^2}{3} = 1$

Intersecting with given tangent.

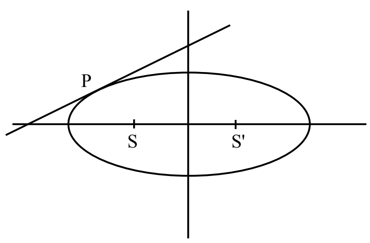
$\frac{x^2 - 6x + 9}{4} + \frac{m^2x^2}{3} = 1$

Now, D = 0 (as it is tangent)

So, $5m^2 = 3$.

5. Official Ans. by NTA (1)

Sol.



Equation of tangent : y = 2x + 6

at P

$\therefore$ P(–8/3, 2/3)

$e = \frac{1}{\sqrt{2}}$

S & S' = (–2, 0) & (2, 0)

Area of Δ SPS' = $\frac{1}{2} \times 4 \times \frac{2}{3}$

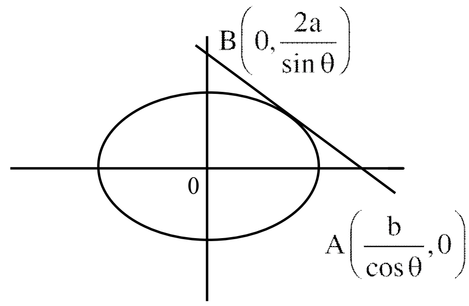
$A = \frac{4}{3}$

$\therefore (5 - e^2)A = (5 - \frac{1}{2})\frac{4}{3} = 6$

6. Official Ans. by NTA (2)

Sol. Tangent

$\frac{x\cos\theta}{b} + \frac{y\sin\theta}{2a} = 1$



So, $\text{area}(\Delta OAB) = \frac{1}{2} \times \frac{b}{\cos\theta} \times \frac{2a}{\sin\theta}$

$= \frac{2ab}{\sin 2\theta} \geq 2ab$

$\Rightarrow k = 2$

7. Official Ans. by NTA (2)

Sol. $12x\cos\theta + 5y\sin\theta = 60$

$\frac{x\cos\theta}{5} + \frac{y\sin\theta}{12} = 1$

is tangent to $\frac{x^2}{25} + \frac{y^2}{144} = 1$

$144x^2 + 25y^2 = 3600$

8. Official Ans. by NTA (3)

Sol. General point on $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is A(2cosθ, 3sinθ)

given B(–3, –5)

midpoint $C\left(\frac{2\cos\theta - 3}{2}, \frac{3\sin\theta - 5}{2}\right)$

$h = \frac{2\cos\theta - 3}{2}; \ k = \frac{3\sin\theta - 5}{2}$

$\Rightarrow \left(\frac{2h+3}{2}\right)^2 + \left(\frac{2k+5}{3}\right)^2 = 1$

$\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0$

9. Official Ans. by NTA (2)

Sol. The point of intersection of the curves $\frac{x^2}{9}+\frac{y^2}{1}=1$ and $x^2 + y^2 = 3$ in the first quadrant is $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$

Now slope of tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$ is

$m_1 = -\frac{1}{3\sqrt{3}}$

And slope of tangent to the circle at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$ is

$m_2 = -\sqrt{3}$

So, if angle between both curves is θ then

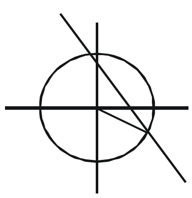
$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right| = \left|\frac{-\frac{1}{3\sqrt{3}}+\sqrt{3}}{1+\left(-\frac{1}{3\sqrt{3}}\left(-\sqrt{3}\right)\right)}\right|$$

$$=\frac{2}{\sqrt{3}}$$

Option (2)

10. Official Ans. by NTA (4)

Sol.



Homogenising

$$x^2+2y^2-2\left(x+y\right)^2=0$$

$$\Rightarrow -x^2-4xy=0 \Rightarrow x^2+4xy=0$$

Lines are x = 0 and $y=-\frac{x}{4}$

$\therefore$ Angle between lines $=\frac{\pi}{2}+\tan^{-1}\frac{1}{4}$

option (4)

11. Official Ans. by NTA (3)

Sol. Given curves are $\frac{x^2}{9}+\frac{y^2}{4}=1$

$$x^2+y^2=\frac{31}{4}$$

let slope of common tangent be m

so tangents are $y = mx \pm \sqrt{9m^2+4}$

$$y = mx \pm \frac{\sqrt{31}}{2}\sqrt{1+m^2}$$

hence $9m^2+4=\frac{31}{4}\left(1+m^2\right)$

$$\Rightarrow 36m^2+16=31+31m^2 \Rightarrow m^2=3$$

12. Official Ans by NTA (1)

Sol. $y^2 = 3x^2$

and $x^2 + y^2 = 4b$

Solve both we get

so $x^2 = b$

$$\frac{x^2}{16}+\frac{3x^2}{b^2}=1$$

$$\frac{b}{16}+\frac{3}{b}=1$$

$b^2 - 16b + 48 = 0$

$(b - 12)(b - 4) = 0$

$b = 12, b > 4$

13. Official Ans. by NTA (2)

Sol. Tangent to parabola

$2y = 2(x + 6) - 20$

$\Rightarrow y = x - 4$

Condition of tangency for ellipse.

$16 = 2(1)^2 + b$

$\Rightarrow b = 14$ **Option (2)**

14. Official Ans. by NTA (3)

Sol. Equation of tangent be

$$\frac{x\cos\theta}{3\sqrt{3}}+\frac{y.\sin\theta}{1}=1, \qquad \theta\in\left(0,\frac{\pi}{2}\right)$$

intercept on x-axis

$OA = 3\sqrt{3}\sec\theta$

intercept on y-axis

$OB = \operatorname{cosec}\theta$

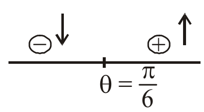
Now, sum of intercept

$= 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta = f(\theta)$ let

$f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$

$$= 3\sqrt{3}\frac{\sin\theta}{\cos^2\theta} - \frac{\cos\theta}{\sin^2\theta}$$

$$= \underbrace{\frac{\cos\theta}{\sin^2\theta}.3\sqrt{3}}_{\oplus}\left[\tan^3\theta - \frac{1}{3\sqrt{3}}\right] = 0 \Rightarrow \theta = \frac{\pi}{6}$$



$\Rightarrow$ at $\theta = \frac{\pi}{6}$, $f(\theta)$ is minimum

HYPERBOLA

1. Official Ans. by NTA (4)

Sol. Tangent to hyperbola of

Slope m = –2 (given)

$y = -2x \pm \sqrt{3(3)}$

$\left(y = mx \pm \sqrt{a^2m^2 - b^2}\right)$

$\Rightarrow y + 2x = \pm 3 \Rightarrow 2x + y = 3 \quad (k > 0)$

For parabola $y^2 = \alpha x$

$$y = mx + \frac{\alpha}{4m}$$

$$\Rightarrow y = -2x + \frac{\alpha}{-8}$$

$$\Rightarrow \frac{\alpha}{-8} = 3$$

$\Rightarrow \alpha = -24.$

2. Official Ans. by NTA (1)

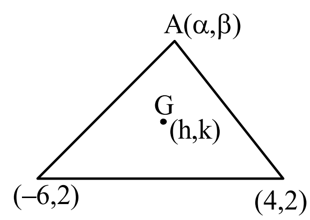
Sol. Given hyperbola is

$16(x + 1)^2 - 9(y - 2)^2 = 164 + 16 - 36 = 144$

$$\Rightarrow \frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$$

Eccentricity, $e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

$\Rightarrow$ foci are (4, 2) and (–6, 2)



Let the centroid be (h, k)

& A(α, β) be point on hyperbola

So $h = \frac{\alpha - 6 + 4}{3}, k = \frac{\beta + 2 + 2}{3}$

$\Rightarrow \alpha = 3h + 2, \beta = 3k - 4$

(α, β) lies on hyperbola so

$16(3h + 2 + 1)^2 - 9(3k - 4 - 2)^2 = 144$

$\Rightarrow 144(h + 1)^2 - 81(k - 2)^2 = 144$

$\Rightarrow 16(h^2 + 2h + 1) - 9(k^2 - 4k + 4) = 16$

$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$

3. Official Ans. by NTA (3)

Sol. $P(-2\sqrt{6}, \sqrt{3})$ lies on hyperbola

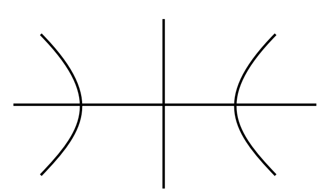
$$\Rightarrow \frac{24}{a^2} - \frac{3}{b^2} = 1 \quad ....(i)$$

$$e = \frac{\sqrt{5}}{2} \Rightarrow b^2 = a^2\left(\frac{5}{4} - 1\right) \Rightarrow 4b^2 = a^2$$

Put in (i) $\Rightarrow \frac{6}{b^2} - \frac{3}{b^2} = 1 \Rightarrow b = \sqrt{3}$

$\Rightarrow a = \sqrt{12}$

$$\frac{x^2}{12} - \frac{y^2}{3} = 1$$



Tangent at P :

$$\frac{-x}{\sqrt{6}}-\frac{y}{\sqrt{3}}=1 \Rightarrow Q\,(0,\ \sqrt{3}\,)$$

Slope of $T=-\frac{1}{\sqrt{2}}$

Normal at P :

$$y-\sqrt{3}=\sqrt{2}(x+2\sqrt{6})$$

$\Rightarrow\ R=(0,5\sqrt{3})$

$QR = 6\sqrt{3}$

4. Official Ans. by NTA (36)

ALLEN Ans. (Bonus)

Sol. Since, point A (sec θ, 2 tan θ)

lies on the hyperbola

$2x^2 - y^2 = 2$

Therefore, $2\sec^2\theta - 4\tan^2\theta = 2$

$\Rightarrow 2 + 2\tan^2\theta - 4\tan^2\theta = 2$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$

Similarly, for point B, we will get $\phi = 0$.

but according to question $\theta+\phi=\frac{\pi}{2}$

which is not possible.

Hence it must be a 'BONUS'.

5. Official Ans. by NTA (2)

Sol. For orthogonal curves $a - c = b - d$

$\Rightarrow a - b = c - d$

6. Official Ans. by NTA (3)

Sol. $f(g(x)) = 2g(x) - 1 = 2\left(\frac{2x-1}{2(x-1)}\right)-1$

$$=\frac{x}{x-1}=1+\frac{1}{x-1}$$

Range of $f(g(x)) = \mathbb{R} - \{1\}$

Range of $f(g(x))$ is not onto

& $f(g(x))$ is one-one

So $f(g(x))$ is one-one but not onto.

7. Official Ans. by NTA (2)

Sol. $K=\frac{4\sqrt{3}}{\sqrt{3}x+y}=\frac{\sqrt{3}x-y}{4\sqrt{3}}$

$\Rightarrow 3x^2 - y^2 = 48$

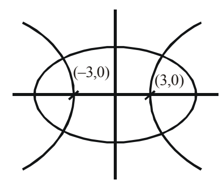
$\Rightarrow \frac{x^2}{16}-\frac{y^2}{48}=1$

Now, $48 = 16(e^2 - 1)$

$\Rightarrow e=\sqrt{4}=2$

8. Official Ans. by NTA (2)

Sol.



For ellipse $e_1=\sqrt{1-\frac{b^2}{a^2}}=\frac{3}{5}$

for hyperbola $e_2=\frac{5}{3}$

Let hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$\because$ it passes through (3,0) $\Rightarrow \frac{9}{a_2}=1$

$\Rightarrow a^2 = 9$

$\Rightarrow b^2 = a^2(e^2 - 1)$

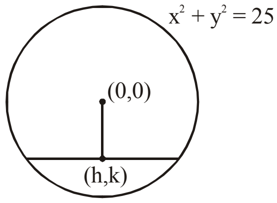
$=9\left(\frac{25}{9}-1\right)=16$

$\therefore$ Hyperbola is

$\frac{x^2}{9}-\frac{y^2}{16}=1$ ... option 2.

9. Official Ans. by NTA (4)

Sol.



Equation of chord

$y - k = -\frac{h}{k}(x - h)$

$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k}$

tangent to $\frac{x^2}{9} - \frac{y^2}{16} = 1$

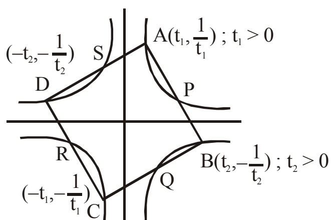
$c^2 = a^2m^2 - b^2$

$\left(\frac{h^2 + k^2}{k}\right)^2 = 9\left(-\frac{h}{k}\right)^2 - 16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

10. Official Ans. by NTA (80)

Sol. $xy = 1, -1$



$\frac{t_1 + t_2}{2} \cdot \frac{\frac{1}{t_1} - \frac{1}{t_2}}{2} = 1$

$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$

$\frac{1}{t_1^2} \times \left(-\frac{1}{t_2^2}\right) = -1 \Rightarrow t_1t_2 = 1$

$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$

$t_1^2 - t_2^2 = 4$

$\Rightarrow t_1^2 + t_2^2 = \sqrt{4^2 + 4} = 2\sqrt{5}$

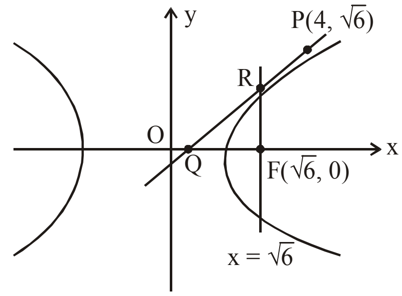
$\Rightarrow t_1^2 = 2 + \sqrt{5} \Rightarrow \frac{1}{t_1^2} = \sqrt{5} - 2$

$AB^2 = (t_1 - t_2)^2 + \left(\frac{1}{t_1} + \frac{1}{t_2}\right)^2$

$= 2\left(t_1^2 + \frac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow \text{Area}^2 = 80$

11. Official Ans. by NTA (3)

Sol.



$\frac{x^2}{4} - \frac{y^2}{2} = 1$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$

$\therefore$ Focus $F(ae, 0) \Rightarrow F\left(\sqrt{6}, 0\right)$

equation of tangent at P to the hyperbola is

$2x - y\sqrt{6} = 2$

tangent meet x-axis at Q(1, 0)

& latus rectum $x = \sqrt{6}$ at $R\left(\sqrt{6}, \frac{2}{\sqrt{6}}\left(\sqrt{6} - 1\right)\right)$

$\therefore$ Area of $\Delta_{QFR} = \frac{1}{2}\left(\sqrt{6} - 1\right) \cdot \frac{2}{\sqrt{6}}\left(\sqrt{6} - 1\right)$

$= \frac{7}{\sqrt{6}} - 2$

PERMUTATION & COMBINATION

1. Official Ans. by NTA (777)

Sol. 15 : Players

6 : Bowlers

7 : Batsman

2 : Wicket keepers

Total number of ways for :

at least 4 bowlers, 5 batsman & 1 wicket keeper

$= {}^6C_4({}^7C_6 \times {}^2C_1 + {}^7C_5 \times {}^2C_2) + {}^6C_5 \times {}^7C_5 \times {}^2C_1$

$= \boxed{777}$

2. Official Ans. by NTA (96)

Sol.

2,4,6,8				
4	4	3	2	1

$= 4 \times 4 \times 3 \times 2 = 96$

3. Official Ans. by NTA (238)

Sol.

Class	10^{th}	11^{th}	12^{th}
Total student	5	6	8
	2	3	5

$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5$

Number of selection	2	2	6

$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6$

	3	2	5

$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5$

$\Rightarrow$ Total number of ways = 23800

According to question

$100\,K = 23800$

$\Rightarrow K = 238$

4. Official Ans. by NTA (3)

Sol. ${}^nP_r = {}^nP_{r+1} \Rightarrow \dfrac{n!}{(n-r)!} = \dfrac{n!}{(n-r-1)!}$

$\Rightarrow (n-r) = 1 \qquad \ldots(1)$

${}^nC_r = {}^nC_{r-1}$

$\Rightarrow \dfrac{n!}{r!(n-r)!} = \dfrac{n!}{(r-1)!(n-r+1)!}$

$\Rightarrow \dfrac{1}{r(n-r)!} = \dfrac{1}{(n-r+1)(n-r)!}$

$\Rightarrow n - r + 1 = r$

$\Rightarrow n + 1 = 2r \qquad \ldots(2)$

$(1) \Rightarrow 2r - 1 - r = 1 \Rightarrow r = 2$

5. Official Ans. by NTA (924)

Sol. $N = 2^{10} \times 5^{10} \times 11^{11} \times 13^{13}$

Now, power of 2 must be zero,

power of 5 can be anything,

power of 13 can be anything.

But, power of 11 should be even.

So, required number of divisors is

$1 \times 11 \times 14 \times 6 = 924$

6. Official Ans. by NTA (52)

Sol. (i) When '0' is at unit place

↑	↑	0
5	4	

Number of numbers = 20

(ii) When 4 or 6 are at unit place

OX		4,6
4	× 4	↑ 2

Number of numbers = 32

So number of numbers = 52

7. Official Ans. by NTA (7744)

So1. 209, 220, 231, ..., 495

Sum $= \dfrac{27}{2}(209 + 495) = 9504$

Number containing 1 at unit place
$\underline{2}\ \underline{3}\ \underline{1}$
$\underline{3}\ \underline{4}\ \underline{1}$
$\underline{4}\ \underline{5}\ \underline{1}$

Number containing 1 at 10^{th} place
$\underline{3}\ \underline{1}\ \underline{9}$
$\underline{4}\ \underline{1}\ \underline{8}$

Required $= 9501 - (231 + 341 + 451 + 319 + 418)$

7744

8. Official Ans. by NTA (100)

Sol.

5	a	b	b	a	5

It is always divisible by 5 and 11.

So, required number = 10 × 10 = 100

9. Official Ans. by NTA (80)

Sol. 3n type $\rightarrow 3, 6, 9 = P$

$3n - 1$ type $\rightarrow 2, 5 = Q$

$3n - 2$ type $\rightarrow 1, 4 = R$

number of subset of S containing one element which are not divisible by 3 $= {}^2C_1 + {}^2C_1 = 4$

number of subset of S containing two numbers whose some is not divisible by 3

$= {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$

number of subsets containing 3 elements whose sum is not divisible by 3

$= {}^3C_2 \times {}^4C_1 + \left({}^2C_2 \times {}^2C_1\right)2 + {}^3C_1\left({}^2C_2 + {}^2C_2\right) = 22$

number of subsets containing 4 elements whose sum is not divisible by 3

$= {}^3C_3 \times {}^4C_1 + {}^3C_2\left({}^2C_2 + {}^2C_2\right) + \left({}^3C_1{}^2C_1 \times {}^2C_2\right)2$

= 4 + 6 + 12 = 22.

number of subsets of S containing 5 elements whose sum is not divisible by 3.

$= {}^3C_3\left({}^2C_2 + {}^2C_2\right) + \left({}^3C_2{}^2C_1 \times {}^2C_2\right) \times 2 = 2 + 12 = 14$

number of subsets of S containing 6 elements whose sum is not divisible by 3 = 4

$\Rightarrow$ Total subsets of Set A whose sum of digits is not divisible by 3 = 4 + 14 + 22 + 22 + 14 + 4 = 80.

10. Official Ans. by NTA (576)

Sol. VOWELS → 2 Vowels
VOWELS → 4 Consonants

All Consonants should not be together

= Total – All consonants together,

= 6! – 3! 4! = 576

11. Official Ans. by NTA (3)

Sol. Total Number of Triangles = ${}^{15}C_3$

i + j + k = 15 (Given)

5 Cases

i	j	k
1	2	12
1	3	11
1	4	10
1	5	9
1	6	8

4 Cases

i	j	k
2	3	10
2	4	9
2	5	8
2	6	7

3 Cases

i	j	k
3	4	8
3	5	7

1 Cases

i	j	k
4	5	6

Number of Possible triangles using the vertices P_i, P_j, P_k such that i + j + k ≠ 15 is equal to ${}^{15}C_3 - 12 = 443$ Option (3)

12. Official Ans. by NTA (77)

Sol. FARMER (6)

A, E, F, M, R, R

A					
E					
F	A	E			
F	A	M			
F	A	R	E		
F	A	R	M	E	R

$\frac{|\underline{5}}{|\underline{2}} - |\underline{4} = 60 - 24 = 36$

$\frac{|\underline{3}}{|\underline{2}} - |\underline{2} = 3 - 2 = 1$

= 1

= 2

= 1

77

13. Official Ans. by NTA (31650)

Sol. If group C has one student then number of groups

${}^{10}C_1[2^9 - 2] = 5100$

If group C has two students then number of groups

${}^{10}C_2[2^8 - 2] = 11430$

If group C has three students then number of groups

$= {}^{10}C_3 \times [2^7 - 2] = 15120$

So total groups = 31650

14. Official Ans. by NTA (1)

Sol.

Indians	Foreigners	Number of ways
2	4	${}^6C_2 \times {}^8C_4 = 1050$
3	6	${}^6C_3 \times {}^8C_6 = 560$
4	8	${}^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

15. Official Ans. by NTA (4)

Sol. $xyz = 2^3 \times 3^1$

Let $x = 2^{\alpha_1} \times 3^{\beta_1}$

$y = 2^{\alpha_2} \times 3^{\beta_2}$

$z = 2^{\alpha_3} \times 3^{\beta_2}$

Now $\alpha_1 + \alpha_2 + \alpha_3 = 3$.

No. of non-negative intergal sol = ${}^5C_2 = 10$

& $\beta_1 + \beta_2 + \beta_3 = 1$

No. of non-negative intergal $sol^n = {}^3C_2 = 3$

Total ways = $10 \times 3 = 30$.

16. Official Ans. by NTA (32)

Sol. We need three digits numbers.

Since $1 + 2 + 3 + 4 + 5 = 15$

So, number of possible triplets for multiple of 15 is $1 \times 2 \times 2$

so Ans. = $4 \times \underline{|3} + 4 \times 3 - 1 \times 2 \times \underline{|2} = 32$

17. Official Ans. by NTA (2)

Sol. $x = {}^5C_3 \times 3! = 60$

$y = {}^{15}C_3 \times 3! = 15 \times 14 \times 13 = 30 \times 91$

$\therefore 2y = 91x$

18. Official Ans. by NTA (45)

Sol. for $3^n + 7^n$ to be divisible by 10

n can be any odd number

$\therefore$ Number of odd two digit numbers = 45

19. Official Ans. by NTA (3)

Sol. **(I)** First possiblity is 1, 1, 1, 1, 1, 2, 3

required number = $\frac{7!}{5!} = 7 \times 6 = 42$

(II) Second possiblity is 1, 1, 1, 1, 2, 2, 2

required number = $\frac{7!}{4!\ 3!} = \frac{7\times 6\times 5}{6} = 35$

Total = 42 + 35 = 77

20. Official Ans. by NTA (4)

Sol. $y + z = 5$

$\frac{1}{y} + \frac{1}{z} = \frac{5}{6} \quad y > z$

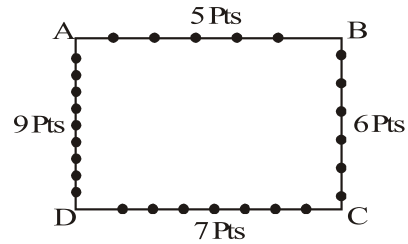
$\Rightarrow y = 3, z = 2$

$\Rightarrow n = 2^x.3^3.5^2 = (2.2.2 \ldots)(3.3.3)(5.5)$

Number of odd divisors = $4 \times 3 = 12$

21. Official Ans by NTA (4)

Sol.



α = Number of triangles

$\alpha = 5\cdot6\cdot7 + 5\cdot7\cdot9 + 5\cdot6\cdot9 + 6\cdot7\cdot9$

$= 210 + 315 + 270 + 378$

$= 1173$

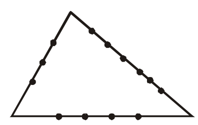
β = Number of Quadrilateral

$\beta = 5\cdot6\cdot7\cdot9 = 1890$

$\beta - \alpha = 1890 - 1173 = 717$

22. Official Ans. by NTA (3)

Sol.



Total Number of triangles formed

$= {}^{14}C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3$

$= 333$ **Option (3)**

23. Official Ans. by NTA (3)

Sol. Total matches between boys of both team

$= {}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both

team = ${}^nC_1\ {}^6C_1 = 6n$

Now, $28 + 6n = 52$

$\Rightarrow n = 4$

24. Official Ans. by NTA (1)

Sol. Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12$.

total numbers when 1 at unit place is 3.

2 at unit place is 6

3 at unit place is 3.

So, sum $= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

$= (1111) \times 24$

$= 26664$

25. Official Ans. by NTA (300)

Sol. 3_ _ = 10 × 10 = 100

_3 _ = 10 × 10 = 100

_ _ 3 = 10 × 10 = $\frac{100}{300}$

26. Official Ans. by NTA (1000)

Sol. Let N be the four digit number

gcd(N,18) = 3

Hence N is an odd integer which is divisible by 3 but not by 9.

4 digit odd multiples of 3

1005, 1011,......., 9999 → 1500

4 digit odd multiples of 9

1017, 1035,......., 9999 → 500

Hence number of such N = 1000

BINOMIAL THEOREM

1. Official Ans. by NTA (2)

Sol. $(1-x)^{100} \cdot (x^2+x+1)^{100} \cdot (1-x)$

$=\left((1-x)(x^2+x+1)\right)^{100}(1-x)$

$=(1^3-x^3)^{100}(1-x)$

$=(1-x^3)^{100}(1-x)$

$=\underbrace{(1-x^3)^{100}}_{\text{No term of } x^{256}}-\underbrace{x(1-x^3)^{100}}_{\text{We find cofficient of } x^{255}}$

Required coefficient $(-1)\times(-{}^{100}C_{85})$

$={}^{100}C_{85}={}^{100}C_{15}$

2. Official Ans. by NTA (21)

Sol. $\left(4^{1/4}+5^{1/6}\right)^{120}$

$T_{r+1}={}^{120}C_r(2^{1/2})^{120-r}(5)^{r/6}$

for rational terms $r=6\lambda \quad 0\le r\le 120$

so total no of forms are 21.

3. Official Ans. by NTA (4)

Sol. $(1-y)^m(1+y)^n$

Coefficient of y $(a_1) = 1.{}^nC_1+{}^mC_1(-1)$

$= n-m = 10$ .....(1)

Coefficient of y^2 (a_2)

$=1.\,{}^nC_2-{}^mC_1.\,{}^nC_1.+1.{}^mC_2=10$

$=\frac{n(n-1)}{2}-m.n+\frac{m(m-1)}{2}=10$

$m^2+n^2-2mn-(n+m)=20$

$(n-m)^2-(n+m)=20$

$n+m=80$ ..... (2)

By equation (1) & (2)

$m = 35, n = 45$

4. Official Ans. by NTA (9)

Sol. $\frac{1}{\alpha(\alpha+1).....(\alpha+20)}=\sum_{k=0}^{20}\frac{A_k}{\alpha+k}$

$A_{14}=\frac{1}{(-14)(-13).....(-1)(1).....(6)}=\frac{1}{14!\cdot 6!}$

$A_{15}=\frac{1}{(-15)(-14).....(-1)(1).....(5)}=\frac{1}{15!\cdot 5!}$

$A_{13}=\frac{1}{(-13).....(-1)(1).....(7)}=\frac{-1}{13!\cdot 7!}$

$\frac{A_{14}}{A_{13}}=\frac{1}{14!\cdot 6!}\times -13!\times 7!=\frac{-7}{14}=-\frac{1}{2}$

$\frac{A_{15}}{A_{13}}=-\frac{1}{15!\times 5!}\times -13!\times 7!=\frac{42}{15\times 14}=\frac{1}{5}$

$100\left(\frac{A_{14}}{A_{13}}+\frac{A_{15}}{A_{13}}\right)^2=100\left(-\frac{1}{2}+\frac{1}{5}\right)^2=9$

5. Official Ans. by NTA (8)

Sol. $\left(2x^r+\frac{1}{x^2}\right)^{10}$

General term = ${}^{10}C_R\,(2x^r)^{10-R}\,x^{-2R}$

$\Rightarrow 2^{10-R}\,{}^{10}C_R=180$ .... (1)

& $(10-R)\,r-2R=0$

$r=\frac{2R}{10-R}$

$r=\frac{2(R-10)}{10-R}+\frac{20}{10-R}$

$\Rightarrow r=-2+\frac{20}{10-R}$ .... (2)

R = 8 or 5 reject equation (1) not satisfied

At R = 8

$2^{10-R}\,{}^{10}C_R=180\Rightarrow \boxed{r=8}$

6. Official Ans. by NTA (96)

Sol. $11^n > 10^n + 9^n$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10+1)^n - (10-1)^n > 10^n$

$\Rightarrow \{ {}^nC_1 \cdot 10^{n-1} + {}^nC_3 10^{n-0} + {}^nC_5 10^{n-5} + \cdots\} > 10^n$

$\Rightarrow 2n \cdot 10^{n-1} + 2\{ {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\} > 10^n$ .... (1)

For n = 5

$10^5 + 2\{ {}^5C_3 10^2 + {}^5C_5\} > 10^5$ (True)

For n = 6, 7, 8, ….. 100

$2n10^{n-1} > 10^n$

$\Rightarrow 2n10^{n-1} + 2\ \{ {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\} > 10^n$

$\Rightarrow 11^n - 9^n > 10^n$ For n = 5, 6, 7, ….. 100

For n = 4, Inequality (1) is not satisfied

$\Rightarrow$ Inequality does not hold good for

N = 1, 2, 3, 4

So, required number of elements

= 96

7. Official Ans. by NTA (3)

Sol. $(a-b)^{-1} + (a-2b)^{-1} + \ldots + (a-nb)^{-1}$

$$= \frac{1}{a}\sum_{r=1}^{n}\left(1 - \frac{rb}{a}\right)^{-1}$$

$$= \frac{1}{a}\sum_{r=1}^{n}\left\{\left(1 + \frac{rb}{a} + \frac{r^2b^2}{a^2}\right) + (\text{terms to be neglected})\right\}$$

$$= \frac{1}{a}\left[n + \frac{n(n+1)}{2}.\frac{b}{a} + \frac{n(n+1)(2n+1)}{6}.\frac{b^2}{a^2}\right]$$

$$= \frac{1}{a}\left[n^3\left(\frac{b^2}{3a^2}\right) + \ldots\right]$$

So $\gamma = \dfrac{b^2}{3a^3}$

8. Official Ans. by NTA (1)

Sol. Coeff. of middle term in $(1+x)^{20} = {}^{20}C_{10}$

& Sum of Coeff. of two middle terms in $(1+x)^{19} = {}^{19}C_9 + {}^{19}C_{10}$

So required ratio $= \dfrac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \dfrac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

9. Official Ans. by NTA (210)

Sol. $$\left(\left(x^{1/3}+1\right) - \left(\frac{x^{1/2}+1}{x^{1/2}}\right)\right)^{10}$$

$$= \left(x^{1/3} - \frac{1}{x^{1/2}}\right)^{10}$$

Now General Term

$$T_{r+1} = {}^{10}C_r\left(x^{1/3}\right)^{10-r}.\left(-\frac{1}{x^{1/2}}\right)^r$$

For independent term

$\dfrac{10-r}{3} - \dfrac{r}{2} = 0 \Rightarrow r = 4$

$\Rightarrow T_5 = {}^{10}C_4 = 210$

10. Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^{12}C_r\left(2^{1/3}\right)^r.\left(3^{1/4}\right)^{12-r}$

T_{r+1} will be rational number

when r = 0, 3, 6, 9, 12

& r = 0, 4, 8, 12

$\Rightarrow$ r = 0, 12

$T_1 + T_{13} = 1 \times 3^3 + 1 \times 2^4 \times 1$

= 24 + 16 = 43

11. Official Ans. by NTA (4)

Sol. $$T_{r+1} = {}^{10}C_r(x\sin\alpha)^{10-r}\left(\frac{a\cos\alpha}{x}\right)^r$$

r = 0, 1, 2, …, 10

T_{r+1} will be independent of x

when $10 - 2r = 0 \Rightarrow r = 5$

$$T_6 = {}^{10}C_5(x\sin\alpha)^5 \times \left(\frac{a\cos\alpha}{x}\right)^5$$

$$= {}^{10}C_5 \times a^5 \times \frac{1}{2^5}(\sin 2\alpha)^5$$

will be greatest when $\sin 2\alpha = 1$

$\Rightarrow {}^{10}C_5\dfrac{a^5}{2^5} = {}^{10}C_5 \Rightarrow a = 2$

12. Official Ans. by NTA (1)

Sol. Let $P=\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$,

Let $x = 10^{100}$

$\Rightarrow P=\left(1+\frac{1}{x}\right)^x$

$\Rightarrow P=1+(x)\left(\frac{1}{x}\right)+\frac{(x)(x-1)}{\lfloor 2}.\frac{1}{x^2}+\frac{(x)(x-1)(x-2)}{\lfloor 3}.\frac{1}{x^3}+...$

(upto $10^{100}+1$ terms)

$\Rightarrow P=1+1+\left(\frac{1}{\lfloor 2}-\frac{1}{\lfloor 2 x^2}\right)+\left(\frac{1}{\lfloor 3}-...\right)+...\text{ so on}$

$\Rightarrow P=2+\left(\text{Positive value less then }\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+...\right)$

Also $e=1+\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+...$

$\Rightarrow \frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+...=e-2$

$\Rightarrow$ P = 2 + (positive value less then e – 2)

$\Rightarrow P \in (2, 3)$

$\Rightarrow$ least integer value of P is 3

13. Official Ans. by NTA (98)

Sol. $1.{}^nC_0 + 3.{}^nC_1 + 5.{}^nC_2 + \ldots + (2n+1).{}^nC_n$

$T_r = (2r + 1){}^nC_r$

$S = \Sigma T_r$

$S = \Sigma(2r+1){}^nC_r = \Sigma 2r{}^nC_r + \Sigma{}^nC_r$

$S = 2(n.2^{n-1}) + 2^n = 2^n(n + 1)$

$2^n(n + 1) = 2^{100}.101 \Rightarrow n = 100$

$2\left[\frac{n-1}{2}\right]=2\left[\frac{99}{2}\right]=98$

14. Official Ans. by NTA (55)

Sol. ${}^nC_7 2^{n-7}\frac{1}{3^7}={}^nC_8 2^{n-8}\frac{1}{3^8}$

$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

15. Official Ans. by NTA (3)

Sol. Coefficient of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$

${}^{11}C_r(x^2)^{11-r}.\left(\frac{1}{bx}\right)^r$

${}^{11}C_r x^{22-3r}.\frac{1}{b^r}$

$22 - 3r = 7$

$r = 5$

$\therefore\ {}^{11}C_5.\frac{1}{b^5}.x^7$

Coefficient of x^{-7} in $\left(x-\frac{b}{bx^2}\right)^{11}$

${}^{11}C_r(x)^{11-r}.\left(-\frac{1}{bx^2}\right)^r$

${}^{11}C_r x^{11-3r}.\frac{(-1)^r}{b^r}$

$11-3r=-7\ \therefore r=6$

${}^{11}C_6.\frac{1}{b^6}x^{-7}$

${}^{11}C_5.\frac{1}{b^5}={}^{11}C_6.\frac{1}{b^6}$

Since $b\neq 0\ \therefore b=1$

16. Official Ans. by NTA (4)

Sol. ${}^{10}C_8\left(25^{(x-1)}+7\right)\times\left(5^{(x-1)}+1\right)^{-1}=180$

$\Rightarrow \frac{25^{x-1}+7}{5^{(x-1)}+1}=4$

$\Rightarrow \frac{t^2+7}{t+1}=4;$

$\Rightarrow t = 1, 3 = 5^{x-1}$

$\Rightarrow$ x – 1 = 0 (one of the possible value).

$\Rightarrow x = 1$

17. Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{20} r^2.{}^{20}C_r$

$\sum\left(4(r-1)+r\right).{}^{20}C_r$

$\sum r(r-1).\frac{20\times 19}{r(r-1)}.{}^{18}C_r+r.\frac{20}{r}.\sum{}^{19}C_{r-1}$

$\Rightarrow 20 \times 19.2^{18} + 20.2^{19}$

$\Rightarrow 420 \times 2^{18}$

18. Official Ans. by NTA (136)

Sol. ${}^{1}P_{1}+2\cdot{}^{2}P_{2}+3\cdot{}^{3}P_{3}+\ldots+15\cdot{}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + \ldots.15 \times 15!$

$=\sum_{r=1}^{15}(r+1-1)r!$

$=\sum_{r=1}^{15}(r+1)!-(r)!$

$= 16! - 1$

$= {}^{16}P_{16} - 1$

$\Rightarrow q = r = 16, s = 1$

${}^{q+s}C_{r-s} = {}^{17}C_{15} = 136$

19. Official Ans. by NTA (49)

Sol. $A_k=\sum_{i=0}^{9}{}^{9}C_{i}\ {}^{12}C_{k-i}+\sum_{i=0}^{8}{}^{8}C_{i}\ {}^{13}C_{k-i}$

$A_k={}^{21}C_k+{}^{21}C_k=2\cdot{}^{21}C_k$

$A_4-A_3=2\left({}^{21}C_4-{}^{21}C_3\right)=2(5985-1330)$

$190\,p = 2(5985 - 1330) \Rightarrow p = 49.$

20. Official Ans. by NTA (1)

Sol. Let $t=\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+\ldots.\infty$

$=\left(2-\frac{1}{2}\right)x^2+\left(2-\frac{1}{3}\right)x^3+\left(2-\frac{1}{4}\right)x^4+\ldots.\infty$

$=2\left(x^2+x^3+x^4+\ldots\infty\right)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\infty\right)$

$=\frac{2x^2}{1-x}-\left(\ell n(1-x)-x\right)$

$\Rightarrow t=\frac{2x^2}{1-x}+x-\ell n(1-x)$

$\Rightarrow t=\frac{x(1+x)}{1-x}-\ell n(1-x)$

21. Official Ans. by NTA (3)

Sol. $\sum_{k=0}^{20}{}^{20}C_k\cdot{}^{20}C_{20-k}$

sum of suffix is const. so summation will be ${}^{40}C_{20}$

22. Official Ans. by NTA (15)

Sol. $3(1 + 6)^{22} + 2\cdot(1 + 9)^{22} - 44 = (3 + 2 - 44) = 18\,.I$

$= -39 + 18.I$

$= (54 - 39) + 18(I - 3)$

$= 15 + 18\, I_1$

$\Rightarrow$ Remainder = 15.

23. Official Ans. by NTA (55)

Sol. $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$

$T_{r+1}=(-1)^r\,.\,{}^{12}C_r\left(\frac{x}{4}\right)^{12-r}\left(\frac{12}{x^2}\right)^r$

$T_{r+1}=(-1)^r\,.\,{}^{12}C_r\left(\frac{1}{4}\right)^{12-r}(12)^r.(x)^{12-3r}$

Term independent of x $\Rightarrow 12 - 3r = 0 \Rightarrow r = 4$

$T_5=(-1)^4\,.\,{}^{12}C_4\left(\frac{1}{4}\right)^8(12)^4=\frac{3^6}{4^4}.k$

$\Rightarrow k = 55$

24. Official Ans. by NTA (315)

Sol. $\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha}(2b)^{\beta}.(4ab)^{\gamma}$

$\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha+\gamma}.b^{\beta+\gamma}.2^{\beta}.4^{\gamma}$

$\alpha+\beta+\gamma = 10$ …(1)

$\alpha+\gamma = 7$ …(2)

$\beta+\gamma = 8$ …(3)

$(2) + (3) - (1) \Rightarrow \gamma = 5$

$\alpha = 2$

$\beta = 3$

so coefficients $=\frac{10!}{2!3!5!}2^3.2^{10}$

$=\frac{10\times9\times8\times7\times6\times5}{2\times3\times2\times5!}\times2^{13}$

$= 315 \times 2^{16} \Rightarrow k = 315$

25. Official Ans. by NTA (924)

Sol. $(x + y)^n \Rightarrow 2^n = 4096 \quad 2^{10} = 1024 \times 2$

$\Rightarrow 2^n = 2^{12} \quad 2^{11} = 2048$

$n = 12 \quad 2^{12} = \underline{4096}$

$${}^{12}C_6 = \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}$$

$= 11 \times 3 \times 4 \times 7$

$= 924$

26. Official Ans. by NTA (2)

Sol. ${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2\right)$

${}^{n+1}C_2 + 2\left({}^3C_3 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2\right)$

$\left\{\text{use } {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r\right\}$

$= {}^{n+1}C_2 + 2\left({}^4C_3 + {}^4C_2 + {}^5C_3 + \ldots\ldots + {}^nC_2\right)$

$= {}^{n+1}C_2 + 2\left({}^5C_3 + {}^5C_2 + \ldots\ldots + {}^nC_2\right)$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$= {}^{n+1}C_2 + 2\left({}^nC_3 + {}^nC_2\right)$

$= {}^{n+1}C_2 + 2.{}^{n+1}C_3$

$$= \frac{(n+1)n}{2} + 2.\frac{(n+1)(n)(n-1)}{2.3}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

27. Official Ans. by NTA (BONUS)

Sol. Bonus

$$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$$

${}^{25}C_k + {}^{25}C_{k+1}$

${}^{26}C_{k+1}$

as ${}^nC_r$ is defined for all values of n as will as r

so ${}^{26}C_{k+1}$ always exists

Now k is unbounded so maximum value is not defined.

28. Official Ans. by NTA (2)

Sol. $(-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots\ldots -15.{}^{15}C_{15})$
$+ ({}^{14}C_1 + {}^{14}C_3 + \ldots + {}^{14}C_{11})$

$$= \sum_{r=1}^{15}(-1)^r.r\,{}^{15}C_r + \left({}^{14}C_1 + {}^{14}C_3 + \ldots + {}^{14}C_{11} + {}^{14}C_{13}\right) - {}^{14}C_3$$

$$= \sum_{r=1}^{15}(-1)^r\,15.\,{}^{14}C_{r-1} + 2^{13} - 14$$

$= 15(-{}^{14}C_0 + {}^{14}C_1 \ldots\ldots - {}^{14}C_{14}) + 2^{13} - 14$

29. Official Ans. by NTA (1)

Sol. $x = 4k + 3$

$\therefore (2020 + x)^{2022} = (2020 + 4k + 3)^{2022}$

$= (4(505 + k) + 3)^{2022}$

$= (4\lambda + 3)^{2022} = (16\lambda^2 + 24\lambda + 9)^{1011}$

$= (8(2\lambda^2 + 3\lambda + 1) + 1)^{1011}$

$= (8p + 1)^{1011}$

$\therefore$ Remainder when divided by 8 = 1

$= 2^{13} - 14$

30. Official Ans. by NTA (45)

Sol. $30({}^{30}C_0) + 29({}^{30}C_1) + \ldots + 2({}^{30}C_{28}) + 1({}^{30}C_{29})$

$= 30({}^{30}C_{30}) + 29({}^{30}C_{29}) + \ldots\ldots + 2({}^{30}C_2) + 1({}^{30}C_1)$

$$= \sum_{r=1}^{30} r\left({}^{30}C_r\right)$$

$$= \sum_{r=1}^{30} r\left(\frac{30}{r}\right)\left({}^{29}C_{r-1}\right)$$

$$= 30\sum_{r=1}^{30} {}^{29}C_{r-1}$$

$= 30\left({}^{29}C_0 + {}^{29}C_1 + {}^{29}C_2 + \ldots + {}^{29}C_{29}\right)$

$= 30\left(2^{29}\right) = 15(2)^{30} = n(2)^m$

$\therefore$ n = 15, m = 30

31. Official Ans. by NTA (2)

Sol. Term independent of t will be the middle term due to exect same magnitude but opposite sign powers of t in the binomial expression given

so $T_6 = {}^{10}C_5\left(tx^2 5\right)^5\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^5$

$T_6 = f(x) = {}^{10}C_5\left(x\sqrt{1-x}\right)$; for maximum

$f'(x) = 0 \Rightarrow x = \frac{2}{3}$ & $f''\left(\frac{2}{3}\right) < 0$

so $f(x)_{max.} = {}^{10}C_5\left(\frac{2}{3}\right).\frac{1}{\sqrt{3}}$

32. Official Ans by NTA (6)

Sol. $A = \sum_{k=0}^{n} {}^nC_k\left[\left(-\frac{1}{2}\right)^k + \left(\frac{-3}{4}\right)^k + \left(\frac{-7}{8}\right)^k + \left(\frac{-15}{16}\right)^k + \left(\frac{-37}{32}\right)^k\right]$

$A = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \left(1-\frac{15}{16}\right)^n + \left(1-\frac{31}{32}\right)^n$

$A = \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \frac{1}{16^n} + \frac{1}{32^n}$

$A = \frac{1}{2^n}\left(\frac{1-\left(\frac{1}{2^n}\right)^5}{1-\frac{1}{2^n}}\right) \Rightarrow A = \frac{\left(1-\frac{1}{2^{5n}}\right)}{(2^n-1)}$

$(2^n - 1)A = 1 - \frac{1}{2^{5n}}$, Given $63A = 1 - \frac{1}{2^{30}}$

Clearly $5n = 30$

$n = 6$

33. Official Ans. by NTA (1)

Sol. $(3^{1/4} + 5^{1/8})^{60}$

${}^{60}C_r(3^{1/4})^{60-r}.(5^{1/8})^r$

${}^{60}C_r(3)^{\frac{60-r}{4}}.5^{\frac{r}{8}}$

For rational terms.

$\frac{r}{8} = k$; $0 \le r \le 60$

$0 \le 8k \le 60$

$0 \le k \le \frac{60}{8}$

$0 \le k \le 7.5$

$k = 0, 1, 2, 3, 4, 5, 6, 7$

$\frac{60-8k}{4}$ is always divisible by 4 for all value of k.

Total rational terms = 8

Total terms = 61

irrational terms = 53

$n - 1 = 53 - 1 = 52$

52 is divisible by 26.

34. Official Ans. by NTA (3)

Sol. $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$(1 - x + x^3)^n = a_0 + a_1x + a_2x^2 \ldots\ldots + a_{3n}x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}$ = Sum of $a_0 + a_2 + a_4 \ldots\ldots$

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ = Sum of $a_1 + a_3 + a_5 \ldots\ldots$

put x = 1

$1 = a_0 + a_1 + a_2 + a_3 \ldots\ldots + a_{3n}$ ....(A)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 \ldots\ldots + (-1)^{3n}a_{3n}$ ....(B)

Solving (A) and (B)

$a_0 + a_2 + a_4 \ldots = 1$

$a_1 + a_3 + a_5 \ldots\ldots = 0$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$

35. Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r}$

$= {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + \ldots\ldots + {}^6C_6 \cdot {}^6C_0$

Now,

$(1+x)^6(1+x)^6$

$= \left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \ldots\ldots + {}^6C_6x^6\right)$
$\left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \ldots\ldots + {}^6C_6x^6\right)$

Comparing coefficeint of x^6 both sides

${}^6C_0 \cdot {}^6C_6 + {}^6C_1 + {}^6C_5 + \ldots\ldots + {}^6C_6 \cdot {}^6C_0 = {}^{12}C_6$

$= 924$

Ans.(4)

36. Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^nC_r(x)^{n-r}\left(\frac{a}{x^2}\right)^r$

$= {}^nC_r\, a^r x^{n-3r}$

${}^nC_2\, a^2 : {}^nC_3\, a^3 : {}^nC_4\, a^4 = 12 : 8 : 3$

After solving

$n = 6,\ a = \frac{1}{2}$

For term independent of 'x' $\Rightarrow n = 3r$

$r = 2$

$\therefore$ Coefficient is ${}^6C_2\left(\frac{1}{2}\right)^2 = \frac{15}{4}$

Nearest integer is 4.

37. Official Ans. by NTA (2)

Sol. $n(E) = 5 + 4 + 4 + 3 + 1 = 17$

So, $P(E) = \frac{17}{36}$

38. Official Ans. by NTA (1)

Sol. ${}^7C_3 x^4 \; x^{(3\log_2^x)} = 4480$

$\Rightarrow \; x^{(4+3\log_2^x)} = 2^7$

$\Rightarrow \; (4+3t)t = 7; t = \log_2^x$

$\Rightarrow \; t = 1, \frac{-7}{3} \Rightarrow x = 2$

39. Official Ans. by NTA (1)

Sol. $y = 4 + \frac{1}{\left(5 + \frac{1}{y}\right)}$

$y - 4 = \frac{y}{(5y+1)}$

$5y^2 - 20y - 4 = 0$

$y = \frac{20+\sqrt{480}}{10}$

$y = \frac{20-\sqrt{480}}{10} \rightarrow$ rejected

$y = 2 + \sqrt{\frac{480}{100}}$

Correct with Option (A)

40. Official Ans. by NTA (4)

Sol. $(2023 - 2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$ (as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17 - 1)^{940}$

$= 17k_2 + 4\,(17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder $= 4$

41. Official Ans. by NTA (2)

Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1x + ... + a_{40}x^{40}$

put $x = 1, -1$

$\Rightarrow a_0 + a_1 + a_2 + ... + a_{40} = 2^{20}$

$a_0 - a_1 + a_2 + ... + a_{40} = 2^{20}$

$\Rightarrow a_1 + a_3 + ... + a_{39} = \frac{4^{20} - 2^{20}}{2}$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$

here $a_{39} = \frac{20!(2)^{19} \times 1}{19!} = 20 \times 2^{19}$

$\Rightarrow \; a_1 + a_3 + ... + a_{37} = 2^{19}(2^{20} - 1 - 20)$

$= 2^{19}(2^{20} - 21)$

Ans. 4

42. Official Ans. by NTA (160)

Sol. $\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3) - 9(r+1) + 8\}$

$= \sum_{r=1}^{10}\left[\{(r+3)! - (r+1)!\} - 8\{(r+1)! - r!\}\right]$

$= (13! + 12! - 2! - 3!) - 8(11! - 1)$

$= (12.13 + 12 - 8).11! - 8 + 8$

$= (160)(11)!$

Hence $\alpha = 160$

43. Official Ans. by NTA (8)

Sol. Let $p'(x) = a(x - 1)(x + 1) = a(x^2 - 1)$

$p(x) = a\int(x^2 - 1)\,dx + c$

$= a\left(\frac{x^3}{3} - x\right) + c$

Now $p(-3) = 0$

$\Rightarrow a\left(-\frac{27}{3} + 3\right) + c = 0$

$\Rightarrow -6a + c = 0$ ...(1)

Now $\int_{-1}^{1}\left(a\left(\frac{x^3}{3} - x\right) + c\right)dx = 18$

$= 2c = 18 \Rightarrow c = 9$ ...(2)

$\Rightarrow$ from (1) & (2) $\Rightarrow -6a + 9 = 0 \Rightarrow a = \frac{3}{2}$

$\Rightarrow p(x) = \frac{3}{2}\left(\frac{x^3}{3} - x\right) + 9$

sum of coefficient

$= \frac{1}{2} - \frac{3}{2} + 9$

$= 8$

44. Official Ans. by NTA (19)

Allen Answer (Bonus)

Sol. BONUS

Instead of ^nC_k it must be $^{10}C_k$ i.e.

$$\sum_{k=0}^{10}(2^2+3k)\,{}^{10}C_k=\alpha.3^{10}+\beta.2^{10}$$

$$\text{LHS}=4\sum_{k=0}^{10}{}^{10}C_k+3\sum_{k=0}^{10}k.\frac{10}{k}.{}^9C_{k-1}$$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + \beta.2^{10}$

$\Rightarrow \alpha = 0, \beta = 19 \Rightarrow \alpha + \beta = 19$

SET

1. Official Ans. by NTA (3)

Sol. $n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$

$100 \geq 89 + 98 - n(A \cup B)$

$n(A \cup B) \geq 87$

$87 \leq n(A \cup B) \leq 89$

Option (3)

2. Official Ans. by NTA (256)

Sol. $A=(-\infty,1)\cup(3,\infty)$

$B=(-\infty,-2)\cup(2,\infty)$

$C=(-\infty,2]\cup[6,\infty)$

So, $A\cap B\cap C=(-\infty,-2)\cup[6,\infty)$

$z\cap(A\cap B\cap C)'=\{-2,-1,0,-1,2,3,4,5\}$

Hence no. of its subsets $= 2^8 = 256$.

3. Official Ans. by NTA (5)

Sol. B and C will contain three digit numbers of the form $9k + 2$ and $9k + \ell$ respectively. We need to find sum of all elements in the set $B \cup C$ effectively.

Now, $S(B \cup C) = S(B) + S(C) - S(B \cap C)$

where S(k) denotes sum of elements of set k.

Also, B = {101, 109, ..., 992}

$\therefore \quad S(B)=\frac{100}{2}(101+992)=54650$

Case-I : If $\ell = 2$

then $B \cap C = B$

$\therefore \quad S(B \cup C) = S(B)$

which is not possible as given sum is

$274 \times 400 = 109600$.

Case-II : If $\ell \neq 2$

then $B \cap C = \phi$

$\therefore \quad S(B \cup C) = S(B) + S(C) = 400 \times 274$

$$\Rightarrow 54650+\sum_{k=11}^{110}9k+\ell=109600$$

$$\Rightarrow 9\sum_{k=11}^{110}k+\sum_{k=11}^{110}\ell=54950$$

$$\Rightarrow 9\left(\frac{100}{2}(11+110)\right)+\ell(100)=54950$$

$\Rightarrow 54450 + 100\ell = 54950$

$\Rightarrow \ell = 5$

4. Official Ans. by NTA (4)

Sol. Equivalence class of (1, –1) is a circle with centre at (0,0) and radius = $\sqrt{2}$

$\Rightarrow x^2 + y^2 = 2$

$S = \{(x,y) | x^2 + y^2 = 2\}$

5. Official Ans. by NTA (3)

Sol. $A \cap B \cap C$ is visible in all three venn diagram

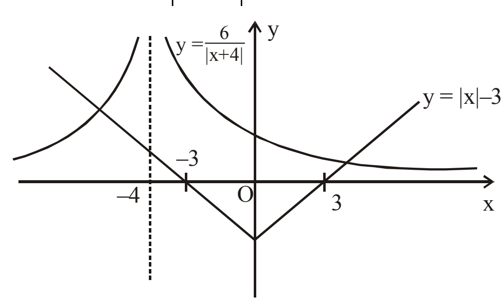
Hence, Option (3)

6. Official Ans. by NTA (2)

Sol. $x \neq -4$

$(|x| - 3)(|x + 4|) = 6$

$$\Rightarrow \quad |x|-3=\frac{6}{|x+4|}$$



No. of solutions = 2

RELATION

1. Official Ans. by NTA (2)

Sol. $x^3 - 3x^2y - xy^2 + 3y^3 = 0$

$\Rightarrow x(x^2 - y^2) - 3y(x^2 - y^2) = 0$

$\Rightarrow (x - 3y)(x - y)(x + y) = 0$

Now, $x = y\ \forall (x,y) \in N \times N$ so reflexive

But not symmetric & transitive

See, (3,1) satisfies but (1,3) does not. Also (3,1) & (1,–1) satisfies but (3, –1) does not

2. Official Ans. by NTA (2)

Sol. Note that (1,2) and (2,3) satisfy $0 < |x - y| \le 1$ but (1,3) does not satisfy it so

$0 \le |x - y| \le 1$ is symmetric but not transitive

So, (2) is correct.

3. Official Ans by NTA (4)

Sol. $A = \{2, 3, 4, 5, ..., 30\}$

$(a, b) \simeq (c, d) \Rightarrow ad = bc$

$(4, 3) \simeq (c, d) \Rightarrow 4d = 3c$

$\Rightarrow \frac{4}{3} = \frac{c}{d}$

$\frac{c}{d} = \frac{4}{3}$ & $c, d \in \{2, 3, ..., 30\}$

(c, d) = {(4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)}

No. of ordered pair = 7

4. Official Ans. by NTA (3)

Sol. A and B are matrices of $n \times n$ order & ARB iff there exists a non singular matrix $P(\det(P) \neq 0)$ such that $PAP^{-1} = B$

For reflexive

$ARA \Rightarrow PAP^{-1} = A$ ...(1) must be true

for P = I, Eq.(1) is true so 'R' is reflexive

For symmetric

$ARB \Leftrightarrow PAP^{-1} = B$ ...(1) is true

for BRA iff $PBP^{-1} = A$ ...(2) must be true

$\because PAP^{-1} = B$

$P^{-1}PAP^{-1} = P^{-1}B$

$IAP^{-1}P = P^{-1}BP$

$A = P^{-1}BP$ ...(3)

from (2) & (3) $PBP^{-1} = P^{-1}BP$

can be true some $P = P^{-1} \Rightarrow P^2 = I\ (\det(P) \neq 0)$

So 'R' is symmetric

For trnasitive

$ARB \Leftrightarrow PAP^{-1} = B$... is true

$BRC \Leftrightarrow PBP^{-1} = C$... is true

now $PPAP^{-1}P^{-1} = C$

$P^2A(P^2)^{-1} = C \Rightarrow ARC$

So 'R' is transitive relation

$\Rightarrow$ Hence R is equivalence

FUNCTION

1. Official Ans. by NTA (3)

Sol. For domain,

$\frac{|[x]| - 2}{|[x]| - 3} \ge 0$

Case I : When $|[x]| - 2 \ge 0$

and $|[x]| - 3 > 0$

$\therefore x \in (-\infty, -3) \cup [4, \infty)$ .....(1)

Case II : When $|[x]| - 2 \le 0$

and $|[x]| - 3 < 0$

$\therefore x \in [-2, 3)$ .....(2)

So, from (1) and (2)

we get

Domain of function

$= (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$\therefore (a + b + c) = -3 + (-2) + 3 = -2\ (a < b < c)$

$\Rightarrow$ Option (3) is correct.

2. Official Ans. by NTA (2)

Sol. $f(x) = \frac{5x + 3}{6x - \alpha} = y$ ......(i)

$5x + 3 = 6xy - \alpha y$

$x(6y - 5) = \alpha y + 3$

$x = \frac{\alpha y + 3}{6y - 5}$

$f^{-1}(x) = \frac{\alpha x + 3}{6x - 5}$ .....(ii)

$f \circ f(x) = x$

$f(x) = f^{-1}(x)$

From eq^n (i) & (ii)

Clearly $(\alpha = 5)$

3. Official Ans. by NTA (4)

Sol. $\left[e^x\right]^2+\left[e^x+1\right]-3=0$

$\Rightarrow \left[e^x\right]^2+\left[e^x\right]+1-3=0$

Let $\left[e^x\right]=t$

$\Rightarrow t^2+t-2=0$

$\Rightarrow t=-2, 1$

$\left[e^x\right]=-2$ (Not possible)

or $\left[e^x\right]=1 \quad \therefore\ 1\le e^x<2$

$\Rightarrow \ \ln(1)\le x<\ln(2)$

$\Rightarrow \ 0\le x<\ln(2)$

$\Rightarrow \ x\in[0, \ln 2)$

4. Official Ans. by NTA (720)

Sol. $f(1)+f(2)=3-f(3)$

$\Rightarrow f(1)+f(2)=3+f(3)=3$

The only possibility is : 0 + 1 + 2 = 3

$\Rightarrow$ Elements 1, 2, 3 in the domain can be mapped with 0, 1, 2 only.

So number of bijective functions.

$=\lfloor 3\times \lfloor 5 = 720$

5. Official Ans. by NTA (1)

Sol. $g: N\to N \quad g(3n+1)=3n+2$

$g(3n+2)=3n+3$

$g(3n+3)=3n+1$

$$g(x)=\begin{bmatrix} x+1 & x=3k+1\\ x+1 & x=3k+2\\ x-2 & x=3k+3\end{bmatrix}$$

$$g(g(x))=\begin{bmatrix} x+2 & x=3k+1\\ x-1 & x=3k+2\\ x-1 & x=3k+3\end{bmatrix}$$

$$g(g(g(x)))=\begin{bmatrix} x & x=3k+1\\ x & x=3k+2\\ x & x=3k+3\end{bmatrix}$$

If $f: N\to N$, f is a one-one function such that $f(g(x))=f(x)\Rightarrow g(x)=x$, which is not the case

If f $f: N\to N$ f is an onto function such that $f(g(x))=f(x)$,

one possibility is

$$f(x)=\begin{bmatrix} n & x=3n+1\\ n & x=3n+2 & n\in N_0\\ n & x=3n+3\end{bmatrix}$$

Here $f(x)$ is onto, also $f(g(x))=f(x)\ \forall\ x\in N$

6. Official Ans. by NTA (2)

Sol. $$\sum_{n=8}^{100}\left[\frac{(-1)^n.n}{2}\right]$$

$=4-5+5-6+6+\ldots-50+50=4$

7. Official Ans. by NTA (3)

Sol. $\therefore\ (gof)^{-1}$ exist $\Rightarrow$ gof is bijective

$\Rightarrow$ 'f' must be one-one and 'g' must be ONTO

8. Official Ans. by NTA (490)

Sol. F(mn) = f(m). f(n)

Put m = 1 **f(n) = f(1).** f(n) $\Rightarrow$ f(1) = 1

Put m = n = 2

$$f(4)=f(2).f(2)\begin{cases} f(2)=1\Rightarrow f(4)=1\\ \text{or}\\ f(2)=2\Rightarrow f(4)=4\end{cases}$$

Put m = 2, n = 3

$$f(6)=f(2).f(3)\begin{cases} \text{when } f(2)=1\\ f(3)=1 \text{ to } 7\\ \\ f(2)=2\\ f(3)=1 \text{ or } 2 \text{ or} 3\end{cases}$$

f(5), f(7) can take any value

Total = (1 × 1 × 7 × 1 × 7 × 1 × 7) + (1 × 1 × 3 × 1 × 7 × 1 × 7)

= 490

9. Official Ans. by NTA (3)

Sol. f(x) = cos λx

$\because\ f\left(\frac{1}{2}\right) = -1$

So, $-1 = \cos\frac{\lambda}{2}$

$\Rightarrow \lambda = 2\pi$

Thus f(x) = cos 2πx

Now k is natural number

Thus f(k) = 1

$$\sum_{k=1}^{20} \frac{1}{\sin k \sin(k+1)} = \frac{1}{\sin 1}\sum_{k=1}^{20}\left[\frac{\sin((k+1)-k)}{\sin k \cdot \sin(k+1)}\right]$$

$$= \frac{1}{\sin 1}\sum_{k=1}^{20}(\cot k - \cot(k+1))$$

$$= \frac{\cot 1 - \cot 21}{\sin 1} = \operatorname{cosec}^2 1 \operatorname{cosec}(21)\cdot \sin 20$$

10. Official Ans. by NTA (4)

Sol. $\frac{1+x}{x} \in (-\infty,-1] \cup [1,\infty)$

$\frac{1}{x} \in (-\infty,-2] \cup [0,\infty)$

$x \in \left[-\frac{1}{2},0\right) \cup (0,\infty)$

$x \in \left[-\frac{1}{2},\infty\right) - \{0\}$

11. Official Ans. by NTA (2)

Sol. $f(m+n) = f(m) + f(n)$

Put m = 1, n = 1

$f(2) = 2f(1)$

Put m = 2, n = 1

$f(3) = f(2) + f(1) = 3f(1)$

Put m = 3, n = 3

$f(6) = 2f(3) \Rightarrow f(3) = 9$

$\Rightarrow f(1) = 3,\ f(2) = 6$

$f(2).f(3) = 6 \times 9 = 54$

12. Official Ans. by NTA (4)

Sol. $f(x) = \log_{\sqrt{5}}$

$\left(3 + \cos\left(\frac{3\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right) - \cos\left(\frac{3\pi}{4}-x\right)\right)$

$f(x) = \log_{\sqrt{5}}\left[3 + 2\cos\left(\frac{\pi}{4}\right)\cos(x) - 2\sin\left(\frac{3\pi}{4}\right)\sin(x)\right]$

$f(x) = \log_{\sqrt{5}}\left[3 + \sqrt{2}(\cos x - \sin x)\right]$

Since $-\sqrt{2} \le \cos x - \sin x \le \sqrt{2}$

$\Rightarrow \log_{\sqrt{5}}\left[3 + \sqrt{2}(-\sqrt{2})\right] \le f(x) \le \log_{\sqrt{5}}\left[3 + \sqrt{2}(\sqrt{2})\right]$

$\Rightarrow \log_{\sqrt{5}}(1) \le f(x) \le \log_{\sqrt{5}}(5)$

So Range of f(x) is [0, 2]

Option (4)

13. Official Ans. by NTA (26)

Sol. k f(k) + 2 = λ (x – 2) (x – 3) (x – 4) (x – 5) ...(1)

put x = 0

we get $\lambda = \frac{1}{60}$

Now put λ in equation (1)

$\Rightarrow kf(k) + 2 = \frac{1}{60}(x-2)(x-3)(x-4)(x-5)$

Put x = 10

$\Rightarrow 10f(10) + 2 = \frac{1}{60}(8)(7)(6)(5)$

$\Rightarrow 52 - 10f(10) = 52 - 26 = 26$

14. Official Ans. by NTA (4)

Sol. f(n + 1) – f(n) = f(1)

⇒ f(n) = nf (1)

⇒ f is one-one

Now, Let $f(g(x_2)) = f(g(x_1))$

$\Rightarrow g(x_2) = g(x_1)$ (as f is one-one)

$\Rightarrow x_1 = x_2$ (as fog is one-one)

⇒ g is one-one

Now, f(g(n)) = g(n) f(1)

may be many-one if

g(n) is many-one

15. Official Ans. by NTA (4)

Sol. $f(x)=\dfrac{5^x}{5^x+5}$ $\quad f(2-x)=\dfrac{5}{5^x+5}$

$f(x)+f(2-x)=1$

$\Rightarrow f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+...+f\left(\frac{39}{20}\right)$

$=\left(f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)\right)+...+\left(f\left(\frac{19}{20}\right)+f\left(\frac{21}{20}\right)+f\left(\frac{20}{20}\right)\right)$

$=19+\frac{1}{2}=\frac{39}{2}$

16. Official Ans. by NTA (1)

Sol. $f(x)=\begin{cases}x+1, & \text{if x is odd}\\ x, & \text{if x is even}\end{cases}$

$\because$ g : A → A such that $g(f(x))=f(x)$

⇒ If x is even then $g(x)=x$ ...(1)

If x is odd then $g(x+1)=x+1$ ...(2)

from (1) and (2) we can say that

$g(x)=x$ if x is even

⇒ If x is odd then g(x) can take any value in set A

so number of $g(x)=10^5\times 1$

17. Official Ans. by NTA (3)

Sol. Domain of $fog(x)=\sin^{-1}(g(x))$

$\Rightarrow |g(x)|\le 1,\ g(2)=\frac{3}{7}$

$\left|\dfrac{x^2-x-2}{2x^2-x-6}\right|\le 1$

$\left|\dfrac{(x+1)(x-2)}{(2x+3)(x-2)}\right|\le 1$

$\dfrac{x+1}{2x+3}\le 1$ and $\dfrac{x+1}{2x+3}\ge -1$

$\dfrac{x+1-2x-3}{2x+3}\le 0$ and $\dfrac{x+1+2x+3}{2x+3}\ge 0$

$\dfrac{x+2}{2x+3}\ge 0$ and $\dfrac{3x+4}{2x+3}\ge 0$

$x\in(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$

18. Official Ans. by NTA (4)

Sol. $\left|\dfrac{f(x)-f(y)}{(x-y)}\right|\le|(x-y)|$

$x-y=h$ let $\Rightarrow x=y+h$

$\lim\limits_{x\to 0}\left|\dfrac{f(y+h)-f(y)}{h}\right|\le 0$

$\Rightarrow |f'(y)|\le 0\Rightarrow f'(y)=0$

$\Rightarrow f(y)=k$ (constant)

and $f(0)=1$ given

So, $f(y)=1\Rightarrow f(x)=1$

19. Official Ans. by NTA (2)

Sol. $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$ ....(1)

replace x by $\frac{1}{x}$

$af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta x$ ....(2)

(1) + (2)

$(a+\alpha)f(x)+(a+\alpha)f\left(\frac{1}{x}\right)=x(b+\beta)+(b+\beta)\frac{1}{x}$

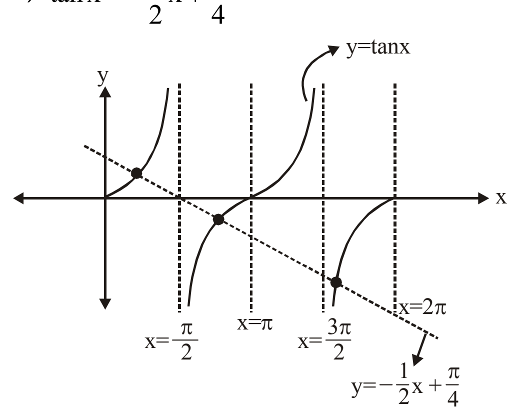
$\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=\dfrac{b+\beta}{a+\alpha}=\frac{2}{1}=2$

20. Official Ans. by NTA (1)

Sol. $x+2\tan x=\frac{\pi}{2}$

$\Rightarrow 2\tan x=\frac{\pi}{2}-x$

$\Rightarrow \tan x=-\frac{1}{2}x+\frac{\pi}{4}$



Number of soluitons of the given eauation is '3'.

Ans. (1)

21. Official Ans. by NTA (3)

Allen Ans. (1 or 2 or 3)

Sol. Given $y = 5^{(\log_a x)} = f(x)$

Interchanging x & y for inverse

$x = 5^{(\log_a y)} = y^{(\log_a 5)}$

option (1) or option (2)

Further, from given relation

$\log_5 y = \log_a x$

$\Rightarrow x = a^{(\log_5 y)} = y^{(\log_5 a)}$

$\Rightarrow x = y^{\left(\frac{1}{\log_a 5}\right)} = f^{-1}(y)$

option (3)

22. Official Ans. by NTA (3)

Sol. $f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$, domain [0, 1]

$f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$, domain [0, 1]

$g(x) - f(x) = \sqrt{1-x} - \sqrt{x}$, domain [0, 1]

$\frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{1-x}}$, domain [0, 1)

$\frac{g(x)}{f(x)} = \frac{\sqrt{1-x}}{\sqrt{x}}$, domain (0, 1]

So, common domain is (0, 1)

23. Official Ans. by NTA (3)

Sol. $f(x) = y = \frac{x-2}{x-3}$

$\therefore x = \frac{3y-2}{y-1}$

$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$

& $g(x) = y = 2x - 3$

$\therefore x = \frac{y+3}{2}$

$\therefore g^{-1}(x) = \frac{x+3}{2}$

$\because f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\therefore x^2 - 5x + 6 = 0 \;<\; x_1,\ x_2$

$\therefore$ sum of roots

$x_1 + x_2 = 5$

INVERSE TRIGONOMETRY FUNCTION

1. Official Ans. by NTA (4)

Sol. $\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$

For equation to be defined,

$x^2 + x \geq 0$

$\Rightarrow x^2 + x + 1 \geq 1$

$\therefore$ only possibility that the equation is defined

$x^2 + x = 0 \quad \Rightarrow x = 0; x = -1$

None of these values satisfy

$\therefore$ No of roots = 0

2. Official Ans. by NTA (2)

Sol. $\underbrace{\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{5}}_{x>0,y>0,xy<1} + \tan^{-1}\frac{5}{12}$

$\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}} = \underbrace{\tan^{-1}\frac{15}{8} + \tan^{-1}\frac{5}{12}}_{x>0,y>0,xy<1}$

$\tan^{-1}\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\cdot\frac{5}{12}} = \tan^{-1}\frac{220}{21}$

$\tan\left(\tan^{-1}\frac{220}{21}\right) = \frac{220}{21}$

3. Official Ans. by NTA (1)

Sol. $O \leq x^2 - x + 1 \leq 1$

$\Rightarrow x^2 - x \leq 0$

$\Rightarrow x \in [0, 1]$

Also, $0 < \sin^{-1}\left(\frac{2x-1}{2}\right) \leq \frac{\pi}{2}$

$\Rightarrow 0 < \frac{2x-1}{2} \leq 1$

$\Rightarrow 0 < 2x - 1 \leq 2$

$1 < 2x \leq 3$

$\frac{1}{2} < x \leq \frac{3}{2}$

Taking intersection

$x \in \left(\frac{1}{2}, 1\right]$

$\Rightarrow \alpha = \frac{1}{2}, \beta = 1$

$\Rightarrow \alpha + \beta = \frac{3}{2}$

4. Official Ans. by NTA (2)

Sol. $\sum_{r=1}^{50}\tan^{-1}\left(\frac{2}{4r^2}\right)=\sum_{r=1}^{50}\tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$\sum_{r=1}^{50}\tan^{-1}(2r+1)-\tan^{-1}(2r-1)$

$\tan^{-1}(101)-\tan^{-1}1 \Rightarrow \tan^{-1}\frac{50}{51}$

5. Official Ans. by NTA (2)

Sol. Given $a=(\sin^{-1}x)^2-(\cos^{-1}x)^2$

$=(\sin^{-1}x+\cos^{-1}x)(\sin^{-1}x-\cos^{-1}x)$

$=\frac{\pi}{2}\left(\frac{\pi}{2}-2\cos^{-1}x\right)$

$\Rightarrow 2\cos^{-1}x=\frac{\pi}{2}-\frac{2a}{\pi}$

$\Rightarrow \cos^{-1}(2x^2-1)=\frac{\pi}{2}-\frac{2a}{\pi}$

$\Rightarrow 2x^2-1=\cos\left(\frac{\pi}{2}-\frac{2a}{\pi}\right)$ option (2)

6. Official Ans. by NTA (4)

Sol. Let $g(x)=\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$g(x)\in[1,\sqrt{2}]$ for $x\in[0,\pi/2]$

$f(x)=\tan^{-1}(\sin x+\cos x)\in\left[\frac{\pi}{4},\tan^{-1}\sqrt{2}\right]$

$\tan(\tan^{-1}\sqrt{2}-\frac{\pi}{4})=\frac{\sqrt{2}-1}{1+\sqrt{2}}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=3-2\sqrt{2}$

7. Official Ans. by NTA (3)

Sol. $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$

$-1\le\frac{x-1}{x+1}\le 1\Rightarrow 0\le x<\infty$ ...(1)

$-1\le\frac{3x^2+x-1}{(x-1)^2}\le 1\Rightarrow x\in\left[\frac{-1}{4},\frac{1}{2}\right]\cup\{0\}$ ...(2)

(1) & (2)

$\Rightarrow$ Domain $=\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$

8. Official Ans. by NTA (3)

Sol. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$

$\Rightarrow(2\pi-5)+(6-2\pi)-(12-4\pi)$

$\Rightarrow 4\pi-11.$

9. Official Ans. by NTA (2)

Sol. $\operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right]$

$\operatorname{cosec}\left[\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right]$

$=\operatorname{cosec}\left[\tan^{-1}\left(\frac{56}{33}\right)\right]=\frac{65}{56}$ option (2)

10. Official Ans. by NTA (1)

Sol. $\tan^{-1}a+\tan^{-1}b=\frac{\pi}{4}$ $\quad 0<a,b<1$

$\Rightarrow\frac{a+b}{1-ab}=1$

$a+b=1-ab$

$(a+1)(b+1)=2$

Now $\left[a-\frac{a^2}{2}+\frac{a^3}{3}+...\right]+\left[b-\frac{b^2}{2}+\frac{b^3}{3}+...\right]$

$=\log_e(1+a)+\log_e(1+b)$

($\because$ expansion of $\log_e(1+x)$)

$=\log_e[(1+a)(1+b)]$

$=\log_e 2$

11. Official Ans. by NTA (3)

Sol. $\frac{\sin^{-1}x}{r}=a,\ \frac{\cos^{-1}x}{r}=b,\ \frac{\tan^{-1}y}{r}=c$

So, $a+b=\frac{\pi}{2r}$

$\cos\left(\frac{\pi c}{a+b}\right)=\cos\left(\frac{\pi\tan^{-1}y}{\frac{\pi}{2r}r}\right)$

$=\cos(2\tan^{-1}y)$, let $\tan^{-1}y=\theta$

$=\cos(2\theta)$

$=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-y^2}{1+y^2}$

12. Official Ans by NTA (3)

Sol. $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$

$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right)=\sin^{-1}x$

$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x$

$x=0,\ 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$ squaring we get

$16(25-9x^2)=625+9(25-16x^2)-150\sqrt{25-16x^2}$

$400=625+225-150\sqrt{25-16x^2}$

$\sqrt{25-16x^2}=3 \Rightarrow 25-16x^2=9$

$\Rightarrow x^2=1$

Put x = 0, 1, –1 in the original equation

We see that all values satisfy the original equation.

Number of solution = 3

13. Official Ans. by NTA (3)

Sol. $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

Divide by 3^{2r}

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r}.2+3}\right)$

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$

Let $\left(\frac{2}{3}\right)^r=t$

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$

$\sum_{r=1}^{k}\tan^{-1}\left(\frac{t-\frac{2t}{3}}{1+t.\frac{2t}{3}}\right)$

$\sum_{r=1}^{k}\left(\tan^{-1}(t)-\tan^{-1}\left(\frac{2t}{3}\right)\right)$

$\sum_{r=1}^{k}\left(\tan^{-1}\left(\frac{2}{3}\right)^r-\tan^{-1}\left(\frac{2}{3}\right)^{r+1}\right)$

$S_k=\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}$

$S_\infty=\lim_{k\to\infty}\left(\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}\right)$

$=\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}(0)$

$\therefore\ S_\infty=\tan^{-1}\left(\frac{2}{3}\right)=\cot^{-1}\left(\frac{3}{2}\right)$

14. Official Ans. by NTA (2)

Sol. Given equation

$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$

Now, $\sin^{-1}\left[x^2+\frac{1}{3}\right]$ is defined if

$-1\le x^2+\frac{1}{3}<2 \Rightarrow \frac{-4}{3}\le x^2<\frac{5}{3}$

$\Rightarrow \boxed{0\le x^2<\frac{5}{3}}$ ....(1)

and $\cos^{-1}\left[x^2-\frac{2}{3}\right]$ is defined if

$-1\le x^2-\frac{2}{3}<2 \Rightarrow \frac{-1}{3}\le x^2<\frac{8}{3}$

$\Rightarrow \boxed{0\le x^2<\frac{8}{3}}$ ....(2)

So, form (1) and (2) we can conclude

$\boxed{0\le x^2<\frac{5}{3}}$

Case - I if $0 \le x^2 < \frac{2}{3}$

$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$

$\Rightarrow x + \pi = x^2$

$\Rightarrow x^2 = \pi$

but $\pi \notin \left[0, \frac{2}{3}\right)$

$\Rightarrow$ No value of 'x'

Case - II if $\frac{2}{3} \le x^2 < \frac{5}{3}$

$\sin^{-1}(1) + \cos^{-1}(0) = x^2$

$\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = x^2$

$\Rightarrow x^2 = \pi$

but $\pi \notin \left[\frac{2}{3}, \frac{5}{3}\right)$

$\Rightarrow$ No value of 'x'

So, number of solutions of the equation is zero.

Ans.(2)

15. Official Ans. by NTA (1)

Sol. $\text{Cot}^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18) + ...$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$= \tan^{-1} 201 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{200}{202}\right)$

$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$

$\alpha = 1.01$

16. Official Ans. by NTA (1)

Sol. $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\frac{8}{31}$

Taking tangent both sides :-

$\frac{(x+1)+(x-1)}{1-(x^2-1)} = \frac{8}{31}$

$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow x = -8, \frac{1}{4}$

But, if $x = \frac{1}{4}$

$\tan^{-1}(x+1) \in \left(0, \frac{\pi}{2}\right)$

& $\cot^{-1}\left(\frac{1}{x-1}\right) \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \text{LHS} > \frac{\pi}{2}$ & $\text{RHS} < \frac{\pi}{2}$

(Not possible)

Hence, $x = -8$

17. Official Ans. by NTA (2)

Sol. $f(x) = \frac{\text{cosec}^{-1} x}{\sqrt{\{x\}}}$

Domain $\in (-\infty, -1] \cup [1, \infty)$

$\{x\} \ne 0$ so $x \ne$ integers

LIMIT

1. Official Ans. by NTA (3)

Sol. $\lim_{x \to 0}\left(2 - \cos x\sqrt{\cos x}\right)^{\frac{x+2}{x^2}}$

form: 1^{∞}

$= e^{\lim_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)\times(x+2)}$

Now $\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$= \lim_{x\to 0} \frac{\sin x\sqrt{\cos 2x} - \cos x \times \frac{1}{2\sqrt{\cos 2x}} \times (-2\sin 2x)}{2x}$$

(by L' Hospital Rule)

$$\lim_{x\to 0} \frac{\sin x \cos 2x + \sin 2x . \cos x}{2x}$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

So, $e^{\lim_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)(x+2)}$

$$= e^{\frac{3}{2}\times 2} = e^3$$

$$\Rightarrow \boxed{a=3}$$

2. Official Ans. by NTA (3)

Sol. $$\lim_{x\to 0} \frac{\alpha x\left(1 + x + \frac{x^2}{2}\right) - \beta\left(x - \frac{x^2}{2} + \frac{x^3}{3}\right) + \gamma x^2 (1-x)}{x^3}$$

$$\lim_{x\to 0} \frac{x(\alpha-\beta) + x^2\left(\alpha + \frac{\beta}{2} + \gamma\right) + x^3\left(\frac{\alpha}{2} - \frac{\beta}{3} - \gamma\right)}{x^3} = 10$$

For limit to exist

$$\alpha - \beta = 0,\ \alpha + \frac{\beta}{2} + \gamma = 0$$

$$\frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10 \ \ \ldots\ldots(i)$$

$$\beta = \alpha,\ \gamma = -3\frac{\alpha}{2}$$

Put in (i)

$$\frac{\alpha}{2} - \frac{\alpha}{3} + \frac{3\alpha}{2} = 10$$

$$\frac{\alpha}{6} + \frac{3\alpha}{2} = 10 \Rightarrow \frac{\alpha + 9\alpha}{6} = 10$$

$$\Rightarrow \alpha = 6$$

$$\alpha = 6, \beta = 6, \gamma = -9$$

$$\alpha + \beta + \gamma = 3$$

3. Official Ans. by NTA (4)

Sol. $$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{\left(\frac{2j}{n} - \frac{1}{n} + 8\right)}{\left(\frac{2j}{n} - \frac{1}{n} + 4\right)}$$

$$\int_0^1 \frac{2x+8}{2x+4}dx = \int_0^1 dx + \int_0^1 \frac{4}{2x+4}dx$$

$$= 1 + 4\frac{1}{2}\left(\ell n|2x+4|\right)\Big|_0^1$$

$$= 1 + 2\ell n\left(\frac{3}{2}\right)$$

4. Official Ans. by NTA (3)

Sol. $$\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}\right)$$

$$\left(\frac{\left(\sqrt[8]{1-\sin x} + \sqrt[8]{1+\sin x}\right)}{\sqrt[8]{1-\sin x} + \sqrt[8]{1+\sin x}}\right)$$

$$\left(\frac{\left(\sqrt[4]{1-\sin x} + \sqrt[4]{1+\sin x}\right)}{\sqrt[4]{1-\sin x} + \sqrt[4]{1+\sin x}}\right)$$

$$\left(\frac{\left(\sqrt[2]{1-\sin x} + \sqrt[2]{1+\sin x}\right)}{\sqrt[2]{1-\sin x} + \sqrt[2]{1+\sin x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{1-\sin x - (1+\sin x)}\right)$$

$$\left(\sqrt[8]{1-\sin x} + \sqrt[8]{1+\sin x}\right)$$

$$\left(\sqrt[4]{1-\sin x} + \sqrt[4]{1+\sin x}\right)$$

$$\left(\sqrt[2]{1-\sin x} + \sqrt[2]{1+\sin x}\right)$$

$$= \lim_{x\to 0} \frac{x}{(-2\sin x)}\left(\sqrt[8]{1-\sin x} + \sqrt[8]{1+\sin x}\right)$$

$$\left(\sqrt[4]{1-\sin x} + \sqrt[4]{1+\sin x}\right)$$

$$\left(\sqrt[2]{1-\sin x} + \sqrt[2]{1+\sin x}\right)$$

$$= \lim_{x\to 0}\left(-\frac{1}{2}\right)(2)\ (2)\ (2)\ \left\{\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right\} = -4$$

5. Official Ans. by NTA (1)

Sol. $$S = \lim_{x\to 2} \sum_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4}$$

$$S = \sum_{n=1}^{9} \frac{2}{4(n^2+3n+2)} = \frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$S = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{11}\right) = \frac{9}{44}$$

6. Official Ans. by NTA (3)

Sol. $\lim_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\frac{1\left(1+\frac{2(x^2+bx+c)}{1!}+\frac{2^2(x^2+bx+c)^2}{2!}+...\right)-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\frac{2(x^2+bx+1)^2}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\frac{2(x-\alpha)^2(x-\beta)^2}{(x-\beta)^2}$

$\Rightarrow 2(\beta-\alpha)^2=2(b^2-4c)$

7. Official Ans. by NTA (1)

Sol. $y=\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+.....$

$=(x^2+x^3+x^4+.......)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+....\right)$

$=\frac{x^2}{1-x}+x-\left(x+\frac{x^2}{2}+\frac{x^2}{3}+...\right)$

$=\frac{x}{1-x}+\ell n(1-x)$

$x=\frac{1}{2}\Rightarrow y=1-\ell n2$

$e^{1+y}=e^{1+1-\ell n2}$

$=e^{2-\ell n2}=\frac{e^2}{2}$

8. Official Ans. by NTA (2)

Sol. $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b \qquad (\infty-\infty)$

$\Rightarrow a>0$

Now, $\lim_{x\to\infty}\frac{(x^2-x+1-a^2x^2)}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow 1-a^2=0\Rightarrow a=1$

Now, $\lim_{x\to\infty}\frac{-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow \frac{-1}{1+a}=b\Rightarrow b=-\frac{1}{2}$

$(a,b)=\left(1,-\frac{1}{2}\right)$

9. Official Ans. by NTA (3)

Sol. $\lim_{x\to0}\frac{\sin^2(\pi\cos^4x)}{x^4}$

$\lim_{x\to0}\frac{1-\cos(2\pi\cos^4x)}{2x^4}$

$\lim_{x\to0}\frac{1-\cos(2\pi-2\pi\cos^4x)}{\left[2\pi(1-\cos^4x)\right]^2}4\pi^2.\frac{\sin^4x}{2x^4}(1+\cos^2x)^2$

$=\frac{1}{2}.4\pi^2.\frac{1}{2}(2)^2=4\pi^2$

10. Official Ans. by NTA (4)

Sol. $\alpha=\lim_{x\to\frac{\pi}{4}}\frac{\tan^3x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$; $\frac{0}{0}$ form

Using L Hopital rule

$\alpha=\lim_{x\to\frac{\pi}{4}}\frac{3\tan^2x\sec^2x-\sec^2x}{-\sin\left(x+\frac{\pi}{4}\right)}$

$\Rightarrow \alpha=-4$

$\beta=\lim_{x\to0}(\cos x)^{\cot x}=e^{\lim_{x\to0}\frac{(\cos x-1)}{\tan x}}$

$\beta=e^{\lim_{x\to0}\frac{-(1-\cos x)}{x^2}.\frac{x^2}{\left(\frac{\tan x}{x}\right)x}}$

$\beta=e^{\lim_{x\to0}\left(\frac{-1}{2}\right).\frac{x}{1}}=e^0\Rightarrow\beta=1$

$\alpha=-4$; $\beta=1$

If $ax^2+bx-4=0$ are the roots then

$16a-4b-4=0$ & $a+b-4=0$

$\Rightarrow a=1$ & $b=3$

11. Official Ans. by NTA (2)

Sol. $\lim_{x\to\frac{\pi}{4}}\frac{\frac{\pi}{4}\int_2^{\sec^2x}f(x)dx}{x^2-\frac{\pi^2}{16}}$

$\lim_{x\to\frac{\pi}{4}}\frac{\pi}{4}.\frac{\left[f(\sec^2x).2\sec x.\sec x\tan x\right]}{2x}$

$\lim_{x\to\frac{\pi}{4}}\frac{\pi}{4}f(\sec^2x).\sec^3x.\frac{\sin x}{x}$

$\frac{\pi}{4}f(2).(\sqrt{2})^3.\frac{1}{\sqrt{2}}\times\frac{4}{\pi}$

$\Rightarrow 2f(2)$

12. Official Ans. by NTA (7)

Sol. $f(n) = x^6 + 2x^4 + x^3 + 2x + 3$

$$\lim_{x\to 1}\frac{x^n f(1) - f(x)}{x-1} = 44$$

$$\lim_{x\to 1}\frac{9x^n - (x^6 + 2x^4 + x^3 + 2x + 3)}{x-1} = 44$$

$$\lim_{x\to 1}\frac{9nx^{n-1} - (6x^5 + 8x^3 + 3x^2 + 2)}{1} = 44$$

$\Rightarrow 9n - (19) = 44$

$\Rightarrow 9n = 63$

$\Rightarrow n = 7$

13. Official Ans. by NTA (1)

Sol. $$\lim_{n\to\alpha}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r(r+1)}\right)\right)$$

$$= \lim_{n\to\alpha}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)\right)$$

$$= \tan\left(\lim_{n\to\alpha}\sum_{r=1}^{n}\left[\tan^{-1}(r+1) - \tan^{-1}(r)\right]\right)$$

$$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1) - \frac{\pi}{4}\right)\right)$$

$$= \tan\left(\frac{\pi}{4}\right) = 1$$

14. Official Ans. by NTA (4)

Sol. Given limit is of 1^∞ form

So, $$l = \exp\left(\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{3}+........+\frac{1}{n}}{n}\right)$$

Now,

$$0 \le 1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{n} \le 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+....+\frac{1}{\sqrt{n}}$$

$$\le 2\sqrt{n}-1$$

So, $l = \exp(0)$ (from sandwich theorem)

$= 1$

15. Official Ans. by NTA (5)

Sol. $$\lim_{x\to 0}\frac{ax - (e^{4x}-1)}{ax(e^{4x}-1)} \qquad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0}\frac{ax - (e^{4x}-1)}{ax\,.\,4x} \qquad \boxed{\text{Use } \lim_{x\to 0}\frac{e^{4x}-1}{4x} = 1}$$

Apply L'Hospital Rule

$$= \lim_{x\to 0}\frac{a - 4e^{4x}}{8ax} \qquad \left(\frac{a-4}{0}\text{ form}\right)$$

limit exists only when $a - 4 = 0 \Rightarrow a = 4$

$$= \lim_{x\to 0}\frac{4 - 4e^{4x}}{32x}$$

$$= \lim_{x\to 0}\frac{1 - e^{4x}}{8x} \qquad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0}\frac{-e^{4x}.4}{8} = -\frac{1}{2} \Rightarrow b = -\frac{1}{2}$$

$$a - 2b = 4 - 2\left(-\frac{1}{2}\right)$$

$= 5$

16. Official Ans. by NTA (1)

Sol. $$L = \lim_{h\to 0} 2\left(\frac{\sqrt{3}\left(\frac{1}{2}\cosh + \frac{\sqrt{3}}{2}\sinh\right) - \left(\frac{\sqrt{3}}{2}\cosh - \frac{\sinh}{2}\right)}{(\sqrt{3}h)(\sqrt{3})}\right)$$

$$L = \lim_{h\to 0}\frac{4\sinh}{3h}$$

$$\Rightarrow L = \frac{4}{3}$$

17. Official Ans by NTA (3)

Sol. $$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-}(x)$$

$$\lim_{x\to 0^+}\frac{\cos^{-1}(1-x^2)\cdot\sin^{-1}(1-x)}{x(1-x)(1+x)}$$

$$\lim_{x\to 0^+}\frac{\cos^{-1}(1-x^2)}{x\cdot 1\cdot 1}\cdot\frac{\pi}{2}$$

Let $1 - x^2 = \cos\theta$

$$\frac{\pi}{2}\lim_{x\to 0^+}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$\frac{\pi}{2}\lim_{\theta\to0^+}\frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}}=\frac{\pi}{\sqrt{2}}$$

Now, $$\lim_{x\to0^-}\frac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$$

$$\lim_{x\to0^-}\frac{\frac{\pi}{2}\left(-\sin^{-1}x\right)}{(1+x)(2+x)(-x)}$$

$$\lim_{x\to0^-}\frac{\frac{\pi}{2}}{1\cdot2}\cdot\frac{\sin^{-1}x}{x}=\frac{\pi}{4}$$

$\Rightarrow$ RHL $\neq$ LHL

Function can't be continuous

$\Rightarrow$ No value of α exist

18. Official Ans. by NTA (1)

Sol. $$E=2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$$

$$E=\frac{2}{\ell n2}\int_0^1\ell n\left(1+\tan\frac{\pi x}{4}\right)dx \quad .....(i)$$

replacing $x\to1-x$

$$E=\frac{2}{\ell n2}\int_0^1\ell n\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$E=\frac{2}{\ell n2}\int_0^1\ell n\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E=\frac{2}{\ell n2}\int_0^1\ell n\left(1+\frac{1+\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

$$E=\frac{2}{\ell n2}\int_0^1\ell n\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E=\frac{2}{\ell n2}\int_0^1\left(\ell n2-\ell n\left(1+\tan\frac{\pi x}{4}\right)\right)dx \quad .....(ii)$$

equation (i) + (ii)

E = 1

19. Official Ans. by NTA (4)

Sol. $$\lim_{x\to0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$$

$$\Rightarrow \lim_{x\to0}\frac{a\left(1+x+\frac{x^2}{2!}...\right)-b\left(1-\frac{x^2}{2!}+...\right)+c\left(1-x+\frac{x^2}{2!}\right)}{\left(\frac{x\sin x}{x}\right)x}=2$$

$a-b+c=0 \quad .....(1)$

$a-c=0 \quad .....(2)$

$\&\ \frac{a+b+c}{2}=2$

$\Rightarrow \boxed{a+b+c=4}$

20. Official Ans. by NTA (1)

Sol. We know that

$r\le[r]<r+1$

and $2r\le[2r]<2r+1$

$3r\le[3r]<3r+1$

$\vdots \quad \vdots \quad \vdots$

$nr\le[nr]<nr+1$

$r+2r+....+nr$

$\le[r]+[2r]+....+[nr]<(r+2r+....+nr)+n$

$$\frac{\frac{n(n+1)}{2}\cdot r}{n^2}\le\frac{[r]+[2r]+......+[nr]}{n^2}<\frac{\frac{n(n+1)}{2}r+n}{n^2}$$

Now,

$$\lim_{n\to\infty}\frac{n(n+1)\cdot r}{2\cdot n^2}=\frac{r}{2}$$

and $$\lim_{n\to\infty}\frac{\frac{n(n+1)r}{2}+n}{n^2}=\frac{r}{2}$$

So, by Sandwich Theorem, we can conclude that

$$\lim_{n\to\infty}\frac{[r]+[2r]+......+[nr]}{n^2}=\frac{r}{2}$$

Ans. (1)

21. Official Ans. by NTA (1)

Sol. $$\lim_{\theta\to0}\frac{\tan\left(\pi\left(1-\sin^2\theta\right)\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$=\lim_{\theta\to0}\frac{-\tan\left(\pi\sin^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$=\lim_{\theta\to0}-\left(\frac{\tan\left(\pi\sin^2\theta\right)}{\pi\sin^2\theta}\right)\left(\frac{2\pi\sin^2\theta}{\sin\left(2\pi\sin^2\theta\right)}\right)\times\frac{1}{2}$$

$$=\frac{-1}{2}$$ **Option (1)**

22. Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 0^+}\dfrac{\cos^{-1}x}{(1-x^2)}\times\dfrac{\sin^{-1}x}{x}=\dfrac{\pi}{2}$

23. Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 0}\dfrac{\left(x+\dfrac{x^3}{3!}...\right)-\left(x-\dfrac{x^3}{3}...\right)}{3x^3}=\dfrac{1}{6}$

So 6L + 1 = 2

CONTINUITY

1. Official Ans. by NTA (2)

Sol. Continuous at x = 0

$f(0^+) = f(0^-) \Rightarrow a - 1 = 0 - e^0$

$\Rightarrow a = 0$

Continuous at x = 1

$f(1^+) = f(1^-)$

$\Rightarrow 2(1) - b = a + (-1)$

$\Rightarrow b = 2 - a + 1 \Rightarrow b = 3$

$\therefore a + b = 3$

2. Official Ans. by NTA (1)

Sol. $\lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+} e^{\frac{\tan(x-2)}{x-2}}=e^1$

$\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-}\dfrac{-\lambda(x-2)(x-3)}{\mu(x-2)(x-3)}=-\dfrac{\lambda}{\mu}$

For continuity $\mu = e = -\dfrac{\lambda}{\mu} \Rightarrow \mu = e, \lambda = -e^2$

$\lambda + \mu = e(-e + 1)$

3. Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to 0} f(x)=b$

$\lim\limits_{x\to 0^+} xe^{\frac{\cot 4x}{\cot 2x}}=e^{\frac{1}{2}}=b$

$\lim\limits_{x\to 0^-}(1+|\sin x|)^{\frac{3a}{|\sin x|}}=e^{3a}=e^{\frac{1}{2}}$

$\lim\limits_{x\to 0^-}(1+|\sin x|)^{\frac{3a}{|\sin x|}}=e^{3a}=e^{\frac{1}{2}}$

$a=\dfrac{1}{6}\Rightarrow 6a=1$

$(6a + b^2) = (1 + e)$

4. Official Ans. by NTA (14)

Sol. $f(x)=\begin{cases} a\sin\dfrac{\pi}{2}(x-1), & x\le 0\\ \dfrac{\tan 2x-\sin 2x}{bx^3}, & x>0\end{cases}$

For continuity at '0'

$\lim\limits_{x\to 0^+} f(x)=f(0)$

$\Rightarrow \lim\limits_{x\to 0^+}\dfrac{\tan 2x-\sin 2x}{bx^3}=-a$

$\Rightarrow \lim\limits_{x\to 0^+}\dfrac{\dfrac{8x^3}{3}+\dfrac{8x^3}{3!}}{bx^3}=-a$

$\Rightarrow 8\left(\dfrac{1}{3}+\dfrac{1}{3!}\right)=-ab$

$\Rightarrow 4 = -ab$

$\Rightarrow 10 - ab = 14$

5. Official Ans. by NTA (1)

Sol. If $f(x)$ is continuous at x = 0, RHL = LHL = $f(0)$

$\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\dfrac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}\cdot\dfrac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$

(Rationalisation)

$\lim\limits_{x\to 0^+}-\dfrac{2\sin^2 x}{x^2}\cdot\left(\sqrt{x^2+1}+1\right)=-4$

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}\dfrac{1}{x}\ell n\left(\dfrac{1+\dfrac{x}{a}}{1-\dfrac{x}{b}}\right)$

$\lim\limits_{x\to 0^-}\dfrac{\ell n\left(1+\dfrac{x}{a}\right)}{\left(\dfrac{x}{a}\right).a}+\dfrac{\ell n\left(1-\dfrac{x}{b}\right)}{\left(-\dfrac{x}{b}\right).b}$

$=\dfrac{1}{a}+\dfrac{1}{b}$

So $\dfrac{1}{a}+\dfrac{1}{b}=-4=k$

$\Rightarrow\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{4}{k}=-4-1=-5$

6. Official Ans. by NTA (2)

Sol. $f(x) = [x]\,|x^2 - 1| + \sin\dfrac{\pi}{[x+3]} - [x+1]$

$$f(x)=\begin{cases} 3-2x^2, & -2<x<-1 \\ x^2, & -1\le x<0 \\ \dfrac{\sqrt{3}}{2}+1 & 0\le x<1 \\ x^2+1+\dfrac{1}{\sqrt{2}}, & 1\le x<2 \end{cases}$$

discontinuous at x = 0 , 1

7. Official Ans. by NTA (3)

Sol. For x = n, $n \in Z$

$$\text{LHL} = \lim_{x\to n^-} f(x) = \lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$$

$= 0$

$$\text{RHL} = \lim_{x\to n^+} f(x) = \lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$$

$= 0$

$f(n) = 0$

$\Rightarrow$ LHL = RHL = $f(n)$

$\Rightarrow$ $f(x)$ is continuous for every real x.

8. Official Ans. by NTA (2)

Sol. $f(x)$ is continuous on R

$\Rightarrow f(1^-) = f(1) = f(1^+)$

$|a + 1 + b| = \lim\limits_{x\to 1}\sin(\pi x)$

$|a + 1 + b| = 0 \Rightarrow a + b = -1$ ...(1)

$\Rightarrow$ Also $f(-1^-) = f(-1) = f(-1^+)$

$$\lim_{x\to -1} 2\sin\left(\frac{-\pi x}{2}\right) = |a-1+b|$$

$|a - 1 + b| = 2$

Either $a - 1 + b = 2$ or $a - 1 + b = -2$

$a + b = 3$ ...(2) or $a + b = -1$ ...(3)

from (1) and (2) $\Rightarrow a + b = 3 = -1$(reject)

from (1) and (3) $\Rightarrow a + b = -1$

9. Official Ans by NTA (1)

Sol. $g[f(x)] = \begin{cases} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x)\ge 0 \end{cases}$

$$g[f(x)] = \begin{cases} x+a+1 & x+a<0\ \&\ x<0 \\ |x-1|+1 & |x-1|<0\ \&\ x\ge 0 \\ (x+a-1)^2+b & x+a\ge 0\ \&\ x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0\ \&\ x\ge 0 \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a)\ \&\ x\in(-\infty,0) \\ |x-1|+1 & x\in\phi \\ (x+a-1)^2+b & x\in[-a,\infty)\ \&\ x\in(-\infty,0) \\ (|x-1|-1)^2+b & x\in R\ \&\ x\in[0,\infty) \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a) \\ (x+a-1)^2+b & x\in[-a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty) \end{cases}$$

g(f(x)) is continuous

at $x = -a$ & at $x = 0$

$1 = b + 1$ & $(a - 1)^2 + b = b$

$b = 0$ & $a = 1$

$\Rightarrow a + b = 1$

10. Official Ans. by NTA (6)

Sol. $\lim\limits_{x\to 0}\dfrac{\cos(\sin x)-\cos x}{x^4} = f(0)$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4} = \frac{1}{K}$$

$$\Rightarrow \lim_{x\to 0} 2\left(\frac{\sin x+x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right) = \frac{1}{K}$$

$$\Rightarrow 2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6} = \frac{1}{K}$$

$\Rightarrow K = 6$

11. Official Ans. by NTA (4)

Sol. $f(x)$ is continuous at x = 0

$\lim\limits_{x\to 0^+} f(x) = f(0) = \lim\limits_{x\to 0^-} f(x)$ ...(1)

$f(0) = b$ ...(2)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\frac{\sin(a+1)x}{2x}+\frac{\sin 2x}{2x}\right)$$

$= \dfrac{a+1}{2}+1$ ...(3)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x\to 0^+} \frac{(x+bx^3-x)}{bx^{5/2}\left(\sqrt{x+bx^3}+\sqrt{x}\right)}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+bx^2}+1\right)} = \frac{1}{2} \ ...(4)$$

Use (2), (3) & (4) in (1)

$$\frac{1}{2} = b = \frac{a+1}{2} + 1$$

$$\Rightarrow b = \frac{1}{2},\ a = -2$$

$$a + b = \frac{-3}{2}$$

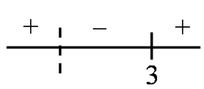
DIFFERENTIABILITY

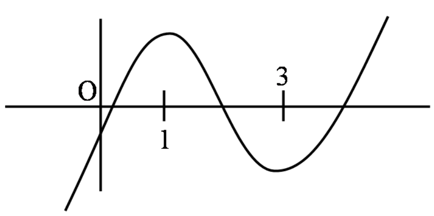
1. Official Ans. by NTA (1)

Sol. $f(x) = x^3 - 6x^2 + 9x - 3$

$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$

$f(1) = 1\ f(3) = -3$





$$g(x) = \begin{cases} f(x) & 0 \le x \le 1 \\ 0 & 1 \le x \le 3 \\ -1 & 3 < x \le 4 \end{cases}$$

g(x) is continuous

$$g'(x) = \begin{cases} 3(x-1)(x-3) & 0 \le x \le 1 \\ 0 & 1 \le x \le 3 \\ -1 & 3 < x \le 4 \end{cases}$$

g(x) is non-differentiable at x = 3

2. Official Ans. by NTA (1)

Sol. For continuity

$$\lim_{x\to 0} \frac{x^3}{4\sin^4 x}\left(\ell n\left(1+2xe^{-2x}\right) - 2\ell n\left(1-xe^{-x}\right)\right)$$

$= \alpha$

$$\lim_{x\to 0} \frac{1}{4x}\left[2xe^{-2x} + 2xe^{-x}\right] = \alpha$$

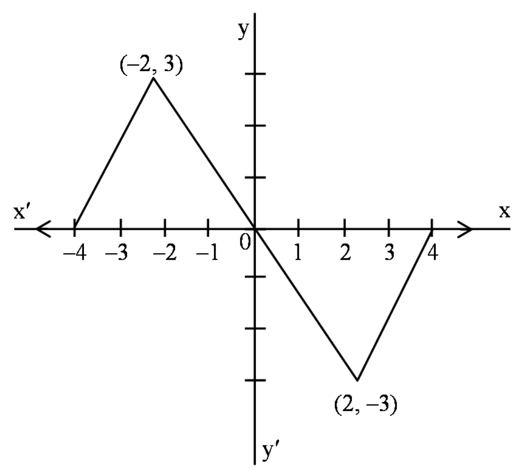
$$= \frac{1}{4}(4) = \alpha = 1$$

3. Official Ans. by NTA (4)

Sol. $$f(x-2)\begin{cases} \frac{3x}{2} & -4 \le x \le -2 \\ -\frac{3x}{2} & -2 < x \le 0 \\ 0 & x \in (-\infty,-4) \cup (0,+\infty) \end{cases}$$

$$f(x-2)\begin{cases} \frac{3x}{2} & 0 \le x \le 2 \\ -\frac{3x}{2}+6 & 2 \le x \le 4 \\ 0 & x \in (-\infty,0) \cup (4,+\infty) \end{cases}$$

$$g(x) = f(x+2) - f(x-2)\begin{cases} \frac{3x}{2}+6 & -4 \le x \le -2 \\ -\frac{3x}{2} & -2 < x < 2 \\ \frac{3x}{2}-6 & 2 \le x \le 4 \\ 0 & x \in (-\infty,-4) \cup (4,\infty) \end{cases}$$



n = 0

m = 4 ⇒ (n + m = 4)

4. Official Ans. by NTA (4)

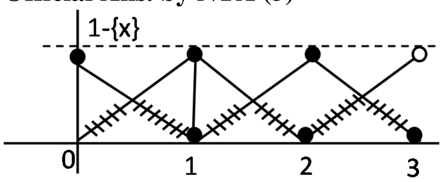
Sol. Apply L'Hopital Rule

$$\lim_{x\to 2}\left(\frac{2xf(2)-4f'(x)}{1}\right)$$

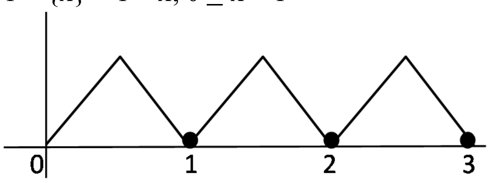
$$=\frac{4(4)-4}{1}=12$$

5. Official Ans. by NTA (5)

Sol.



$1-\{x\}=1-x;\ 0\le x<1$

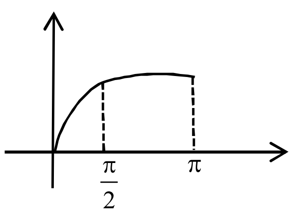


Non differentiable at

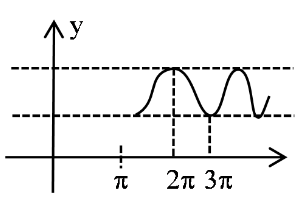
$x=\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2}$

6. Official Ans. by NTA (2)

Sol. Graph of $\max\{\ \sin t:\ 0\le t\le x\ \}$ in $x\in[0,\pi]$

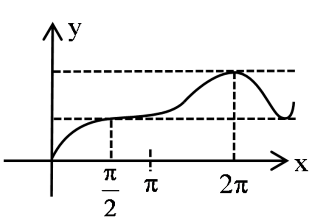


& graph of cos for $x\in[\pi,\infty)$



So graph of

$$f(x)=\begin{cases}\max\{\sin t:0\le t\le x, & 0\le x\le\pi\\ 2+\cos x & x>h\end{cases}$$

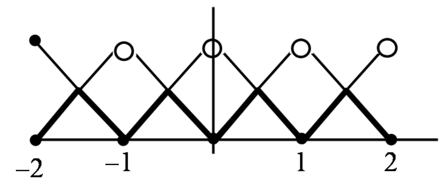


f(x) is differentiable everywhere in $(0,\infty)$

7. Official Ans. by NTA (1)

Sol. $\min\{x-[x],\ 1-x+[x]\}$

$h(x)=\min\{x-[x],\ 1-[x-[x])\}$



$\Rightarrow$ always continuous in [–2, 2]

but non differentiable at 7 Points

8. Official Ans. by NTA (3)

Sol. $f(x)=|(x-3)(x+1)|\ .\ e^{(3x-2)^2}$

$$f(x)=\begin{cases}(x-3)(x+1).e^{(3x-2)^2} & ;\ x\in(3,\infty)\\ -(x-3)(x+1).e^{(3x-2)^2} & ;\ x\in[-1,3]\\ (x-3).(x+1).e^{(3x-2)^2} & ;\ x\in(-\infty,-1)\end{cases}$$

Clearly, non-differentiable at x = –1 & x = 3.

9. Official Ans. by NTA (2)

Sol. $f(x)=|2x+1|-3|x+2|+|x^2+x-2|$

$=|2x+1|-3|x+2|+|x+2||x-1|$

$=|2x+1|+|x+2|(|x-1|-3)$

Critical points are $x=\frac{-1}{2}$, –2, –1

but x = –2 is making a zero.

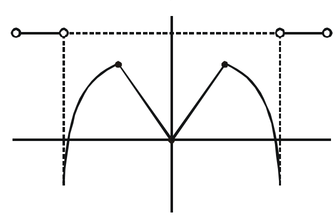
twice in product so, points of non differentability are $\boxed{x=\frac{-1}{2}\text{ and }x=-1}$

$\therefore$ Number of points of non-differentiability = $\boxed{2}$

10. Official Ans. by NTA (5)

Sol. $f(x)=\begin{cases}\min\{|x|,\ 2-x^2\} & ,\ -2\le x\le 2\\ [|x|] & ,\ 2<|x|\le 3\end{cases}$

$\Rightarrow x\in[-3,-2)\cup(2,3]$



Number of points of non-differentiability in (–3, 3) = 5

11. Official Ans. by NTA (2)

Sol. $f(g(x))=\begin{cases} g(x)+2, & g(x)<0 \\ (g(x))^2, & g(x)\ge 0 \end{cases}$

$$=\begin{cases} x^3+2, & x<0 \\ x^6, & x\in[0,1) \\ (3x-2)^2, & x\in[1,\infty) \end{cases}$$

$$(f\text{o}g(x))'=\begin{cases} 3x^2, & x<0 \\ 6x^5, & x\in(0,1) \\ 2(3x-2)\times 3, & x\in(1,\infty) \end{cases}$$

At 'O'

L.H.L. $\neq$ R.H.L. (Discontinuous)

At '1'

L.H.D. = 6 = R.H.D.

$\Rightarrow$ $f$og(x) is differentiable for $x\in\mathbb{R}-\{0\}$

12. Official Ans. by NTA (4)

Sol. $f(x)=\begin{cases} \dfrac{1}{|x|}, & |x|\ge 1 \\ ax^2+b, & |x|<1 \end{cases}$

at x = 1 function must be continuous

So, 1 = a + b ...(1)

differentiability at x = 1

$$\left(-\frac{1}{x^2}\right)_{x=1}=(2ax)_{x=1}$$

$\Rightarrow -1=2a\Rightarrow a=-\dfrac{1}{2}$

$(1)\Rightarrow b=1+\dfrac{1}{2}=\dfrac{3}{2}$

13. Official Ans. by NTA (3)

Sol. If $f(x+y)=f(x).f(y)$ & $f'(0)=3$ then

$f(x)=a^x\Rightarrow f'(x)=a^x.\ell na$

$\Rightarrow f'(0)=\ell na=3\Rightarrow a=e^3$

$\Rightarrow f(x)=(e^3)^x=e^{3x}$

$$\lim_{x\to 0}\frac{f(x)-1}{x}=\lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\times 3\right)=1\times 3=3$$

METHOD OF DIFFERENTIATION

1. Official Ans. by NTA (39)

Sol. $f(x)=\begin{cases} \dfrac{P(x)}{\sin(x-2)}, & x\neq 2 \\ 7, & x=2 \end{cases}$

P"(x) = const. $\Rightarrow$ P(x) is a 2 degree polynomial

$f(x)$ is cont. at x = 2

$f(2^+)=f(2^-)$

$$\lim_{x\to 2^+}\frac{P(x)}{\sin(x-2)}=7$$

$$\lim_{x\to 2^+}\frac{(x-2)(ax+b)}{\sin(x-2)}=7\Rightarrow \boxed{2a+b=7}$$

P(x) = (x – 2)(ax + b)

$P(3)=(3-2)(3a+b)=9\Rightarrow \boxed{3a+b=9}$

$\boxed{a=2,\ b=3}$

P(5) = (5 – 2)(2.5 + 3) = 3.13 = 39

2. Official Ans. by NTA (3)

Sol. $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\dfrac{1-x}{x}}\right)\right)$

$$\cot^{-1}\sqrt{\frac{1-x}{x}}=\sin^{-1}\sqrt{x}$$

or $f(x)=\cos(2\tan^{-1}\sqrt{x})$

$$=\cos\tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$$

$$f(x)=\frac{1-x}{1+x}$$

Now $f'(x)=\dfrac{-2}{(1+x)^2}$

or $f'(x)(1-x)^2=-2\left(\dfrac{1-x}{1+x}\right)^2$

or $(1-x)^2f'(x)+2(f(x))^2=0$.

3. Official Ans. by NTA (40)

Sol. $\ln(x + y) = 4xy$ (At $x = 0, y = 1$)

$x + y = e^{4xy}$

$$\Rightarrow 1+\frac{dy}{dx}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)$$

At $x = 0$ $\boxed{\frac{dy}{dx}=3}$

$$\frac{d^2y}{dx^2}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)^2+e^{4xy}\left(4x\frac{d^2y}{dx^2}+4y\right)$$

At $x = 0$, $\frac{d^2y}{dx^2} = e^0(4)^2 + e^0(24)$

$$\Rightarrow \frac{d^2y}{dx^2}=40$$

4. Official Ans. by NTA (17)

Sol. $y^{\frac{1}{4}}+\frac{1}{y^{\frac{1}{4}}}=2x$

$$\Rightarrow \left(y^{\frac{1}{4}}\right)^2-2xy^{\left(\frac{1}{4}\right)}+1=0$$

$$\Rightarrow y^{\frac{1}{4}}=x+\sqrt{x^2-1} \text{ or } x-\sqrt{x^2-1}$$

So, $\frac{1}{4}\frac{1}{y^{\frac{3}{4}}}\frac{dy}{dx}=1+\frac{x}{\sqrt{x^2-1}}$

$$\Rightarrow \frac{1}{4}\frac{1}{y^{3/4}}\frac{dy}{dx}=\frac{y^{\frac{1}{4}}}{\sqrt{x^2-1}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4y}{\sqrt{x^2-1}} \quad \ldots(1)$$

Hence, $\frac{d^2y}{dx^2}=4\frac{\left(\sqrt{x^2-1}\right)y'-\frac{yx}{\sqrt{x^2-1}}}{x^2-1}$

$$\Rightarrow (x^2-1)y''=4\frac{(x^2-1)y'-xy}{\sqrt{x^2-1}}$$

$$\Rightarrow (x^2-1)y''=4\left(\sqrt{x^2-1}y'-\frac{xy}{\sqrt{x^2-1}}\right)$$

$$\Rightarrow (x^2-1)y''=4\left(4y-\frac{xy'}{4}\right) \text{ (from I)}$$

$$\Rightarrow (x^2-1)y''+xy'-16y=0$$

So, $|\alpha-\beta| = 17$

5. Official Ans. by NTA (1)

Sol. $$y(x) = \cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right]$$

$$y(x) = \cot^{-1}\left(\tan\frac{x}{2}\right)=\frac{\pi}{2}-\frac{x}{2}$$

$$y'(x)=\frac{-1}{2}$$

6. Official Ans. by NTA (1)

Sol. $$\lim_{x\to 0^+}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}=\lim_{x\to 0^+}\frac{(\sin x)2x}{3x^2}$$

$$=\lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)\times\frac{2}{3}=\frac{2}{3}$$

7. Official Ans. by NTA (2)

Sol. $f'(a) = 2$, $f(a) = 4$

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$$

$$\Rightarrow \lim_{x\to a}\frac{f(a)-af'(x)}{1} \quad \text{(Lopitals rule)}$$

$= f(a) - af'(a)$

$= 4 - 2a$

8. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Since, slope is maximum so,

$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$x = 2, 3$

at $x = 2$

$\frac{d^3y}{dx^3}=12x-30$

at $x = 2$, $\frac{d^3y}{dx^3}<0$

So, maxima

$y = \frac{1}{2} \times 16 - 5 \times 8 + 18 \times 4 - 19 \times 2$

$= 8 - 40 + 72 - 38 = 80 - 78 = 2$

point (2, 2)

9. Official Ans. by NTA (2)

Sol. $f(x)f''(x)-\left(f'(x)\right)^2=0$

$\frac{f''(x)}{f'(x)}=\frac{f'(x)}{f(x)}$

$\ln\left(f'(x)\right)=\ln f(x)+\ln c$

$f'(x)=cf(x)$

$\frac{f'(x)}{f(x)}=c$

$\ln f(x) = cx + k_1$

$f(x) = ke^{cx}$

$f(0) = 1 = k$

$f'(0)=c=2$

$f(x) = e^{2x}$

$f(1) = e^2 \in (6, 9)$

10. Official Ans by NTA (1)

Sol. $l nf(x + 1) = l n(xf(x))$

$l nf(x + 1) = l nx + l nf(x)$

$\Rightarrow g(x + 1) = l nx + g(x)$

$\Rightarrow g(x + 1) - g(x) = l nx$

$\Rightarrow g''(x + 1) - g''(x) = -\frac{1}{x^2}$

Put x = 1, 2, 3, 4

$g''(2) - g''(1) = -\frac{1}{1^2}$ ...(1)

$g''(3) - g''(2) = -\frac{1}{2^2}$ ...(2)

$g''(4) - g''(3) = -\frac{1}{3^2}$ ...(3)

$g''(5) - g''(4) = -\frac{1}{4^2}$ ...(4)

Add all the equation we get

$g''(5) - g''(1) = -\frac{1}{1^2}-\frac{1}{2^2}-\frac{1}{3^2}-\frac{1}{4^2}$

$|g''(5)-g''(1)|=\frac{205}{144}$

11. Official Ans. by NTA (481)

Sol. $f(x)=\sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ at x= 1 ; $2^{2x} = 4$

for $\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$;

Let $\tan^{-1} x = \theta$; $\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore \sin\left(\cos^{-1}\cos 2\theta\right)=\sin 2\theta$

$\left\{\begin{array}{ll}\text{If} \quad x>1\Rightarrow\frac{\pi}{2}>\theta>\frac{\pi}{4} \\ \therefore \quad \pi>2\theta>\frac{\pi}{2}\end{array}\right\}$

$=2\sin\theta\cos\theta=\frac{2\tan\theta}{1+\tan^2\theta}$

$=\frac{2x}{1+x^2}$

Hence, $f(x) = \frac{2\cdot 2^x}{1+2^{2x}}$

$\therefore f'(x)=\frac{(1+2^{2x})(2.2^x\ln 2)-2^{2x}\cdot 2\cdot\ln 2\cdot 2\cdot 2^x}{(1+2^{2x})}$

$\therefore f^1(1)=\frac{20\ln 2-32\ln 2}{25}=-\frac{12}{25}\ln 2$

So, a = 25, b = 12 $\Rightarrow |a^2 - b^2| = 25^2 - 12^2$

$= 625 - 144$

$= 481$

AOD (TANGENT & NORMAL)

1. Official Ans. by NTA (3)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $x^2 + y^2 = ab$

$\frac{2x_1}{a^2}+\frac{2y_1y'}{b^2}=0$

$\Rightarrow y_1'=\frac{-x_1}{a^2}\frac{b^2}{y_1}$ …(1)

$\therefore 2x_1 + 2y_1y' = 0$

$\Rightarrow y_2'=\frac{-x_1}{y_1}$ …(2)

Here (x_1y_1) is point of intersection of both curves

$$\therefore\ x_1^2=\frac{a^2b}{a+b}\ ,\ y_1^2=\frac{ab^2}{a+b}$$

$$\therefore\ \tan\theta=\left|\frac{y_1'-y_2'}{1+y_1'y_2'}\right|=\left|\frac{\frac{-x_1b^2}{a^2y_1}+\frac{x_1}{y_1}}{1+\frac{x_1^2b^2}{a^2y_1^2}}\right|$$

$$\tan\theta=\left|\frac{-b^2x_1y_1+a^2x_1y_1}{a^2y_1^2+b^2x_1^2}\right|$$

$$\tan\theta=\left|\frac{a-b}{\sqrt{ab}}\right|$$

2. Official Ans. by NTA (3)

Sol. $a+b+c=2$ ...(1)

and $\left.\frac{dy}{dx}\right|_{(0,0)}=1$

$$2ax+b\big|_{(0,0)}=1$$

$b=1$

Curve passes through origin

So, $c=0$

and $a=1$

3. Official Ans. by NTA (1)

Sol. Slope of tangent at $P(t, t^3)=\left.\frac{dy}{dx}\right]_{(t,t^3)}$

$=(3x^2)_{x=t}=3t^2$

So equation tangent at $P(t, t^3)$:

$y-t^3=3t^2(x-t)$

for point of intersection with $y=x^3$

$x^3-t^3=3t^2x-3t^3$

$\Rightarrow\ (x-t)(x^2+xt+t^2)=3t^2(x-t)$

for $x\neq t$

$x^2+xt+t^2=3t^2$

$\Rightarrow\ x^2+xt-2t^2=0\ \Rightarrow (x-t)(x+2t)=0$

So for Q : $x=-2t$, $Q(-2t, -8t^3)$

ordinate of required point : $\frac{2t^3-8t^3}{2+1}=-2t^3$

4. Official Ans. by NTA (4)

Sol. $x=y^4$ $xy=k$

for intersection $y^5=k$ ...(1)

Also $x=y^4$

$$\Rightarrow\ 1=4y^3\frac{dy}{dx}\ \Rightarrow\ \frac{dy}{dx}=\frac{1}{4y^3}$$

for $xy=k\Rightarrow\ x=\frac{k}{y}$

$$\Rightarrow\ 1=-\frac{k}{y^2}\cdot\frac{dy}{dx}$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{-y^2}{k}$$

$\because$ Curve cut orthogonally

$$\Rightarrow\ \frac{1}{4y^3}\times\left(\frac{-y^2}{k}\right)=-1$$

$$\Rightarrow\ y=\frac{1}{4k}$$

$\therefore$ from (1) $y^5=k$

$$\Rightarrow\ \frac{1}{(4k)^5}=k$$

$\Rightarrow 4=(4k)^6$

5. Official Ans. by NTA (406)

Sol. $y(x)=\int_0^x(2t^2-15t+10)\,dt$

$$y'(x)\big]_{x=a}=\left[2x^2-15x+10\right]_a=2a^2-15a+10$$

Slope of normal $=-\frac{1}{3}$

$\Rightarrow\ 2a^2-15a+10=3\Rightarrow a=7$

& $a=\frac{1}{2}$ (rejected)

$$b=y(7)=\int_0^7(2t^2-15t+10)dt$$

$$=\left[\frac{2t^3}{3}-\frac{15t^2}{2}+10t\right]_0^7$$

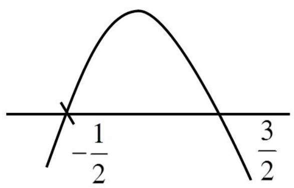
$\Rightarrow\ 6b=4\times7^3-45\times49+60\times7$

$|a+6b|=406$

AOD (MONOTONICITY)

1. Official Ans. by NTA (3)

Sol. $f'(x)\begin{cases}-4x^2+4x+3 & x>0\\ 3e^x(1+x) & x\le 0\end{cases}$



For x > 0, $f'(x) = -4x^2 + 4x + 3$

f(x) is increasing in $\left(-\frac{1}{2},\frac{3}{2}\right)$

For x ≤ 0, $f'(x) = 3e^x(1 + x)$

$f'(x) > 0 \ \forall \ x \in (-1, 0)$

⇒ f(x) is increasing in (–1, 0)

So, in complete domain, f(x) is increasing in $\left(-1,\frac{3}{2}\right)$

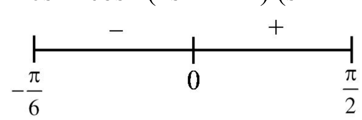
2. Official Ans. by NTA (4)

Sol. $f(x) = 3\sin^4x + 10\sin^3x + 6\sin^2x - 3,\ x\in\left[-\frac{\pi}{6},\frac{\pi}{2}\right]$

$f'(x) = 12\sin^3x\cos x + 30\sin^2x\cos x + 12\sin x\cos x$

$= 6\sin x\cos x\,(2\sin^2x + 5\sin x + 2)$

$= 6\sin x\cos x\,(2\sin x + 1)(\sin + 2)$



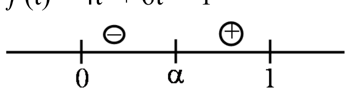
Decreasing in $\left(-\frac{\pi}{6},0\right)$

3. Official Ans. by NTA (3)

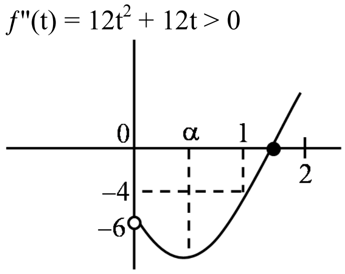
Sol. Let $e^x = t > 0$

$f(t) = t^4 + 2t^3 - t - 6 = 0$

$f'(t) = 4t^3 + 6t^2 - 1$



$f''(t) = 12t^2 + 12t > 0$



$f(0) = -6,\ f(1) = -4,\ f(2) = 24$

⇒ Number of real roots = 1

4. Official Ans. by NTA (2)

Sol. $f(x) = x^2 + ax + 1$

$f'(x) = 2x + a$

when $f(x)$ is increasing on [1, 2]

$2x + a \ge 0 \quad \forall\ x \in [1, 2]$

$a \ge -2x\ \forall\ x \in [1, 2]$

R = –4

when $f(x)$ is decreasing on [1, 2]

$2x + a \le 0\ \ \forall\ x \in [1, 2]$

$a \le -2\ \ \forall\ x \in [1, 2]$

S = –2

$|R - S| = |-4 + 2| = 2$

5. Official Ans. by NTA (2)

Sol. $f(0) = 0 \quad f(1) = 1$ and $f(2) = 2$

Let $h(x) = f(x) - x$ has three roots

By Rolle's theorem $h'(x) = f'(x) - 1$ has at least two roots

$h''(x) = f''(x) = 0$ has at least one roots

6. Official Ans. by NTA (1)

Sol. $f(x) = x^3 - 6x^2 + ax + b$

$f(2) = 8 - 24 + 2a + b = 0$

$2a + b = 16 \quad \ldots(1)$

$f(4) = 64 - 96 + 4a + b = 0$

$4a + b = 32 \quad \ldots(2)$

Solving (1) and (2)

a = 8, b = 0

$\boxed{f(x) = x^3 - 6x^2 + 8x}$

$f(x) = x^3 - 6x^2 + 8x$

$f'(x) = 3x^2 - 12x + 8$

$f''(x) = 6x - 12$

⇒ f'(x) is ↑ for x > 2, and f'(x) is ↓ for x < 2

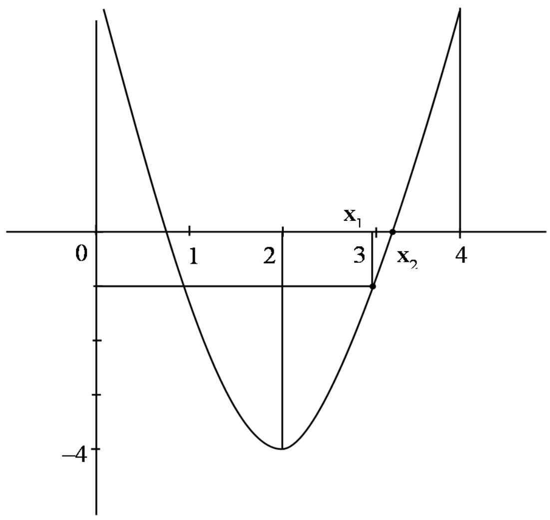
$f'(2) = 12 - 24 + 8 = -4$

$f'(4) = 48 - 48 + 8 = 8$

$f'(x) = 3x^2 - 12x + 8$

vertex (2, – 4)

$f'(2) = -4,\ f'(4) = 8,\ f'(3) = 27 - 36 + 8$



$f'(x_1) = -1$, then $x_1 = 3$

$f'(x_2) = 0$

Again $f'(x) < 0$ for $x \in (2, x_4)$

$f'(x) > 0$ for $x \in (x_4, 4)$

$x_4 \in (3, 4)$

$f(x) = x^3 - 6x^2 + 8x$

$f(3) = 27 - 54 + 24 = -3$

$f(4) = 64 - 96 + 32 = 0$

For $x_4(3, 4)$

$f(x_4) < -3\sqrt{3}$

and $f'(x_3) > -4$

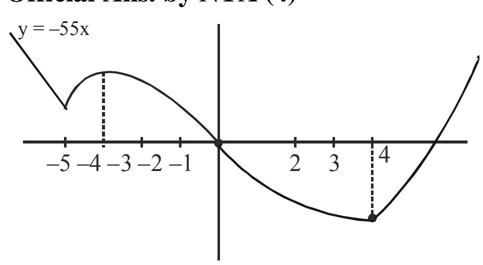
$2f'(x_3) > -8$

So, $2f'(x_3) = \sqrt{3}\ f(x_4)$

Correct Ans. (1)

7. Official Ans. by NTA (4)

Sol.



$$f'(x) = \begin{cases} -55; & x < -5 \\ 6(x-5)(x+4); & -5 < x < 4 \\ 6(x-3)(x+2); & x > 4 \end{cases}$$

f(x) is increasing in

$x \in (-5, -4) \cup (4, \infty)$

8. Official Ans. by NTA (1)

Sol. $f(x) = \dfrac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$

$f'(x) = (2x^2 - x) - 2\cos x + 2\cos x - \sin x(2x-1)$

$= (2x - 1)(x - \sin x)$

for $x > 0$, $x - \sin x > 0$

$x < 0$, $x - \sin x < 0$

for $x \in (-\infty, 0] \cup \left[\dfrac{1}{2}, \infty\right)$, $f'(x) \geq 0$

for $x \in \left[0, \dfrac{1}{2}\right]$, $f'(x) \leq 0$

$\Rightarrow f(x)$ increases in $\left[\dfrac{1}{2}, \infty\right)$.

9. Official Ans. by NTA (1)

Sol. $f(1) = f(2)$

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$\Rightarrow 3a - b = 7$ ......(1)

Also $f^1\left(\dfrac{4}{3}\right) = 0$ (given)

$\Rightarrow \left(3x^2 - 2ax + b\right)_{x=\frac{4}{3}} = 0$

$\Rightarrow \dfrac{16}{3} - \dfrac{8a}{3} + b = 0$

$\Rightarrow 8a - 3b - 16 = 0$ ....(2)

Solving (1) and (2)

$a = 5, b = 8$

10. Official Ans. by NTA (2)

Sol. Let $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10 = f(x)$

Now $f(-2) = -34$ and $f(-1) = 3$

Hence $f(x)$ has a root in $(-2,-1)$

Further $f'(x) = 10x^4 + 20x^3 + 20x^2 + 20x + 10$

$= 10x^2\left[\left(x^2 + \dfrac{1}{x^2}\right) + 2\left(x + \dfrac{1}{x}\right) + 20\right]$

$= 10x^2\left[\left(x + \dfrac{1}{x} + 1\right)^2 + 17\right] > 0$

Hence $f(x)$ has only one real root, so $|a| = 2$

11. Official Ans by NTA (1)

Sol. $f(x)=3\ell n(x-1)-3\ell n(x+1)-\dfrac{2}{x-1}$

$f'(x)=\dfrac{3}{x-1}-\dfrac{3}{x+1}+\dfrac{2}{(x-1)^2}$

$f'(x)=\dfrac{4(2x-1)}{(x-1)^2(x+1)}$

$f'(x)\ge 0$

$\Rightarrow\ x\in(-\infty,-1)\cup\left[\dfrac{1}{2},1\right)\cup(1,\infty)$

12. Official Ans. by NTA (2)

Sol. $f(x)=\begin{cases}-x\left(2-\sin\left(\dfrac{1}{x}\right)\right) & x<0\\ 0 & x=0\\ x\left(2-\sin\left(\dfrac{1}{x}\right)\right)\end{cases}$

$f'(x)=\begin{cases}-\left(2-\sin\dfrac{1}{x}\right)-x\left(-\cos\dfrac{1}{x}\cdot\left(-\dfrac{1}{x^2}\right)\right) & x<0\\ \left(2-\sin\dfrac{1}{x}\right)+x\left(-\cos\dfrac{1}{x}\left(-\dfrac{1}{x^2}\right)\right) & x>0\end{cases}$

$f'(x)=\begin{cases}-2+\sin\dfrac{1}{x}-\dfrac{1}{x}\cos\dfrac{1}{x} & x<0\\ 2-\sin\dfrac{1}{x}+\dfrac{1}{x}\cos\dfrac{1}{x} & x>0\end{cases}$

$f'(x)$ is an oscillating function which is non-monotonic in $(-\infty, 0)\cup(0, \infty)$.

Option (2)

AOD (MAXIMA & MINIMA)

1. Official Ans. by NTA (4)

Sol. $A=\begin{bmatrix}1 & -x & 2x+1\\ -x & 1 & -x\\ 2x+1 & -x & 1\end{bmatrix}$

$|A| = 4x^3 - 4x^2 - 4x = f(x)$

$f'(x) = 4(3x^2 - 2x - 1) = 0$

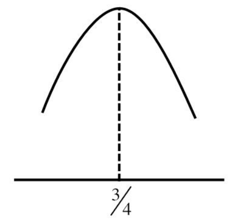
$\Rightarrow x = 1\ ;\ x = \dfrac{-1}{3}$

$\therefore\ \underbrace{f(1)=-4}_{\text{min}}\ ;\underbrace{f\left(-\dfrac{1}{3}\right)=\dfrac{20}{27}}_{\text{max.}}$

Sum $= -4+\dfrac{20}{27}=-\dfrac{88}{27}$

2. Official Ans. by NTA (1)

Sol.



$\dfrac{-B}{2A}=\dfrac{3}{4}$

$\Rightarrow\ \dfrac{-(6)}{2a}=\dfrac{3}{4}$

$\Rightarrow a=\dfrac{-6\times 4}{6}\ \Rightarrow a=-4$

$\therefore\ g(x) = 4x^2 - 6x + 15$

Local max. at $x=\dfrac{-B}{2A}=-\dfrac{(-6)}{2(-4)}$

$=\dfrac{-3}{4}$

3. Official Ans. by NTA (3)

Sol. $f(x) = x^3 - 3x^2 - \dfrac{3}{2}f''(2)\,x + f''(1)$ .....(i)

$f'(x) = 3x^2 - 6x - \dfrac{3}{2}f''(2)$ .....(ii)

$f''(x) = 6x - 6$ .....(iii)

Now is 3rd equation

$f''(2) = 12 - 6 = 6$

$f''(11 = 0)$

Use (ii)

$f'(x) = 3x^2 - 6x - \dfrac{3}{2}f''(2)$

$f'(x) = 3x^2 - 6x - \dfrac{3}{2}\times 6$

$f'(x) = 3x^2 - 6x - 9$

$f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$\Rightarrow x = -1\ \&\ 3$

Use (iii)

$f''(x) = 6x - 6$

$f''(-1) = -12 < 0$ maxima

$f''(3) = 12 > 0$ minima.

Use (i)

$f(x) = x^3 - 3x^2 - \dfrac{3}{2}f''(2)\,x + f''(1)$

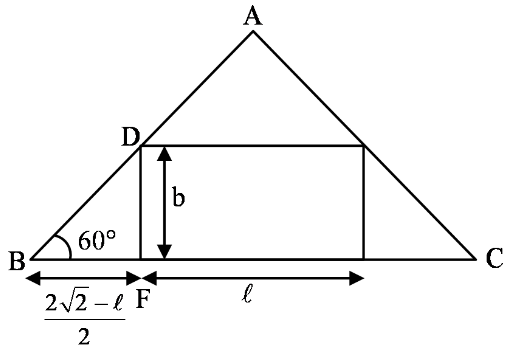
$f(x) = x^3 - 3x^2 - \dfrac{3}{2}\times 6\times x + 0$

$f(x) = x^3 - 3x^2 - 9x$

$f(3) = 27 - 27 - 9\times 3 = -27$

4. Official Ans. by NTA (3)

Sol.



In ΔDBF

$$\tan 60^\circ = \frac{2b}{2\sqrt{2}-\ell} \Rightarrow b = \frac{\sqrt{3}\left(2\sqrt{2}-\ell\right)}{2}$$

A = Area of rectangle = $\ell \times b$

$$A = \ell \times \frac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right)$$

$$\frac{dA}{d\ell} = \frac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right) - \frac{\ell.\sqrt{3}}{2} = 0$$

$$\boxed{\ell = \sqrt{2}}$$

$$A = \ell \times b = \sqrt{2} \times \frac{\sqrt{3}}{2}\left(\sqrt{2}\right) = \sqrt{3}$$

$$\Rightarrow \boxed{A^2 = 3}$$

5. Official Ans. by NTA (36)

Sol. Let x + y = 36

x is perimeter of square and y is perimeter of circle side of square = x/4

radius of circle = $\frac{y}{2\pi}$

$$\text{Sum Areas} = \left(\frac{x}{4}\right)^2 + \pi\left(\frac{y}{2\pi}\right)^2$$

$$= \frac{x^2}{16} + \frac{(36-x)^2}{4\pi}$$

For min Area :

$$x = \frac{144}{\pi+4}$$

$$\Rightarrow \text{Radius} = y = 36 - \frac{144}{\pi+4}$$

$$\Rightarrow k = \frac{36\pi}{\pi+4}$$

$$\left(\frac{4}{\pi}+1\right)k = 36$$

6. Official Ans. by NTA (3)

Sol. $f(x) = \left(\frac{2}{x}\right)^{x^2}$; $x > 0$

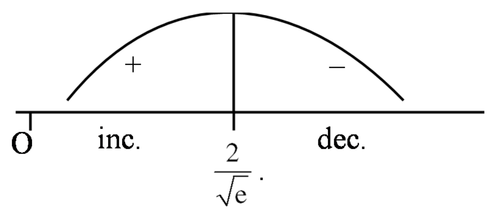
$\ell n\, f(x) = x^2 (\ell n2 - \ell nx)$

$f'(x) = f(x)\ \{-x + (\ell n\, 2 - \ell n\, x)2x\}$

$$f'(x) = \underbrace{f(x)}_{+}.\underbrace{x}_{+}\underbrace{(2\ell n 2 - 2\ell n x - 1)}_{g(x)}$$

$g(x) = 2\ell\, n^2 - 2\ell\, n\, x - 1$

$$= \ell n\frac{4}{x^2} - 1 = 0 \Rightarrow x = \frac{2}{\sqrt{e}}$$

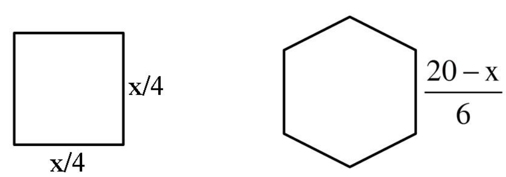


$$LM = \frac{2}{\sqrt{e}}$$

$$\text{Local maximum value} = \left(\frac{2}{2/\sqrt{e}}\right)^{\frac{4}{e}} \Rightarrow e^{\frac{2}{e}}$$

7. Official Ans. by NTA (4)

Sol. Let the wire is cut into two pieces of length x and 20 – x.



Area of square $= \left(\frac{x}{4}\right)^2$ Area of regular hexagon

$$= 6 \times \frac{\sqrt{3}}{4}\left(\frac{20-x}{6}\right)^2$$

$$\text{Total area} = A(x) = \frac{x^2}{16} + \frac{3\sqrt{3}}{2}\frac{(20-x)^2}{36}$$

$$A'(x) = \frac{2x}{16} + \frac{3\sqrt{3}\times 2}{2\times 36}(20-x)(-1)$$

$$A'(x) = 0 \text{ at } x = \frac{40\sqrt{3}}{3+2\sqrt{3}}$$

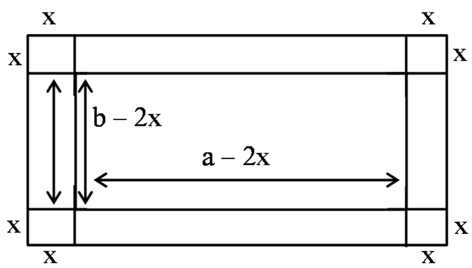
Length of side of regular Hexagon $= \frac{1}{6}(20 - x)$

$$= \frac{1}{6}\left(20 - \frac{4.\sqrt{3}}{3+2\sqrt{3}}\right)$$

$$= \frac{10}{2+2\sqrt{3}}$$

8. Official Ans. by NTA (3)

Sol.



$V = \ell.\ b.\ h \quad = (a - 2x)(b - 2x)\,x$

$\Rightarrow V(x) = (2x - a)(2x - b)\,x$

$\Rightarrow V(x) = 4x^3 - 2(a + b)x^2 + abx$

$\Rightarrow \frac{d}{dx} v(x) = 12x^2 - 4(a + b)x + ab$

$\frac{d}{dx}(v(x)) = 0$

$\Rightarrow 12x^2 - 4(a + b)x + ab = 0 <^{\alpha}_{\beta}$

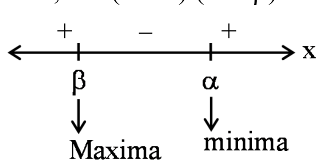
$$\Rightarrow x = \frac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{2(12)}$$

$$= \frac{(a+b) \pm \sqrt{a^2 + b^2 - ab}}{6}$$

Let $x = \alpha = \frac{(a+b) + \sqrt{a^2 + b^2 - ab}}{6}$

$$\beta = \frac{(a+b) - \sqrt{a^2 + b^2 - ab}}{6}$$

Now, $12(x - \alpha)(x - \beta) = 0$



$\therefore\ x = \beta$

$$= \frac{a + b - \sqrt{a^2 + b^2 - ab}}{b}$$

9. Official Ans. by NTA (22)

Sol. $F'(x) = a(x - 1)(x + 3)$

$F''(x) = 6a(x + 1)$

$F'(x) = 3a(x + 1)^2 + b$

$F'(1) = 0 \Rightarrow b = -12a$

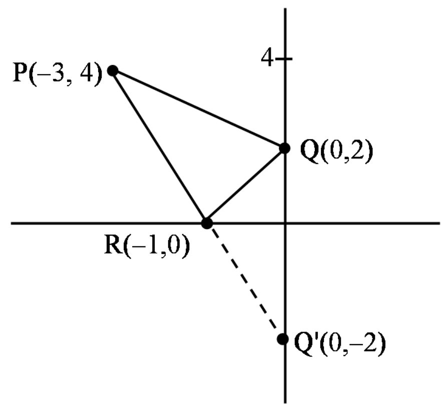
$F(x) = a(x + 1)^3 - 12ax + c$

$= (x + 1)^3 - 12x - 6$

$F(3) = 64 - 36 - 6 = 22$

10. Official Ans. by NTA (1250)

Sol.



$50(PR^2 + RQ^2)$

$50(20 + 5)$

$50(25)$

$= 1250$

11. Official Ans. by NTA (9)

Sol. Let $f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$

$\Rightarrow f'(x) = 0 \Rightarrow \sin x = 2/3$

$\therefore\ f(x)_{min} = \frac{4}{2/3} + \frac{1}{1 - 2/3} = 9$

$f(x)max \to \infty$

$f(x)$ is continuous function

$\therefore\ \alpha_{min} = 9$

12. Official Ans. by NTA (144)

Sol. Let $f(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

as $\lim_{x \to 0} \frac{f(x)}{x^3} = 1$ non-zero finite

So, $d = e = f = 0$

and $f(x) = x^3(x^3 + ax^2 + bx + c)$

Hence, $\lim_{x\to 0}\frac{f(x)}{x^3}=c=1$

Now, as $f(x) = x^6 + ax^5 + bx^4 + x^3$

and $f'(x)=0$ at $x=1$ and $x=-1$

i.e., $f'(x)=6x^5+5ax^4+4bx^3+3x^2$

$f'(1)=0$

$\Rightarrow 6 + 5a + 4b + 3 = 0$

$\Rightarrow 5a + 4b = -9$

& $f'(-1)=0$

$\Rightarrow -6 + 5a - 4b + 3 = 0$

$\Rightarrow 5a - 4b = 3$

Solving both we get,

$a=\frac{-6}{10}=\frac{-3}{5};\ b=\frac{-3}{2}$

$\therefore\ f(x)=x^6-\frac{3}{5}x^5-\frac{3}{2}x^4+x^3$

$\therefore\ 5f(2)=5\left[64-\frac{3}{5}\cdot 32-\frac{3}{2}\cdot 16+8\right]$

$= 320 - 96 - 120 + 40$

$= 144$

13. Official Ans. by NTA (2)

Sol. A.M. ≥ G.M.

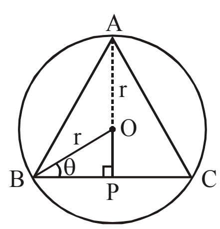
$f(x)=a^{a^x}+a^{1-a^x}=a^{a^x}+\frac{a}{a^{a^x}}\ge 2\sqrt{a}$

14. Official Ans. by NTA (3)

Sol. $h = r\sin\theta + r$

base = BC = $2r\cos\theta$

$\theta\in\left[0,\frac{\pi}{2}\right)$



Area of $\Delta ABC=\frac{1}{2}(BC).h$

$\Delta=\frac{1}{2}(2r\cos\theta).(r\sin\theta+r)$

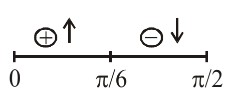
$= r^2 (\cos\theta).(1 + \sin\theta)$

$\frac{d\Delta}{d\theta}=r^2\left[\cos^2\theta-\sin\theta-\sin^2\theta\right]$

$= r^2[1 - \sin\theta - 2\sin^2\theta]$

$=\underbrace{r^2[1+\sin\theta]}_{\text{positive}}[1-2\sin\theta]=0$

$\Rightarrow \theta=\frac{\pi}{6}$



$\Rightarrow \Delta$ is maximum where $\theta=\frac{\pi}{6}$

$\Delta_{max.}=\frac{3\sqrt{3}}{4}r^2$ = area of equilateral Δ with BC = $\sqrt{3}r$.

15. Official Ans. by NTA (2)

Sol. $f(x) = (4a - 3)(x + \log_e 5) + (a - 7)\sin x$

$f'(x) = (4a - 3)(1) + (a - 7)\cos x = 0$

$\Rightarrow\ \cos x=\frac{3-4a}{a-7}$

$-1\le\frac{3-4a}{a-7}<1$

$\frac{3-4a}{a-7}+1\ge 0$

$\frac{3-4a+a-7}{a-7}\ge 0$

$\frac{-3a-4}{a-7}\ge 0$

$\frac{3a+4}{a-7}\le 0$

$\frac{3-4a}{a-7}<1$

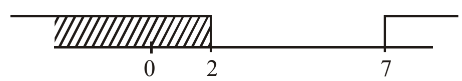
$\frac{3-4a}{a-7}-1<0$

$\frac{3-4a-a+7}{a-7}<0$

$\frac{-5a+10}{a-7}<0$

$\frac{5a-10}{a-7}>0$

$\frac{5(a-2)}{a-7}>0$



$\alpha\in\left[-\frac{4}{3},2\right)$

Check end point $\left[-\frac{4}{3},2\right)$

INDEFINITE INTEGRATION

1. Official Ans. by NTA (15)

Sol. $I=\int\frac{dx}{\left[\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right]^2}$

$\int\frac{dt}{\left(t^2+\frac{3}{4}\right)^2}\quad\left(\text{Put } x+\frac{1}{2}=t\right)$

$=\frac{\sqrt{3}}{2}\int\frac{\sec^2\theta\,d\theta}{\frac{9}{16}\sec^4\theta}\quad\left(\text{Put } t=\frac{\sqrt{3}}{2}\tan\theta\right)$

$=\frac{4\sqrt{3}}{9}\int(1+\cos 2\theta)\,d\theta$

$$=\frac{4\sqrt{3}}{9}\left[\theta+\frac{\sin 2\theta}{2}\right]+c$$

$$=\frac{4\sqrt{3}}{9}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{\sqrt{3}(2x+1)}{3+(2x+1)^2}\right]+c$$

$$=\frac{4\sqrt{3}}{9}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{3}\left(\frac{2x+1}{x^2+x+1}\right)+c$$

Hence, $9\left(\sqrt{3}a+b\right)=15$

2. Official Ans. by NTA (7)

Sol. $\int\frac{2e^x}{4e^x+7e^{-x}}dx\ +3\int\frac{e^{-x}}{4e^x+7e^{-x}}dx$

$$=\int\frac{2e^{2x}}{4e^{2x}+7}dx+3\int\frac{e^{-2x}}{4+7e^{-2x}}dx$$

Let $4e^{2x}+7=T$ Let $4+7e^{-2x}=t$

$8e^{2x}dx=dT$ $-14e^{-2x}dx=dt$

$2e^{2x}dx=\frac{dT}{4}$ $e^{-2x}dx=-\frac{dt}{14}$

$$\int\frac{dT}{4T}-\frac{3}{14}\int\frac{dt}{t}$$

$$=\frac{1}{4}\log T-\frac{3}{14}\log t+C$$

$$=\frac{1}{4}\log(4e^{2x}+7)-\frac{3}{14}\log\left(4+7e^{-2x}\right)+C$$

$$=\frac{1}{14}\left[\frac{1}{2}\log\left(4e^x+7e^{-x}\right)+\frac{13}{2}x\right]+C$$

$$u=\frac{13}{2},v=\frac{1}{2}\Rightarrow u+v=7$$

Aliter :

$2e^x+3e^{-x}=A(4e^x+7e^{-x})+B(4e^x-7e^{-x})+\lambda$

$2=4A+4B$; $3=7A-7B$; $\lambda=0$

$$A+B=\frac{1}{2}$$

$$A-B=\frac{3}{7}$$

$$A=\frac{1}{2}\left(\frac{1}{2}+\frac{3}{7}\right)=\frac{7+6}{28}=\frac{13}{28}$$

$$B=A-\frac{3}{7}=\frac{13}{28}-\frac{3}{7}=\frac{13-12}{28}=\frac{1}{28}$$

$$\int\frac{13}{28}dx+\frac{1}{28}\int\frac{4e^x-7e^{-x}}{4e^x+7e^{-x}}dx$$

$$\frac{13}{28}x+\frac{1}{28}\ell n|4e^x+7e^{-x}|+C$$

$$u=\frac{13}{2};v=\frac{1}{2}$$

$\Rightarrow$ $u+v=7$

3. Official Ans. by NTA (3)

Sol. $\int\frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$

$$=\int\frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4}.(x-1)^2}$$

put $\frac{x+2}{x-1}=t$

$$=-\frac{1}{3}\int\frac{dt}{t^{5/4}}$$

$$=\frac{4}{3}.\frac{1}{t^{1/4}}+C$$

$$=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$$

4. Official Ans. by NTA (3)

Sol. $=\int\frac{\frac{\sin x}{\cos^3 x}}{1+\tan^3 x}dx=\int\frac{\tan x.\sec^2 x}{(\tan x+1)(1+\tan^2 x-\tan x)}dx$

Let $\tan x=t\Rightarrow\sec^2x.dx=dt$

$$=\int\frac{t}{(t+1)(t^2-t+1)}dt$$

$$=\int\left(\frac{A}{t+1}+\frac{B(2t-1)}{t^2-t+1}+\frac{C}{t^2-t+1}\right)dx$$

$\Rightarrow A(t^2-t+1)+B(2t-1)(t^2-t+1)+C(t+1)$
$=t$

$\Rightarrow t^2(A+2B)+t(-A+B+C)+A-B+C=1$

$\therefore\ A+2B=0$ …(1)

$-A+B+C=1$ …(2)

$A-B+C=0$ …(3)

$\Rightarrow C=\frac{1}{2}\Rightarrow A-B=-\frac{1}{2}$ …(4)

$A+2B=0$

$$A-B=-\frac{1}{2}$$

$$\Rightarrow 3B=\frac{1}{2}\Rightarrow B=\frac{1}{6}$$

$$A=-\frac{1}{3}$$

$$I=-\frac{1}{3}\int\frac{dt}{1+t}+\frac{1}{6}\int\frac{2t-1}{t^2-t+1}dt+\frac{1}{2}\int\frac{dt}{t^2-t+1}$$

$$= -\frac{1}{3}\ell n|(1+\tan x)| + \frac{1}{6}\ell n|\tan^2 x - \tan x + 1| +$$

$$\frac{1}{2}.\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\left(\tan x-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right)$$

$$= -\frac{1}{3}\ell n|(1+\tan x)| + \frac{1}{6}\ell n|\tan^2 x - \tan x + 1| +$$

$$\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$$

$$\alpha=-\frac{1}{3},\ \beta=\frac{1}{6},\ \gamma=\frac{1}{\sqrt{3}}$$

$$18(\alpha+\beta+\gamma^2)=18\left(-\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right)=3$$

5. Official Ans. by NTA (3)

Sol. $\int\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx$

$$=\int\frac{\cos x-\sin x}{\sqrt{9-(\sin x+\cos x)^2}}dx$$

Let sinx + cosx = t

$$\int\frac{dt}{\sqrt{9-t^2}}=\sin^{-1}\frac{t}{3}+c$$

$$=\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right)+c$$

So a = 1, b = 3.

6. Official Ans. by NTA (4)

Sol. $I=\int\frac{\sin\theta.\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$

$$\Rightarrow I=\int\frac{\sin\theta.2\sin\theta\cos\theta.\sin^2\theta(\sin^4\theta+\sin^2\theta+1)(2\sin^4\theta+3\sin^2\theta+6)^{1/2}}{2\sin^2\theta}d\theta$$

$$=\int\sin^2\theta\,.\cos\theta(\sin^4\theta+\sin^2\theta+1)(2\sin^4\theta+3\sin^2\theta+6)^{1/2}d\theta$$

Let sinθ = t ⇒ cosθ dθ = dt

$$\therefore\ I=\int t^2(t^4+t^2+1)(2t^4+3t^2+6)^{1/2}dt$$

$$=\int(t^5+t^3+t)t(2t^4+3t^2+6)^{1/2}dt$$

$$=\int(t^5+t^3+t)(t^2)^{1/2}(2t^4+3t^2+6)^{1/2}dt$$

$$=\int(t^5+t^3+t)(2t^6+3t^4+6t^2)^{1/2}dt$$

Let $2t^6 + 3t^4 + 6t^2 = u^2$

$\Rightarrow 12(t^5 + t^3 + t)\,dt = 2u\,du$

$$\therefore\ I=\int(u^2)^{1/2}\cdot\frac{2udu}{12}$$

$$=\int\frac{u^2}{6}du=\frac{u^3}{18}+C$$

$$=\frac{(2t^6+3t^4+6t^2)^{3/2}}{18}+C$$

when t = sinθ

and $t^2 = 1 - \cos^2\theta$ will give option (4)

7. Official Ans. by NTA (2)

Sol. $\int\frac{e^{3\log_e 2x}+5e^{2\log_e 2x}}{e^{4\log_e x}+5e^{3\log_e x}-7e^{2\log_e x}}dx, x>0$

$$=\int\frac{(2x)^3+5(2x)^2}{x^4+5x^3-7x^2}dx=\int\frac{4x^2(2x+5)}{x^2(x^2+5x-7)}dx$$

$$=4\int\frac{d(x^2+5x-7)}{(x^2+5x-7)}=4\log_e|x^2+5x-7|+c$$

option (2)

8. Official Ans by NTA (6)

Sol. $\int\frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}+\int\frac{dx}{x^4+3x^2+1}$

$$\int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}+\frac{1}{2}\int\frac{(x^2+1)-(x^2-1)dx}{x^4+3x^2+1}$$

Put $\tan^{-1}\left(x+\frac{1}{x}\right)=t$

$$\int\frac{dt}{t}+\frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5}-\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put $x-\frac{1}{x}=y$, $x+\frac{1}{x}=z$

$$\log_e t+\frac{1}{2}\int\frac{dy}{y^2+5}-\frac{1}{2}\int\frac{dz}{z^2+1}$$

$$= \log_e \tan^{-1}\left(x+\frac{1}{x}\right) + \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)$$

$$-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

$$\alpha = 1, \beta = \frac{1}{2\sqrt{5}}, \gamma = \frac{1}{\sqrt{5}}, \delta = \frac{-1}{2}$$

or

$$\alpha = 1, \beta = \frac{-1}{2\sqrt{5}}, \gamma = \frac{-1}{\sqrt{5}}, \delta = \frac{-1}{2}$$

$$10(\alpha + \beta\gamma + \delta) = 10\left(1+\frac{1}{10}-\frac{1}{2}\right) = 6$$

9. Official Ans. by NTA (1)

Sol. $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}dx$

$(2x-1)^2 + 5 = t^2$

$2(2x-1)\,2dx = 2t\,dt$

$2\sqrt{t^2-5}dx = t\,dt$

So $\int \frac{\sqrt{t^2-5}\cos t}{2\sqrt{t^2-5}}dt = \frac{1}{2}\sin t + c$

$= \frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

10. Official Ans. by NTA (4)

Sol. $f(x) = \int \frac{(5x^8+7x^6)dx}{x^{14}(x^{-5}+x^{-7}+2)^2}$

Let $x^{-5} + x^{-7} + 2 = t$

$(-5x^{-6} - 7x^{-8})dx = dt$

$\Rightarrow f(x) = \int -\frac{dt}{t^2} = \frac{1}{t}+c$

$f(x) = \frac{x^7}{x^2+1+2x^7}$

$f(1) = \frac{1}{4}$

DEFINITE INTEGRATION

1. Official Ans. by NTA (2)

Sol. $a > 0$

Let $n \le a < n+1, n \in W$

$\therefore a = [a] + \{a\}$

$\Downarrow$ G.I.F $\quad \Downarrow$ Fractional part

Here $[a] = n$

Now, $\int_0^a e^{x-[x]}dx = 10e-9$

$\Rightarrow \int_0^n e^{\{x\}}dx + \int_n^a e^{x-[x]}dx = 10e-9$

$\therefore n\int_0^1 e^x dx + \int_n^a e^{x-n}dx = 10e-9$

$\Rightarrow n(e-1)+(e^{a-n}-1) = 10e-9$

$\therefore \boxed{n=0}$ and $\{a\} = \log_e 2$

So, $a = [a]+\{a\} = (10+\log_e 2)$

$\Rightarrow$ Option (2) is correct.

2. Official Ans. by NTA (2)

ALLEN Ans. (3)

Sol. Let $I = 2\int_0^1 \underbrace{\ln\left(\sqrt{1-x}+\sqrt{1+x}\right)}_{(I)}\ \underbrace{1}_{(II)}dx$

(I.B.P.)

$$\therefore I = 2\left[\left(x.\ln\left(\sqrt{1-x}+\sqrt{1-x}\right)\right)_0^1\right.$$

$$\left.-\int_0^1 x.\left(\frac{1}{\sqrt{1-x}+\sqrt{1+x}}\right)\cdot\left(\frac{1}{2\sqrt{1+x}}-\frac{1}{2\sqrt{1-x}}\right)dx\right]$$

$$= 2\left(\ln\sqrt{2}-0\right)-\frac{2}{2}\int_0^1 \frac{x\sqrt{1-x}-\sqrt{1+x}\ dx}{\left(\sqrt{1-x}+\sqrt{1+x}\right)\sqrt{1-x^2}}$$

$$= (\log_e 2) - \int_0^1 \frac{x\cdot\left(2-2\sqrt{1-x^2}\right)}{-2x\sqrt{1-x^2}}dx \quad \text{(After rationalisation)}$$

$$= (\log_e 2)+\int_0^1\left(\frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)dx$$

$$= (\log_e 2) + (\sin^{-1}x)_0^1 - 1$$

$$= \log_e 2 + \left(\frac{\pi}{2}-0\right)-1$$

$$\therefore I = (\log_e 2) + \frac{\pi}{2} - 1$$

$\Rightarrow$ Option (3) is correct.

3. Official Ans. by NTA (2)

Sol. $g(t)=\int_{-\pi/2}^{\pi/2}\left(\cos\frac{\pi}{4}t+f(x)\right)dx$

$g(t)=\pi\cos\frac{\pi}{4}t+\int_{-\pi/2}^{\pi/2}f(x)dx$

$g(t)=\pi\cos\frac{\pi}{4}t$

$g(1)=\frac{\pi}{\sqrt{2}},g(0)=\pi$

4. Official Ans. by NTA (1)

Sol. $I=\int_0^{100\pi}\frac{\sin^2 x}{e^{\left\{x/\pi\right\}}}dx=100\int_0^{\pi}\frac{\sin^2 x}{e^{x/\pi}}dx$

$100\int_0^{\pi}e^{-x/\pi}\frac{(1-\cos 2x)}{2}dx$

$=50\left\{\int_0^{\pi}e^{-x/\pi}dx-\int_0^{\pi}e^{-x/\pi}\cos 2x\,dx\right\}$

$I_1=\int_0^{\pi}e^{-x/\pi}dx=\left[-\pi e^{-x/\pi}\right]_0^{\pi}=\pi(1-e^{-1})$

$I_2=\int_0^{\pi}e^{-x/\pi}\cos 2x\,dx$

$=-\pi e^{-x/\pi}\cos 2x\Big]_0^{\pi}-\int-\pi e^{-x/\pi}(-2\sin 2x)dx$

$=\pi(1-e^{-1})-2\pi\int_0^{\pi}e^{-x/\pi}\sin 2x\,dx$

$=\pi(1-e^{-1})-2\pi\left\{-\pi e^{-x/\pi}\sin 2x\Big]_0^{\pi}-\int_0^{\pi}-\pi e^{-x/\pi}2\cos 2x dx\right\}$

$=\pi(1-e^{-1})-4\pi^2 I_2$

$\Rightarrow I_2=\frac{\pi(1-e^{-1})}{1+4\pi^2}$

$\therefore I=50\left\{\pi(1-e^{-1})-\frac{\pi(1-e^{-1})}{1+4\pi^2}\right\}$

$=\frac{200(1-e^{-1})\pi^3}{1+4\pi^2}$

5. Official Ans. by NTA (3)

Sol. Let $I=\int_{\pi/24}^{5\pi/24}\frac{(\cos 2x)^{1/3}}{(\cos 2x)^{1/3}+(\sin 2x)^{1/3}}dx$ ….(i)

$\Rightarrow I=\int_{\pi/24}^{5\pi/24}\frac{\left(\cos\left\{2\left(\frac{\pi}{4}-x\right)\right\}\right)^{\frac{1}{3}}}{\left(\cos\left\{2\left(\frac{\pi}{4}-x\right)\right\}\right)^{\frac{1}{3}}+\left(\sin\left\{2\left(\frac{\pi}{4}-x\right)\right\}\right)^{\frac{1}{3}}}dx$

$\left\{\int_a^b f(x)\,dx=\int_a^b f(a+b-x)dx\right\}$

So $I=\int_{\pi/24}^{5\pi/24}\frac{(\sin 2x)^{1/3}}{(\sin 2x)^{1/3}+(\cos 2x)^{1/3}}dx$ ….(ii)

Hence $2I=\int_{\pi/24}^{5\pi/24}dx$ [(i) + (ii)]

$\Rightarrow 2I=\frac{4\pi}{24}\Rightarrow \boxed{I=\frac{\pi}{12}}$

6. Official Ans. by NTA (1)

Sol. $f:[0,\infty)\to[0,\infty)$, $f(x)=\int_0^x[y]\,dy$

Let $x=n+f$, $f\in(0,1)$

So $f(x)=0+1+2+\ldots+(n-1)+\int_n^{n+f}n\,dy$

$f(x)=\frac{n(n-1)}{2}+nf$

$=\frac{[x]([x]-1)}{2}+[x]\{x\}$

Note $\lim_{x\to n^+}f(x)=\frac{n(n-1)}{2}$,

$\lim_{x\to n^-}f(x)=\frac{(n-1)(n-2)}{2}+(n-1)=\frac{n(n-1)}{2}$

$f(x)=\frac{n(n-1)}{2}$ $(n\in N_0)$

so $f(x)$ is cont. $\forall$ $x\geq 0$ and diff. except at integer points

7. Official Ans. by NTA (3)

Sol. $f(x)=\int_0^1 (5+(1-t))\,dt+\int_1^x (5+(t-1))\,dt$

$=6-\frac{1}{2}+\left(4t+\frac{t^2}{2}\right)\Big|_1^x$

$=\frac{11}{2}+4x+\frac{x^2}{2}-4-\frac{1}{2}$

$=\frac{x^2}{2}+4x+1$

$f(2^+)=2+8+1=11$

$f(2)=f(2^-)=5\times2+1=11$

$\Rightarrow$ continuous at x = 2

Clearly differentiable at x = 1

Lf'(2) = 5

Rf'(2) = 6

$\Rightarrow$ not differentiable at x = 2

8. Official Ans. by NTA (2)

Sol. Let $I=\int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$

$\because \log\left(x+\sqrt{x^2+1}\right)$ is an odd function

$\therefore I=0$

9. Official Ans. by NTA (2)

Sol. $I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{(1+e^{x\cos x})(\sin^4x+\cos^4x)}$ ...(1)

Using $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$I=\int_{-\pi/4}^{\pi/4}\frac{dx}{(1+e^{-x\cos x})(\sin^4x+\cos^4x)}$

Add (1) and (2)

$2I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{\sin^4x+\cos^4x}$

$2I=2\int_0^{\frac{\pi}{4}}\frac{dx}{\sin^4x+\cos^4x}$

$I=\int_0^{\frac{\pi}{4}}\frac{(1+\tan^2x)\sec^2x}{\tan^4x+1}dx$

$I=\int_0^{\frac{\pi}{4}}\frac{\left(1+\frac{1}{\tan^2x}\right)\sec^2x}{\left(\tan x-\frac{1}{\tan x}\right)^2+2}dx$

$\tan x-\frac{1}{\tan x}=t$

$\left(1+\frac{1}{\tan^2x}\right)\sec^2x\,dx=dt$

$I=\int_{-\infty}^{0}\frac{dt}{t^2+2}=\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_{-\infty}^{0}$

$I=0-\frac{1}{\sqrt{2}}\left(-\frac{\pi}{2}\right)=\frac{\pi}{2\sqrt{2}}$

10. Official Ans. by NTA (1)

Sol. For domain

$\log_5\left(\log_3\left(18x-x^2-77\right)\right)>0$

$\log_3\left(18x-x^2-77\right)>1$

$18x-x^2-77>3$

$x^2-18x+80<0$

$x\in(8,10)$

$\Rightarrow a=8$ and $b=10$

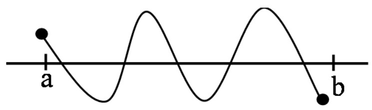
$I=\int_a^b\frac{\sin^3x}{\sin^3x+\sin^3(a+b-x)}dx$

$I=\int_a^b\frac{\sin^3(a+b-x)}{\sin^3x+\sin^3(a+b-x)}$

$2I=(b-a)\Rightarrow I=\frac{b-a}{2}$ $(\because a=8 \text{ and } b=10)$

$I=\frac{10-8}{2}=1$

11. Official Ans. by NTA (3)

Sol. 

$$f(x)=\int_a^x g(t)dt$$

$f(x) \rightarrow 5$

$f'(x) \rightarrow 4$

$g(x) \rightarrow 4$

$g'(x) \rightarrow 3$

12. Official Ans. by NTA (5)

Sol. $I = 2\int_0^{\pi/2} \sin^3 x\, e^{-\sin^2 x} dx$

$= 2\int_0^{\pi/2} \sin x\, e^{-\sin^2 x} dx +$

$\int_0^{\pi/2} \underbrace{\cos x}_{I}\ \underbrace{e^{-\sin^2 x}(-\sin 2x)}_{II}\, dx$

$= 2\int_0^{\pi/2} \sin x e^{-\sin^2 x} dx + \left[\cos x\, e^{-\sin^2 x}\right]_0^{\pi/2}$

$+ \int_0^{\pi/2} \sin x\, e^{-\sin^2 x}\, dx$

$= 3\int_0^{\pi/2} \sin x\, e^{-\sin^2 x} dx - 1$

$= \frac{3}{2}\int_{-1}^{0} \frac{e^{\alpha} d\alpha}{\sqrt{1+\alpha}} - 1$ (Put $-\sin^2 x = t$)

$= \frac{3}{2e}\int_0^1 \frac{e^x}{\sqrt{x}} dx - 1$ (put $1 + \alpha = x$)

$= \frac{3}{2e}\int_0^1 \underbrace{e^x}_{II} \underbrace{\frac{1}{\sqrt{x}}}_{II} dx - 1$

$= 2 - \frac{3}{e}\int_0^1 e^x \sqrt{x}\, dx$

Hence, $\alpha + \beta = \boxed{5}$

13. Official Ans. by NTA (2)

Sol. $I = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right)^2\right)^{1/2} dx$

$I = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left|\frac{4x}{x^2-1}\right| dx \Rightarrow I = 2.4\int_0^{1/\sqrt{2}} \left|\frac{x}{x^2-1}\right| dx$

$\Rightarrow I = -4\int_0^{1/\sqrt{2}} \frac{2x}{x^2-1} dx \Rightarrow I = -4\ln\left|x^2-1\right|\Big|_0^{1/\sqrt{2}}$

$\Rightarrow I = 4 \ln 2 \Rightarrow I = \ln 16$

14. Official Ans. by NTA (2)

Sol. $L = \lim_{n\to\infty} \frac{1}{n}.\sum_{r=0}^{2n-1} \frac{1}{1+4\left(\frac{r}{n}\right)^2}$

$\Rightarrow L = \int_0^2 \frac{1}{1+4x^2} dx$

$\Rightarrow L = \frac{1}{2}\tan^{-1}(2x)\Big|_0^2 \Rightarrow L = \frac{1}{2}\tan^{-1} 4$

15. Official Ans. by NTA (2)

Sol. $I = \int_0^5 \frac{x+[x]}{e^{x-[x]}} dx$

$\int_0^1 \frac{x}{e^x} dx + \int_1^2 \frac{x+1}{e^{x-1}} dx + \int_2^3 \frac{x+2}{e^{x-2}} dx + \ldots\ldots \int_4^5 \frac{x+4}{e^{x-4}} dx$

$\Downarrow \qquad \Downarrow \qquad \Downarrow$

$x = t + 1 \qquad x = z + 2 \qquad x = y + 4$

$\int_0^1 \frac{t+2}{e^t} dt + \int_0^1 \frac{z+4}{e^z} dz + \ldots\ldots + \int_0^1 \frac{y+8}{e^y} dx$

$\Rightarrow \int_0^5 \frac{5x+20}{e^x} dt = 5\int_0^1 \frac{x+4}{e^x} dx$

$\Rightarrow 5\int_0^1 (x+4)e^{-x} dx$

$\Rightarrow 5e^{-x}(-x-5)\big|_0^1 \Rightarrow -\frac{30}{e} + 25$

$\alpha = -30$

$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$

$(\alpha + \beta)^2 = 5^2 = 25$

16. Official Ans. by NTA (3)

Sol. $I=\int_0^{\pi/2}\frac{(1+\sin^2 x)}{(1+\pi^{\sin x})}+\frac{\pi^{\sin x}(1+\sin^2 x)}{(1+\pi^{\sin x})}dx$

$I=\int_0^{\pi/2}(1+\sin^2 x)dx$

$I=\frac{\pi}{2}+\frac{\pi}{2}\cdot\frac{1}{2}=\frac{3\pi}{4}$

17. Official Ans. by NTA (1)

Sol. $U_n=\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r$

$L=\lim_{n\to\infty}(U_n)^{-4/n^2}$

$\log L=\lim_{n\to\infty}\frac{-4}{n^2}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)^r$

$\Rightarrow \log L=\lim_{n\to\infty}\sum_{r=1}^{n}-\frac{4r}{n}\cdot\frac{1}{n}\log\left(1+\frac{r^2}{n^2}\right)$

$\Rightarrow \log L\Rightarrow -4\int_0^1 x\log(1+x^2)dx$

put $1+x^2=t$

Now, $2xdx=dt$

$=-2\int_1^2\log(t)dt=-2[t\log t-t]_1^2$

$\Rightarrow \log L=-2(2\log 2-1)$

$\therefore\ L=e^{-2(2\log 2-1)}$

$=e^{-2\left(\log\left(\frac{4}{e}\right)\right)}$

$=e^{\log\left(\frac{4}{e}\right)^{-2}}$

$=\left(\frac{e}{4}\right)^2=\frac{e^2}{16}$

18. Official Ans. by NTA (3)

Sol. Let $I=\int_6^{16}\frac{\log_e x^2}{\log_e x^2+\log_e(x^2-44x+484)}dx$

$I=\int_6^{16}\frac{\log_e x^2}{\log_e x^2+\log_e(x-22)^2}dx$ ...(1)

We know

$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ (king)

So

$I=\int_6^{16}\frac{\log_e(22-x)^2}{\log_e(22-x)^2+\log_e(22-(22-x))^2}$

$I=\int_0^{16}\frac{\log_e(22-x)^2}{\log_e x^2+\log_e(22-x)^2}dx$ ...(2)

(1) + (2)

$2I=\int_6^{16}1.dx=10$

$I=5$

19. Official Ans. by NTA (1)

Sol. $I=\int_0^1\frac{\sqrt{x}}{(1+x)(1+3x)(3+x)}dx$

Let $x=t^2\ \Rightarrow\ dx=2t.dt$

$I=\int_0^1\frac{t(2t)}{(t^2+1)(1+3t^2)(3+t^2)}dt$

$I=\int_0^1\frac{(3t^2+1)-(t^2+1)}{(3t^2+1)(t^2+1)(3+t^2)}dt$

$I=\int_0^1\frac{dt}{(t^2+1)(3+t^2)}-\int_0^1\frac{dt}{(1+3t^2)(3+t^2)}$

$=\frac{1}{2}\int_0^1\frac{(3+t^2)-(t^2+1)}{(t^2+1)(3+t^2)}dt+\frac{1}{8}\int_0^1\frac{(1+3t^2)-3(3+t^2)}{(1+3t^2)(3+t^2)}dt$

$=\frac{1}{2}\int_0^1\frac{dt}{1+t^2}-\frac{1}{2}\int_0^1\frac{dt}{t^2+3}$

$+\frac{1}{8}\int_0^1\frac{dt}{t^2+3}-\frac{3}{8}\int_0^1\frac{dt}{(1+3t^2)}$

$$=\frac{1}{2}\int_0^1\frac{dt}{t^2+1}-\frac{3}{8}\int_0^1\frac{dt}{t^2+3}-\frac{3}{8}\int_0^1\frac{dt}{1+3t^2}$$

$$=\frac{1}{2}\left(\tan^{-1}(t)\right)_0^1-\frac{3}{8\sqrt{3}}\left(\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right)_0^1$$

$$-\frac{3}{8\sqrt{3}}\left(\tan^{-1}\left(\sqrt{3}t\right)\right)_0^1$$

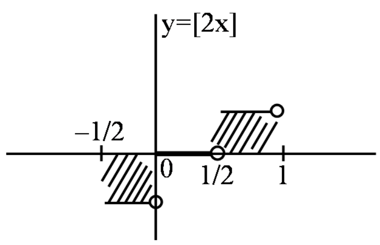
$$=\frac{1}{2}\left(\frac{\pi}{4}\right)-\frac{\sqrt{3}}{8}\left(\frac{\pi}{6}\right)-\frac{\sqrt{3}}{8}\left(\frac{\pi}{3}\right)$$

$$=\frac{\pi}{8}-\frac{\sqrt{3}}{16}\pi$$

$$=\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$$

20. Official Ans. by NTA (5)

Sol. $I=\int_{-1/2}^{1}\left([2x]+|x|\right)dx$



$$=\int_{-1/2}^{1}[2x]\,dx+\int_{-1/2}^{1}|x|\,dx$$

$$=0+\int_{-1/2}^{0}(-x)\,dx+\int_0^1 x\,dx$$

$$=\left(-\frac{x^2}{2}\right)_{-1/2}^{0}+\left(\frac{x^2}{2}\right)_0^1$$

$$=\left(0+\frac{1}{8}\right)+\frac{1}{2}$$

$$=\frac{5}{8}$$

$8I = 5$

21. Official Ans. by NTA (4)

Sol. $x\phi(x)=\int_5^x 3t^2-2\phi'(t)\,dt$

$x\phi(x) = x^3 - 125 - 2[\phi(x) - \phi(5)]$

$x\phi(x) = x^3 - 125 - 2\phi(x) - 2\phi(5)$

$\phi(0) = 4 \Rightarrow \phi(5) = -\frac{133}{2}$

$\phi(x) = \frac{x^3+8}{x+2}$

$\phi(2) = 4$

22. Official Ans. by NTA (2)

Sol. $\pi^2\left[\int_0^1\sin\frac{\pi x}{2}dx+\int_1^2\sin\frac{\pi x}{2}(x-1)\,dx\right]$

$$=\pi^2\left[-\frac{2}{\pi}\left(\cos\frac{\pi x}{2}\right)+\left((x-1)\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}\right)\right)_1^2-\int_1^2-\frac{2}{\pi}\cos\frac{\pi x}{2}dx\right]$$

$$=\pi^2\left[0+\frac{2}{\pi}+\frac{2}{\pi}+\frac{2}{\pi}.\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_1^2\right]$$

$= 4\pi - 4 = 4(\pi - 1)$

23. Official Ans. by NTA (4)

Sol. $f(x)=x+\int_0^{\pi/2}\sin x\cos y f(y)\,dy$

$$f(x)=x+\sin x\underbrace{\int_0^{\pi/2}\cos y f(y)\,dy}_{K}$$

$\Rightarrow f(x) = x + K\sin x$

$\Rightarrow f(y) = y + K\sin y$

Now $K=\int_0^{\pi/2}\cos y\left(y+K\sin y\right)dy$

$$K=\underbrace{\int_0^{\pi/2}y\cos y\,dy}_{\text{Apply IBP}}+\underbrace{\int_0^{\pi/2}\cos y\sin y\,dy}_{\text{Put }\sin y=t}$$

$$K=\left(y\sin y\right)_0^{\pi/2}-\int_0^{\pi/2}\sin dy+K\int_0^1 t\,dt$$

$\Rightarrow K=\frac{\pi}{2}-1+K\left(\frac{1}{2}\right)$

$\Rightarrow K = \pi - 2$

So $f(x) = x + (\pi - 2)\sin x$

Option (4)

24. Official Ans. by NTA (2)

Sol. $\int_1^3 \left(\left[(x-1)^2\right]-3\right)dx$

$=\int_1^2 [x^2]-3\int_1^3 dx$

$=\int_1^3 0\,dx+\int_1^{\sqrt{2}} 1.dx+\int_{\sqrt{2}}^{\sqrt{3}} 2.dx+\int_{\sqrt{3}}^{2} 3.dx-6$

$=\sqrt{2}-1+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})-6$

$=-\sqrt{2}-\sqrt{3}-1$

25. Official Ans. by NTA (2)

Sol. $f'(x)=f'(2-x)$

$f(x) = -f(2-x) + c$

put $x = 0$

$f'(0)=-f'(2)+c$

$c = f(0) + f(2) = 1 + e^2$

so, $f(x) + f(2-x) = 1 + e^2$

$I=\int_0^2 f(x)\,dx$

$I=\int_0^2 f(2-x)\,dx$

$2I=\int_0^2 (f(x)+f(2-x))\,dx$

$2I=(1+e^2)\int_0^2 dx$

$I = 1 + e^2$

26. Official Ans. by NTA (3)

Sol. $\int_{-a}^{0}(-2x+2)dx+\int_0^2 (x+2-x)dx+\int_2^a (2x-2)dx=22$

$x^2-2x\Big|_0^{-a}+2x\Big|_0^2+x^2-2x\Big|_0^a=22$

$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$\int_3^{-3}(x+[x])dx=-(-3-2-1+1+2)=3$

27. Official Ans. by NTA (3)

Sol. $I=\int_{-1}^{1} x^2e^{[x^3]}dx$

$=\int_{-1}^{0} x^2e^{[x^3]}dx+\int_0^1 x^2e^{[x^3]}dx$

$=\int_{-1}^{0} x^2e^{-1}dx+\int_0^1 x^2e^{0}dx$

$=\frac{1}{e}\times\frac{x^3}{3}\Big|_{-1}^{0}+\frac{x^3}{3}\Big|_0^1$

$=\frac{1}{e}\times\left(0-\left(\frac{-1}{3}\right)\right)+\frac{1}{3}$

$=\frac{1}{3e}+\frac{1}{3}=\frac{1+e}{3e}$

28. Official Ans. by NTA (1)

Sol. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+...+\frac{n}{(2n-1)^2}\right]$

$=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{n^2+2nr+r^2}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{(r/n)^2+2(r/n)+1}$

$=\int_0^1\frac{dx}{(x+1)^2}=\left[\frac{-1}{(x+1)}\right]_0^1=\frac{1}{2}$

29. Official Ans. by NTA (19)

Sol. $\int_{-2}^{2} 3\left|x^2-x-2\right|dx$

$=3\int_{-2}^{2}\left|x^2-x-2\right|dx$

$=3\left[\int_{-2}^{-1}(x^2-x-2)\,dx+\int_{-1}^{2}-(x^2-x-2)\,dx\right]$

$=3\left[\left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)\Big|_{-2}^{-1}-\left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)_{-1}^{2}\right]$

$=3\left[7-\frac{2}{3}\right]$

$= 19$

30. Official Ans. by NTA (3)

Sol. $f(x)=\int_1^x \frac{\log_e t}{(1+t)}dt$

$f\left(\frac{1}{x}\right)=\int_1^{1/x}\frac{\ell nt}{1+t}dt$, let $t=\frac{1}{y}$

$=+\int_1^x \frac{\ell ny}{1+y}\cdot\frac{y}{y^2}dy$

$=\int_1^x \frac{\ell ny}{y(1+y)}dy$

hence

$f(x)+f\left(\frac{1}{x}\right)=\int_1^x \frac{(1+t)\ell nt}{t(1+t)}dt=\int_1^x \frac{\ell nt}{t}dt$

$=\frac{1}{2}\ell n^2(x)$

so $f(e)+f\left(\frac{1}{e}\right)=\frac{1}{2}$ …(3)

31. Official Ans. by NTA (1)

Sol. $f(x)=\int_0^x e^t f(t)\,dt+e^x \Rightarrow f(0)=1$

differentiating with respect to x

$f'(x) = e^x f(x) + e^x$

$f'(x) = e^x(f(x) + 1)$

$\int_0^x \frac{f'(x)}{f(x)+1}dx=\int_0^x e^x\,dx$

$\ell n(f(x)+1)\Big|_0^x=e^x\Big|_0^x$

$\ell n(f(x) + 1) - \ell n(f(0) + 1) = e^x - 1$

$\ell n\left(\frac{f(x)+1}{2}\right)=e^x-1$ $\{$as $f(0) = 1\}$

$f(x)=2e^{(e^x-1)}-1$

32. Official Ans. by NTA (1)

Sol. $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx=I_{n,m}$

Now Let $x=\frac{1}{y+1}\Rightarrow dx=-\frac{1}{(y+1)^2}dy$

so

$I_{m,n}=-\int_\infty^0 \frac{1}{(y+1)^{m-1}}\frac{y^{n-1}}{(y+1)^{n-1}}\frac{dy}{(y+1)^2}=\int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}dy$

similarly $I_{m,n}=\int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}dy$

Now $2I_{m,n}=\int_0^\infty \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy$

$=\int_0^\infty \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy$

$=\int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy+\underbrace{\int_1^\infty \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy}_{\text{substitute } y=\frac{1}{t}}$

$\Rightarrow 2I_{m,n}=\int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy-\int_1^0 \frac{t^{n-1}+t^{m-1}}{t^{m+n-2}}\frac{t^{m+n}}{(1+t)^{m+n}}\frac{dt}{t^2}$

$\Rightarrow$ Hence $2I_{m,n}=2\int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}dy\Rightarrow\alpha=1$

33. Official Ans. by NTA (2)

Sol. Put $2x = t \Rightarrow 2dx = dt$

$\Rightarrow I=\frac{1}{2}\int_0^{2\pi}|\sin t|dt$

$=\int_0^\pi |\sin t|dt$

$= 2$

34. Official Ans. by NTA (1)

Sol. $I=\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx$ (using king)

$I=\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^{-x}}dx=\int_{-\pi/2}^{\pi/2}\frac{3^x\cos^2 x}{1+3^x}dx$

$2I=\int_{-\pi/2}^{\pi/2}\frac{(1+3^x)\cos^2 x}{1+3^x}dx$

$=\int_{-\pi/2}^{\pi/2}\cos^2 x dx=2\int_0^{\pi/2}\cos^2 x\,dx$

$\Rightarrow I=\int_0^{\pi/2}\cos^2 x\,dx=\frac{\pi}{4}$

35. Official Ans. by NTA (4)

Sol. $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx = \int_{\pi/4}^{\pi/2} \cot^{n-2} x(\operatorname{cosec}^2 x - 1)\,dx$

$= -\left.\frac{\cot^{n-1} x}{n-1}\right]_{\pi/4}^{\pi/2} - I_{n-2}$

$= \frac{1}{n-1} - I_{n-2}$

$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$

$\Rightarrow I_2 + I_4 = \frac{1}{3}$

$I_3 + I_5 = \frac{1}{4}$

$I_4 + I_6 = \frac{1}{5}$

$\therefore \frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

36. Official Ans. by NTA (1)

Sol. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}dx$, period of $\{x\} = 1$

$\sum_{n=1}^{100} \int_0^1 e^{\{x\}}\,dx = \sum_{n=1}^{100} \int_0^1 e^x dx$

$\sum_{n=1}^{100} (e-1) = 100(e-1)$

37. Official Ans by NTA (3)

Sol. $I = \int_0^{10} [x]\cdot e^{[x]-x+1}$

$I = \int_0^1 0dx + \int_1^2 1\cdot e^{2-x} + \int_2^3 2\cdot e^{3-x} + ... + \int_9^{10} 9\cdot e^{10-x}dx$

$\Rightarrow \; I = \sum_{n=0}^{9} \int_n^{n+1} n\cdot e^{n+1-x}dx$

$= -\sum_{n=0}^{9} n\left(e^{n+1-x}\right)_n^{n+1}$

$= -\sum_{n=0}^{9} n\cdot(e^0 - e^1)$

$= (e-1)\sum_{n=0}^{9} n$

$= (e-1)\cdot\frac{9\cdot 10}{2}$

$= 45(e-1)$

38. Official Ans by NTA (3)

Sol. $\int_0^1 (x^2 + bx + c)dx = 1$

$\frac{1}{3} + \frac{b}{2} + c = 1 \;\Rightarrow\; \frac{b}{2} + c = \frac{2}{3}$

$3b + 6c = 4$ ...(1)

$P(2) = 5$

$4 + 2b + c = 5$

$2b + c = 1$ ...(2)

From (1) & (2)

$b = \frac{2}{9}$ & $c = \frac{5}{9}$

$9(b + c) = 7$

39. Official Ans. by NTA (16)

Sol. $f(x) + f(x + 1) = 2$

$\Rightarrow$ $f(x)$ is periodic with period = 2

$I_1 = \int_0^8 f(x)dx = 4\int_0^2 f(x)dx$

$= 4\int_0^1 \left(f(x) + f(1+x)\right)dx = 8$

Similarly $I_2 = 2 \times 2 = 4$

$I_1 + 2I_2 = 16$

40. Official Ans. by NTA (2)

Sol. $f(x) = e^{-x}\sin x$

Now, $F(x) = \int_0^x f(t)dt \quad \Rightarrow F'(x) = f(x)$

$I = \int_0^1 \left(F'(x) + f(x)\right)e^x\,dx = \int_0^1 \left(f(x) + f(x)\right)\cdot e^x\,dx$

$= 2\int_0^1 f(x)\cdot e^x\,dx = 2\int_0^1 e^{-x}\sin x\cdot e^x\,dx$

$= 2\int_0^1 \sin x\,dx$

$= 2(1 - \cos 1)$

$I = 2\left\{1 - \left(1 - \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \frac{1}{\lfloor 8}.........\right)\right\}$

$$I = 1 - \frac{2}{\lfloor 4} + \frac{2}{\lfloor 6} - \frac{2}{\lfloor 9} + ...$$

$$1 - \frac{2}{\lfloor 4} < I < 1 - \frac{2}{\lfloor 4} + \frac{2}{\lfloor 6}$$

$$\frac{11}{12} < I < \frac{331}{360}$$

$$\Rightarrow I \in \left[\frac{11}{12}, \frac{331}{360}\right]$$

$$\Rightarrow I \in \left[\frac{330}{360}, \frac{331}{360}\right]$$ **Ans. (2)**

41. Official Ans. by NTA (1)

Sol. Let $I = \int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \int_0^{10} \frac{[\sin 2\pi x]}{e^{\{x\}}} dx$

Function $f(x) = \frac{[\sin 2\pi x]}{e^{\{x\}}}$ is periodic with period '1'

Therefore

$$I = 10\int_0^1 \frac{[\sin 2\pi x]}{e^{\{x\}}} dx$$

$$= 10\int_0^1 \frac{[\sin 2\pi x]}{e^x} dx$$

$$= 10\left(\int_0^{1/2} \frac{[\sin 2\pi x]}{e^x} dx + \int_{1/2}^1 \frac{[\sin 2\pi x]}{e^x} dx\right)$$

$$= 10\left(0 + \int_{1/2}^1 \frac{(-1)}{e^x} dx\right)$$

$$= -10\int_{1/2}^1 e^{-x} dx$$

$$= 10\left(e^{-1} - e^{-1/2}\right)$$

Now,

$10 \cdot e^{-1} - 10 \cdot e^{-1/2} = \alpha e^{-1} + \beta e^{-1/2} + \gamma$ (given)

$\Rightarrow \alpha = 10, \beta = -10, \gamma = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$ **Ans. (1)**

42. Official Ans. by NTA (1)

Sol. $I_n = \int_1^e x^{19}(\log|x|)^n dx$

$$I_n = \left|(\log|x|)^{19} \frac{x^{20}}{20}\right|_1^e - \int n(\log|x|)^{n-1} . \frac{1}{x} . \frac{x^{20}}{20} dx$$

$20I_n = e^{20} - nI_{n-1}$

$\therefore 20I_{10} = e^{20} - 10I_9$

$20I_9 = e^{20} - 9I_8$

$\Rightarrow 20I_{10} = 10I_9 + 9I_8$

$\alpha = 10, \beta = 9$

43. Official Ans. by NTA (Bonus)

Sol. $g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3} \frac{\sin^\alpha x}{(\sin^\alpha x + \cos^\alpha x)}$ .....(i)

$g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3} \frac{\cos^\alpha x}{(\sin^\alpha x + \cos^\alpha x)}$ .....(ii)

(1) + (2)

$2g(\alpha) = \frac{\pi}{6}$

$g(\alpha) = \frac{\pi}{12}$

Constant and even function

Due to typing mistake it must be bonus.

44. Official Ans. by NTA (512)

Sol. $I = 2\int_0^4 f(x^2)dx$ {Even funtion}

$$= 2\int_0^4 \left(4x^3 - g(4-x)\right)dx$$

$$= 2\left(\left.\frac{4x^4}{4}\right|_0^4 - \int_0^4 g(4-x)dx\right)$$

$= 2(256 - 0) = 512$

45. Official Ans. by NTA (3)

Sol. $\frac{1}{3} \le f(t) \le 1 \ \forall\ t \in [0,1]$

$0 \le f(t) \le \frac{1}{2} \ \forall\ t \in (1,3]$

Now, $g(3) = \int_0^3 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt$

$\because \int_0^1 \frac{1}{3} dt \le \int_0^1 f(t)\,dt \le \int_0^1 1.\,dt$ ....(1)

and $\int_1^3 0\,dt \le \int_1^3 f(1)\,dt \le \int_1^3 \frac{1}{2} dt$ ....(2)

Adding, we get

$\frac{1}{3} + 0 \le g(3) \le 1 + \frac{1}{2}(3-1)$

$\frac{1}{3} \le g(3) \le 2$

DIFFERENTIAL EQUATION

1. Official Ans. by NTA (1)

Sol. We have

$$\frac{dy}{dx}=\frac{x\left(\frac{y}{x}\cdot\tan\frac{y}{x}-1\right)}{x\tan\frac{y}{x}}$$

$$\therefore \frac{dy}{dx}=\frac{y}{x}-\cot\left(\frac{y}{x}\right)$$

Put $\frac{y}{x}=v$

$\Rightarrow y=vn$

$$\therefore \frac{dy}{dx}=v+\frac{ndv}{dx}$$

Now, we get

$$v+n\frac{dv}{dx}=v-\cot(v)$$

$$\Rightarrow \int(\tan)dv=-\int\frac{dx}{x}$$

$$\therefore \ell n\left|\sec\left(\frac{y}{x}\right)\right|=-\ell n|x|+c$$

$$\text{As}\left(\frac{1}{2}\right)=\left(\frac{y}{x}\right)\Rightarrow \boxed{C=0}$$

$$\therefore \sec\left(\frac{y}{x}\right)=\frac{1}{x}$$

$$\Rightarrow \cos\left(\frac{y}{x}\right)=x$$

$$\therefore \boxed{y=x\cos^{-1}(x)}$$

So, required bounded area

$$=\int_0^{1/\sqrt{2}}\underset{(II)}{x}\,(\underset{(I)}{\cos^{-1}x})dx=\left(\frac{\pi-1}{8}\right)$$

(I.B.P.)

$\therefore$ option (1) is correct.

2. Official Ans. by NTA (2)

Sol. $e^x\sqrt{1-y^2}dx+\frac{y}{x}dy=0$

$$\Rightarrow e^x\sqrt{1-y^2}dx+\frac{-y}{x}dy$$

$$\Rightarrow \int\frac{-y}{\sqrt{1-y^2}}dy=\int\underset{II}{e^x}\ \underset{1}{x}\ dx$$

$$\Rightarrow \sqrt{1-y^2}=e^x(x-1)+c$$

Given : At $x=1,\ y=-1$

$\Rightarrow 0=0+c \Rightarrow c=0$

$$\therefore \sqrt{1-y^2}=e^x(x-1)$$

At $x=3$ $\quad 1-y^2=(e^3\,2)^2\Rightarrow y^2=1-4e^6$

3. Official Ans. by NTA (1)

Sol. $I=\int_{-\pi/2}^{\pi/2}([x]+[-\sin x])dx$ .....(i)

$$I=\int_{-\pi/2}^{\pi/2}([-x]+[\sin x])dx \quad ....(2)$$

(King property)

$$2I=\int_{-\pi/2}^{\pi/2}\left(\underbrace{[x]+[-x]}_{-1}\right)+\left(\underbrace{[\sin x]+[-\sin x]}_{-1}\right)dx$$

$$2I=\int_{-\pi/2}^{\pi/2}(-2)dx=-2(\pi)$$

$I=-\pi$

4. Official Ans. by NTA (4)

Sol. $I=\sum_{r=0}^{n-1}f\left(\frac{5r}{n}\right)\frac{1}{n}$

$$I=\int_0^1 f(5x)dx$$

$$I=\int_0^1(5x+1)dx$$

$$I=\left[\frac{5x^2}{2}+x\right]_0^1$$

$$I=\frac{5}{2}+1=\frac{7}{2}$$

5. Official Ans. by NTA (2)

Sol. $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx=\sqrt{e^{2x}-1}\,dy$

Put $\cos^{-1}(e^{-x})\ \theta$, $\theta\in[0,\pi]$

$\cos\theta=e^{-x}\Rightarrow 2\cos^2\frac{\theta}{2}-1=e^{-x}$

$$\cos\frac{\theta}{2}=\sqrt{\frac{e^{-x}+1}{2}}=\sqrt{\frac{e^x+1}{2c^x}}$$

$$\sqrt{\frac{e^x+1}{2c^x}}dx=\sqrt{e^{2x}-1}\,dy$$

$$\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{e^x}\sqrt{e^x-1}}=\int dy$$

Put $e^x=t,\ \frac{dt}{dx}=e^x$

$$\frac{1}{\sqrt{2}}\int\frac{dt}{e^x\sqrt{e^x}\sqrt{e^x-1}}=\int dy$$

$$\int\frac{dt}{t\sqrt{t^2-t}}=\sqrt{2}y$$

Put $t=\frac{1}{z},\frac{dt}{dz}=-\frac{1}{z^2}$

$$\int \frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}-\frac{1}{z}}} = \sqrt{2}y$$

$$-\int \frac{dz}{\sqrt{1-z}} = \sqrt{2}y$$

$$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$$

$$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}y + c$$

$$2\left(1-e^{-x}\right)^{1/2} = \sqrt{2y} + c \xrightarrow{(0,-1)} \Rightarrow c = \sqrt{2}$$

$2(1-e^{-x})^{1/2} = \sqrt{2}(y+1)$, passes through $(\alpha, 0)$

$$2(1-e^{-\alpha})^{1/2} = \sqrt{2}$$

$$\sqrt{1-e^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1-e^{-\alpha} = \frac{1}{2}$$

$$e^{-\alpha} = \frac{1}{2} \Rightarrow e^{\alpha} = 2$$

6. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} + 2\sin^2 x = 1 + y\cos 2x$

$$\Rightarrow \frac{dy}{dx} + (-\cos 2x)y = \cos 2x$$

$$\text{I.F.} = e^{\int -\cos 2x\, dx} = e^{-\frac{\sin 2x}{2}}$$

Solution of D.E.

$$y\left(e^{-\frac{\sin 2x}{2}}\right) = \int (\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx + c$$

$$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + c$$

Given

$$y\left(\frac{\pi}{4}\right) = 0$$

$$\Rightarrow 0 = -e^{-1/2} + c \Rightarrow \boxed{c = e^{-1/2}}$$

$$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + e^{-1/2}$$

at x = 0

$$y = -1 + e^{-1/2}$$

$$\Rightarrow y(0) = -1 + e^{-1/2} \Rightarrow (y(0)+1)^2 = e^{-1}$$

7. Official Ans. by NTA (4)

Sol. $y + 1 = Y \Rightarrow dy = dY$

$x + 2 = X \Rightarrow dx = dX$

$$\Rightarrow \left(Xe^{\frac{Y}{X}} + Y\right)dX = XdY$$

$$\Rightarrow XdY - YdX = Xe^{Y/X}dX$$

$$\Rightarrow d\left(\frac{Y}{X}\right)e^{-\frac{Y}{X}} = \frac{dX}{X}$$

$$-e^{-Y/X} = \ell|X| + c$$

$$(3, 2) \rightarrow -e^{-2/3} = \ell|3| + c$$

$$-e^{-\frac{Y}{X}} = \ell n|X| - e^{-\frac{2}{3}} - \ell n3$$

$$e^{-\frac{Y}{X}} = e^{2/3} + \ell n3 - \ell n|X| > 0$$

$$\ell n|X| < \left(e^{2/3} + \ell n3\right)$$

Let $\lambda = \left(e^{2/3} + \ell n3\right)$

$|x + 2| < e^{\lambda}$

$-e^{\lambda} < x + 2 < e^{\lambda}$

$-e^{\lambda} - 2 < x < e^{\lambda} - 2$

$\alpha \qquad \beta$

$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$

Although x = –2 should be excluded from domain but according to the given problem it will the most appropriate solution.

8. Official Ans. by NTA (4)

Sol. $\frac{dy - dx}{e^{y-x}} = x\,dx$

$$\Rightarrow \frac{dy - dx}{e^{y-x}} = x\,dx$$

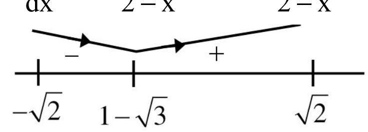
$$\Rightarrow -e^{x-y} = \frac{x^2}{2} + c$$

At x = 0, y = 0 ⇒ c = –1

$$\Rightarrow e^{x-y} = \frac{2-x^2}{2}$$

$$\Rightarrow y = x - \ell n\left(\frac{2-x^2}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{2x}{2-x^2} = \frac{2+2x-x^2}{2-x^2}$$



So minimum value occurs at $x = 1-\sqrt{3}$

$$y\left(1-\sqrt{3}\right)=\left(1-\sqrt{3}\right)-\ell n\left(\frac{2-\left(4-2\sqrt{3}\right)}{2}\right)$$

$$=\left(1-\sqrt{3}\right)-\ell n\left(\sqrt{3}-1\right)$$

9. Official Ans. by NTA (4)

Sol. Let $e^y = t$

$$\Rightarrow \frac{dt}{dx}-(2\sin x)t=-\sin x\cos^2 x$$

I.F. $= e^{2\cos x}$

$$\Rightarrow t\cdot e^{2\cos x}=\int e^{2\cos x}.(-\sin x\cos^2 x)\,dx$$

$$\Rightarrow e^y\cdot e^{2\cos x}=\int e^{2z}\cdot z^2dz, z=e^{2\cos x}$$

$$\Rightarrow e^y\cdot e^{2\cos x}=\frac{1}{2}.\cos^2 x\cdot e^{2\cos x}-\frac{1}{2}\cos x\cdot e^{2\cos x}+\frac{e^{2\cos x}}{4}+C$$

at $x=\frac{\pi}{2}, y=0 \Rightarrow C=\frac{3}{4}$

$$\Rightarrow e^y=\frac{1}{2}\cos^2 x-\frac{1}{2}\cos x+\frac{1}{4}+\frac{3}{4}\cdot e^{-2\cos x}$$

$$\Rightarrow y=\log\left[\frac{\cos^2 x}{2}-\frac{\cos x}{2}+\frac{1}{4}+\frac{3}{4}e^{-2\cos x}\right]$$

Put x = 0

$$\Rightarrow y=\log\left[\frac{1}{4}+\frac{3}{4}e^{-2}\right]\Rightarrow \alpha=\frac{1}{4},\beta=\frac{3}{4}$$

10. Official Ans. by NTA (1)

Sol. $xdy = (y + x^3\cos x)dx$

$xdy = ydx + x^3\cos x dx$

$$\frac{xdy-ydx}{x^2}=\frac{x^3\cos x\,dx}{x^2}$$

$$\frac{d}{dx}\left(\frac{y}{x}\right)=\int x\cos x\,dx$$

$$\Rightarrow \frac{y}{x}=x\sin x-\int 1.\sin x\,dx$$

$$\frac{y}{x}=x\sin x+\cos x+C$$

$\Rightarrow 0 = -1 + C \Rightarrow C = 1, x = \pi, y = 0$

so $\frac{y}{x} = x\sin x + \cos x + 1$

$y = x^2\sin x + x\cos x + x \qquad x=\frac{\pi}{2}$

$$y\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}+\frac{\pi}{2}$$

11. Official Ans. by NTA (1)

Sol. $y'=\frac{2y}{x\ell nx}$

$$\Rightarrow \frac{dy}{y}=\frac{2dx}{x\ell nx}$$

$\Rightarrow \ell n|y| = 2\ell n|\ell nx| + C$

put $x = 2, y = (\ell n2)^2$

$\Rightarrow c = 0$

$\Rightarrow y = (\ell nx)^2$

$\Rightarrow f(e) = 1$

12. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}=e^{3x}.e^{4y}\Rightarrow\int e^{-4y}dy=\int e^{3x}dx$

$$\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+C\Rightarrow-\frac{1}{4}-\frac{1}{3}=C\Rightarrow C=-\frac{7}{12}$$

$$\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}-\frac{7}{12}\Rightarrow e^{-4y}=\frac{4e^{3x}-7}{-3}$$

$$e^{4y}=\frac{3}{7-4e^{3x}}\Rightarrow 4y=\ell n\left(\frac{3}{7-4e^{3x}}\right)$$

$$4y=\ell n\left(\frac{3}{6}\right)\text{ when } x=-\frac{2}{3}\ell n2$$

$$y=\frac{1}{4}\ell n\left(\frac{1}{2}\right)=-\frac{1}{4}\ell n2$$

13. Official Ans. by NTA (16)

Sol. F(3) = 0

$$e^xF(x)=\int_3^x\left(3t^2+2t+4F'(t)\right)dt$$

$$e^xF(x)+e^xF'(x)=3x^2+2x+4F'(x)$$

$$\left(e^x-4\right)\frac{dy}{dx}+e^xy=\left(3x^2+2x\right)$$

$$\frac{dy}{dx}+\frac{e^x}{(e^x-4)}y=\frac{(3x^2+2x)}{(e^x-4)}$$

$$ye^{\int\frac{e^x}{(e^x-4)}dx}=\int\frac{(3x^2+2x)}{(e^x-4)}e^{\int\frac{e^x}{e^x-4}dx}dx$$

$$y.(e^x-4)=\int(3x^2+2x)dx+c$$

$$y\left(e^x-4\right)=x^3+x^2+c$$

Put $x = 3 \Rightarrow c = -36$

$$F(x)=\frac{(x^3+x^2-36)}{(e^x-4)}$$

$$F'(x)=\frac{(3x^2+2x)(e^x-4)-(x^3+x^2-36)e^x}{(e^x-4)^2}$$

Now put value of $x = 4$ we will get $\alpha = 12$ & $\beta = 4$

14. Official Ans. by NTA (2)

Sol. $\sec y\dfrac{dy}{dx} = 2\sin x\cos y$

$\sec^2 y dy = 2\sin x dx$

$\tan y = -2\cos x + c$

$c = 2$

$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \dfrac{\pi}{2}$

$\tan y = 2$

$\sec^2 y\dfrac{dy}{dx} = 2\sin x$

$5\dfrac{dy}{dx} = 2$

15. Official Ans. by NTA (2)

Sol. $(x - x^3)dy = (y + yx^2 - 3x^4)dx$

$\Rightarrow xdy - ydx = (yx^2 - 3x^4)dx + x^3dy$

$\Rightarrow \dfrac{xdy - ydx}{x^2} = (ydx + xdy) - 3x^2dx$

$\Rightarrow d\left(\dfrac{y}{x}\right) = d(xy) - d(x^3)$

Integrate

$\Rightarrow \dfrac{y}{x} = xy - x^3 + c$

given $f(3) = 3$

$\Rightarrow \dfrac{3}{3} = 3 \times 3 - 3^3 + c$

$\Rightarrow c = 19$

$\therefore \dfrac{y}{x} = xy - x^3 + 19$

at $x = 4$, $\dfrac{y}{4} = 4y - 64 + 19$

$15y = 4 \times 45$

$\Rightarrow y = 12$

16. Official Ans. by NTA (2)

Sol. $\int e^{-y}dy = \int e^{\alpha x}\,dx$

$\Rightarrow e^{-y} = \dfrac{e^{\alpha x}}{\alpha} + c$ ...(i)

Put $(x,y) = (\ell n2, \ell n2)$

$\dfrac{-1}{2} = \dfrac{2^\alpha}{\alpha} + C$ ...(ii)

Put $(x,y) \equiv (0, -\ell n2)$ in (i)

$-2 = \dfrac{1}{\alpha} + C$ ...(iii)

(ii) – (iii)

$\dfrac{2^\alpha - 1}{\alpha} = \dfrac{3}{2}$

$\Rightarrow \alpha = 2$ (as $\alpha \in \mathbb{N}$)

17. Official Ans. by NTA (4)

Sol. $(y + 1)\tan^2x\, dx + \tan x\, dy + y\, dx = 0$

or $\dfrac{dy}{dx} + \dfrac{\sec^2 x}{\tan x}.y = -\tan x$

$IF = e^{\int \frac{\sec^2 x}{\tan x}dx} = e^{\ln\tan x} = \tan x$

$\therefore y\tan x = -\int \tan^2 x dx$

or $y\tan x = -\tan x + x + C$

or $y = -1 + \dfrac{x}{\tan x} + \dfrac{C}{\tan x}$

or $\lim\limits_{x\to 0} xy = -x + \dfrac{x^2}{\tan x} + \dfrac{Cx}{\tan x} = 1$

or $C = 1$

$y(x) = \cot x + x\cot x - 1$

$y\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{4}$

18. Official Ans. by NTA (3)

Sol. $2x^2dy + (e^y - 2x)dx = 0$

$\dfrac{dy}{dx} + \dfrac{e^y - 2x}{2x^2} = 0 \Rightarrow \dfrac{dy}{dx} + \dfrac{e^y}{2x^2} - \dfrac{1}{x} = 0$

$e^{-y}\dfrac{dy}{dx} - \dfrac{e^{-y}}{x} = -\dfrac{1}{2x^2} \Rightarrow$ Put $e^{-y} = z$

$\dfrac{-dz}{dx} - \dfrac{z}{x} = -\dfrac{1}{2x^2} \Rightarrow xdz + zdx = \dfrac{dx}{2x}$

$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$

$xe^{-y} = \frac{1}{2}\log_e x + c$, passes through $(e,1)$

$\Rightarrow C = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

19. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} - 2xy = 2(2\sin x - 5)x - 2\cos x$

$IF = e^{-x^2}$

so,

$y.e^{-x^2} = \int e^{-x^2}(2x(2\sin x - 5) - 2\cos x)dx$

$\Rightarrow y.e^{-x^2} = e^{-x^2}(5 - 2\sin x) + c$

$\Rightarrow y = 5 - 2\sin x + c.e^{x^2}$

Given at x = 0, y = 7

$\Rightarrow 7 = 5 + c \Rightarrow c = 2$

So, $y = 5 - 2\sin x + 2e^{x^2}$

Now at $x = \pi$,

$y = 5 + 2e^{\pi^2}$

20. Official Ans. by NTA (3)

Sol. $y + \frac{xdy}{dx} = x^2$ (given)

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$

$If = e^{\int \frac{1}{x}dx} = x$

Solution of DE

$\Rightarrow y.x = \int x.x\ dx$

$\Rightarrow xy = \frac{x^3}{3} + \frac{c}{3}$

Passes through (–2,2), so

$-12 = -8 + c \Rightarrow c = -4$

$\therefore 3xy = x^3 - 4$

ie. $3x.f(x) = x^3 - 4$

21. Official Ans. by NTA (4)

Sol. $\alpha.\ R = \frac{|3(2) + 4(-3) - 5|}{5} = \frac{11}{5}$

$(x-h)^2 = \frac{11}{5}(y-k)$

differentiate w.r.t 'x' : –

$2(x-h) = \frac{11}{5}\frac{dy}{dx}$

again differentiate

$2 = \frac{11}{5}\frac{d^2y}{dx^2}$

$\frac{11d^2y}{dx^2} = 10$.

22. Official Ans. by NTA (4)

Sol. $(2x - 10y^3)\,dy + ydx = 0$

$\Rightarrow \frac{dx}{dy} + \left(\frac{2}{y}\right)x = 10y^2$

I. F. $= e^{\int \frac{2}{y}dy} = e^{2\ell n(y)} = y^2$

Solution of D.E. is

$\therefore\ x.\,y = \int (10y^2)y^2.dy$

$xy^2 = \frac{10y^5}{5} + C \Rightarrow xy^2 = 2y^5 + C$

It passes through $(0, 1) \rightarrow 0 = 2 + C \Rightarrow C = -2$

$\therefore$ Curve is $\boxed{xy^2 = 2y^5 - 2}$

Now, it passes through $(2,\beta)$

$2\beta^2 = 2\beta^5 - 2 \Rightarrow \beta^5 - \beta^2 - 1 = 0$

$\therefore \beta$ is root of an equation $\boxed{y^5 - y^2 - 1 = 0}$ Ans.

23. Official Ans. by NTA (4)

Sol. $\int_0^x \sqrt{1-\left(f'(t)\right)^2}\,dt = \int_0^x f(t)\,dt \quad 0 \le x \le 1$

differentiating both the sides

$\sqrt{1-\left(f'(x)\right)^2} = f(x)$

$\Rightarrow 1 - (f'(x))^2 = f^2(x)$

$\frac{f'(x)}{\sqrt{1-f^2(x)}} = 1$

$\sin^{-1} f(x) = x + C$

$\because f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$

Now $\lim_{x\to 0} \frac{\int_0^x \sin t\,dt}{x^2}\left(\frac{0}{0}\right) = \frac{1}{2}$

24. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2^y}$

$2^y \frac{dy}{dx} = 2^x\left(2^y - 1\right)$

$\int \frac{2^y}{2^y - 1}\,dy = \int 2^x\,dx$

$\frac{\ell n\left(2^y - 1\right)}{\ell n\,2} = \frac{2^x}{\ell n\,2} + C$

$\Rightarrow \log_2(2^y - 1) = 2^x \log_2 e + C$

$\because y(0) = 1 \Rightarrow 0 = \log_2 e + C$

$C = -\log_2 e$

$\Rightarrow \log_2(2^y - 1) = (2^x - 1)\log_2 e$

put $x = 1$, $\log_2(2^y - 1) = \log_2 e$

$2^y = e + 1$

$y = \log_2(e + 1)$ Ans.

25. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = \frac{2^x(y+2^y)}{2^x\left(1+2^y \ell n 2\right)}$

$\Rightarrow \int \frac{(1+2^y)\ell n 2}{(y+2^y)}\,dy = \int dx$

$\Rightarrow \ell n|y + 2^y| = x + c$

$x = 0;\ y = 0 \Rightarrow c = 0$

$\Rightarrow x = \ell n|y + 2^y|$

$\Rightarrow$ at $y = 1$, $x = \ell n 3$

$\because 3 \in (e, e^2) \Rightarrow x \in (1, 2)$

26. Official Ans. by NTA (2)

Sol. Let, $y = tx$

$\frac{dy}{dx} = t + x\frac{dt}{dx}$

$\therefore\ tx\left(t + x\frac{dt}{dx}\right) = x\left(t^2 + \frac{\varphi(t^2)}{\varphi'(t^2)}\right)$

$t^2 + xt\frac{dt}{dx} = t^2 + \frac{\varphi(t^2)}{\varphi'(t^2)}$

$\int \frac{t\varphi'(t^2)}{\varphi(t^2)}\,dt = \int \frac{dx}{x}$

Let $\varphi(t^2) = p$

$\therefore \varphi'(t^2)2t\,dt = dp$

$\Rightarrow \int \frac{dy}{2p} = \int \frac{dx}{x}$

$\frac{1}{2}\ell n\varphi(t^2) = \ell nx + \ell nc$

$\varphi(t^2) = x^2 k$

$\varphi\left(\frac{y^2}{x^2}\right) = kx^2,\ \varphi(1) = k$

$\varphi\left(\frac{y^2}{4}\right) = 4\varphi(1)$

27. Official Ans. by NTA (4)

Sol. $x^2dy+\left(y-\frac{1}{x}\right)dx=0$: x > 0, y(1) = 1

$x^2dy+\frac{(xy-1)}{x}dx=0$

$x^2dy=\frac{(xy-1)}{x}dx$

$\frac{dy}{dx}=\frac{1-xy}{x^3}$

$\frac{dy}{dx}=\frac{1}{x^3}-\frac{y}{x^2}$

$\frac{dy}{dx}=\frac{1}{x^2}.y=\frac{1}{x^3}$

If $e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$

$ye^{-\frac{1}{x}}=\int\frac{1}{x^3}.e^{-\frac{1}{x}}$

$ye^{-\frac{1}{x}}=e^{-x}\left(1+\frac{1}{x}\right)+C$

$1.e^{-1}=e^{-1}(2)+C$

$C=-e^{-1}=-\frac{1}{e}$

$ye^{-\frac{1}{x}}=e^{-\frac{1}{x}}\left(1+\frac{1}{x}\right)-\frac{1}{e}$

$y\left(\frac{1}{2}\right)=3-\frac{1}{e}\times e^2$

$y\left(\frac{1}{2}\right)=3-e$

28. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}+\frac{y}{x}=bx^3$

I.F. $=e^{\frac{1}{x}dx}=x$

So, solution of D.E. is given by

$y.x=\int b.x^3.x\,dx+c$

$y=\frac{c}{x}+\frac{bx^4}{5}$

Passes through (1, 2)

$2=c+\frac{b}{5}$ ...(1)

$\int_1^2 f(x)dx=\frac{62}{5}$

$\left[c\ln x+\frac{bx^5}{25}\right]_1^2=\frac{62}{5}$

$c\ln 2+\frac{31b}{25}=\frac{62}{5}$ ...(2)

By equation (1) & (2)

c = 0 and b = 10

29. Official Ans. by NTA (4)

Sol. $\frac{dP}{dt}=0.5P-450$

$\Rightarrow\int_0^t\frac{dp}{P-900}=\int_0^t\frac{dt}{2}$

$\Rightarrow\left[\ell n|P(t)-900|\right]_0^t=\left[\frac{t}{2}\right]_0^t$

$\Rightarrow\ell n|P(t)-900|-\ell n|P(0)-900|=\frac{t}{2}$

$\Rightarrow\ell n|P(t)-900|-\ell n|50|=\frac{t}{2}$

for P(t) = 0

$\Rightarrow\ell n\left|\frac{900}{50}\right|=\frac{t}{2}\Rightarrow t=2\ell n18$

30. Official Ans. by NTA (4)

Sol. Given

y (0) = 0

& $\frac{dy}{dx}=\frac{(x-2)^2+y+4}{x-2}$

$\Rightarrow\frac{dy}{dx}-\frac{y}{x-2}=(x-2)+\frac{4}{x-2}$

$\Rightarrow$ I.F. $=e^{-\int\frac{1}{x-2}dx}=\frac{1}{x-2}$

Solution of L.D.E.

$\Rightarrow y\frac{1}{x-2}=\int\frac{1}{x-2}\left((x-2)+\frac{4}{x-2}\right).dx$

$\Rightarrow\frac{y}{x-2}=x-\frac{4}{x-2}+C$

Now, at x = 0, y = 0 ⇒ C = –2

y = x (x – 2) – 4 – 2 (x – 2)

⇒ $y=x^2-4x$

This curve passes through (5, 5)

31. Official Ans. by NTA (1)

Sol. $(2xy^2 - y)dx + xdy = 0$

$2xy^2\,dx - y\,dx + x\,dy = 0$

$$2x\,dx = \frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$$

Now integrate

$$x^2 = \frac{x}{y} + c$$

Now point of intersection of lines are (2, 1)

$$4 = \frac{2}{1} + c \qquad \Rightarrow c = 2$$

$$x^2 = \frac{x}{y} + 2$$

Now $y(1) = -1$

$\Rightarrow |y(1)| = 1$

32. Official Ans. by NTA (3)

Sol. $$\frac{dy}{dx} = \frac{xy^2 + y}{x}$$

$$\frac{xdy - ydx}{y^2} = xdx$$

$$-d\left(\frac{x}{y}\right) = xdx$$

$$-\frac{x}{y} = \frac{x^2}{2} + c$$

$\because$ curve intersects the line $x + 2y = 4$ at $x = -2$

$\Rightarrow$ point of intersection is $(-2, 3)$

$\therefore$ curve passes through $(-2, 3)$

$$\Rightarrow \frac{2}{3} = 2 + c \;\Rightarrow\; c = -\frac{4}{3}$$

$$\Rightarrow \frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$$

Now put (3, y)

$$\Rightarrow \frac{-3}{y} = \frac{19}{6}$$

$$\Rightarrow y = \frac{-18}{19}$$

33. Official Ans. by NTA (1)

Sol. $$\frac{dB}{dt} = \lambda B \Rightarrow \int_{1000}^{1200} \frac{dB}{B} = \lambda \int_0^2 dt \Rightarrow \lambda = \frac{1}{2}\ell n\left(\frac{6}{5}\right)$$

$$\int_{1000}^{2000} \frac{dB}{B} = \frac{1}{2}\ell n\left(\frac{6}{5}\right)\int_0^T dt \;\Rightarrow\; T = \frac{2\ell n 2}{\ell n\left(\frac{6}{5}\right)}$$

$\Rightarrow k = 2\ell n 2$

34. Official Ans. by NTA (1)

Sol. Put $e^{\sin y} = t$

$$\Rightarrow e^{\sin y}\cos y\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \text{D.E is } \frac{dt}{dx} + t\cos x = \cos x$$

$$\text{I.F.} = e^{\int \cos x\,dx} = e^{\sin x}$$

$$\Rightarrow \text{solution is } t.e^{\sin x} = \int \cos x e^{\sin x}$$

$$\Rightarrow e^{\sin y}e^{\sin x} = e^{\sin x} + c$$

$\because x = 0, y = 0 \Rightarrow c = 0$

$$\Rightarrow e^{\sin y} = 1$$

$\Rightarrow y = 0$

$$\Rightarrow 1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right) =$$

35. Official Ans. by NTA (2)

Sol. $$y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) = ax + \frac{a^{3/2}}{2} \quad ...(1)$$

$\Rightarrow 2yy' = a$

put in equation (1)

$$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$$

$$(y^2 - 2xyy') = \frac{(2yy')^{3/2}}{2}$$

squaring

$$(y^2 - 2xyy')^2 = \frac{y^3(y')^3}{2}$$

$\therefore$ order = 1

degree = 3

Degree – order = 3 – 1 = 2

36. Official Ans by NTA (2)

Sol. $\frac{dy}{dx}+(\tan x)y=\sin x;\ 0\le x\le\frac{\pi}{3}$

$\text{I.F.}=e^{\int \tan x\,dx}=e^{\ell n \sec x}=\sec x$

$y\sec x=\int \tan x dx$

$y\sec x=\int \tan x dx$

$y\sec x=\ell n|\sec x|+C$

$x=0,\ y=0 \Rightarrow \quad \therefore c=0$

$y\sec x=\ell n|\sec x|$

$y=\cos x\cdot \ell n|\sec x|$

$y|_{x=\frac{\pi}{4}}=\left(\frac{1}{\sqrt{2}}\right)\cdot \ell n\sqrt{2}$

$y|_{x=\frac{\pi}{4}}=\frac{1}{2\sqrt{2}}\log_e 2$

37. Official Ans by NTA (2)

Sol. $\frac{dy}{dx}=\frac{y^2-x^2}{2xy},\quad x\in(0,\infty)$

put $y=vx$

$x\frac{dv}{dx}+v=\frac{v^2-1}{2v}$

$\frac{2v}{v^2+1}dv=-\frac{dx}{x}$

Integrate,

$ln(v^2+1)=-lnx+C$

$ln\left(\frac{y^2}{x^2}+1\right)=-lnx+C$

put $x=1,\ y=1,\ C=ln2$

$ln\left(\frac{y^2}{x^2}+1\right)=-lnx+ln2$

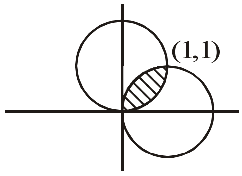
$\Rightarrow x^2+y^2-2x=0$ (Curve C_1)

Similarly,

$\frac{dy}{dx}=\frac{2xy}{x^2-y^2}$

Put $y=vx$

$x^2+y^2-2y=0$



required area $=2\int_0^1\left(\sqrt{2x-x^2}-x\right)dx=\frac{\pi}{2}-1$

38. Official Ans. by NTA (4)

Sol. $\frac{dy}{dx}+2y\tan x=\sin x$

$\text{I.F.}=e^{\int 2\tan x dx}=e^{2\ell n \sec x}$

I.F. $=\sec^2 x$

$y.(\sec^2 x)=\int \sin x.\sec^2 x dx$

$y.(\sec^2 x)=\int \sec x\tan x dx$

$y.(\sec^2 x)=\sec x+C$

$x=\frac{\pi}{3};\ y=0$

$\Rightarrow C=-2$

$\Rightarrow y=\frac{\sec x-2}{\sec^2 x}=\cos x-2\cos^2 x$

$y=t-2t^2 \Rightarrow \frac{dy}{dt}=1-4t=0 \Rightarrow t=\frac{1}{4}$

$\therefore \max=\frac{1}{4}-\frac{1}{8}=\frac{2-1}{8}=\frac{1}{8}$

39. Official Ans. by NTA (2)

Sol. $\cos x(3\sin x+\cos x+3)dy$

$=(1+y\sin x(3\sin x+\cos x+3))dx$

$\frac{dy}{dx}-(\tan x)y=\frac{1}{(3\sin x+\cos x+3)\cos x}$

$\text{I.F.}=e^{\int -\tan x\,dx}=e^{\ell n|\cos x|}=|\cos x|$

$=\cos x\ \forall\ x\in\left[0,\frac{\pi}{2}\right)$

Solution of D.E.

$y(\cos x)=\int(\cos x)\cdot\frac{1}{\cos x(3\sin x+\cos x+3)}dx+C$

$y(\cos x)=\int\frac{dx}{3\sin x+\cos x+3}dx+C$

$y(\cos x)=\int\frac{\left(\sec^2\frac{x}{2}\right)}{2\tan^2\frac{x}{2}+6\tan\frac{x}{2}+4}dx+C$

Now

Let $I_1 = \int \frac{\left(\sec^2\frac{x}{2}\right)}{2\left(\tan^2\frac{x}{2}+3\tan\frac{x}{2}+2\right)}dx + C$

Put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$I_1 = \int \frac{dt}{t^3+3t+2} = \int \frac{dt}{(t+2)(t+1)}$

$= \int\left(\frac{1}{t+1}-\frac{1}{t+2}\right)dt$

$= \ell n\left|\left(\frac{t+1}{t+2}\right)\right| = \ell n\left|\left(\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right)\right|$

So solution of D.E.

$y(\cos x) = \ell n\left|\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right| + C$

$\Rightarrow y(\cos x) = \ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right) + C \quad \text{for } 0 \le x < \frac{\pi}{2}$

Now, it is given y(0) = 0

$\Rightarrow 0 = \ell n\left(\frac{1}{2}\right) + C \qquad \Rightarrow \boxed{C = \ell n2}$

$\Rightarrow y(\cos x) = \ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right) + \ell n2$

For $x = \frac{\pi}{3}$

$y\left(\frac{1}{2}\right) = \ell n\left(\frac{1+\frac{1}{\sqrt{3}}}{2+\frac{1}{\sqrt{3}}}\right) + \ell n2$

$y = 2\ell n\left(\frac{2\sqrt{3}+10}{11}\right)$ **Ans.(2)**

40. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} - \frac{y}{2x} = \frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}$

$IF = e^{-\int\frac{dx}{2d}} = e^{-\frac{1}{2}\ln x} = \frac{1}{x^{1/2}}$

$y.x^{-1/2} = \int \frac{x^{9/4}\cdot x^{-1/2}}{x^{5/4}\left(x^{3/4}+1\right)}dx$

$\int \frac{x^{1/2}}{(x^{3/4}+1)}dx$

$x = t^4 \Rightarrow dx = 4t^3\,dt$

$\int \frac{t^2\cdot 4t^3\,dt}{(t^3+1)}$

$4\int \frac{t^2\left(t^3+1-1\right)}{(t^3+1)}dt$

$4\int t^2dt - 4\int \frac{t^2}{t^3+1}dt$

$\frac{4t^3}{3} - \frac{4}{3}\ln\left(t^3+1\right) + C$

$yx^{-1/2} = \frac{4x^{3/4}}{3} - \frac{4}{3}\ln\left(x^{3/4}+1\right) + C$

$1 - \frac{4}{3}\log_e 2 = \frac{4}{3} - \frac{4}{3}\log_e 2 + C$

$\Rightarrow C = -\frac{1}{3}$

$y = \frac{4}{3}x^{5/4} - \frac{4}{3}\sqrt{x}\ln\left(x^{3/4}+1\right) - \frac{\sqrt{x}}{3}$

$y(16) = \frac{4}{3}\times 32 - \frac{4}{3}\times 4\ln 9 - \frac{4}{3}$

$= \frac{124}{3} - \frac{32}{3}\ln 3 = 4\left(\frac{31}{3} - \frac{8}{3}\ln 3\right)$

41. Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = (1+y)(x-1)$

$\frac{dy}{(y+1)} = (x-1)dx$

Integrate $\ln(y+1) = \frac{x^2}{2} - x + c$

$(0,0) \Rightarrow c = 0 \Rightarrow y = e^{\left(\frac{x^2}{2}-x\right)} - 1$

42. Official Ans. by NTA (1)

Sol. $I = \int_0^{\sqrt{\pi/2}} \left([x^2] + [-\cos x]\right)dx$

$= \int_0^1 0\,dx + \int_1^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}} (-1)\,dx$

$= \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} = -1$

$\Rightarrow |I| = 1$

43. Official Ans. by NTA (3)

Sol. $y^2 = 4ax + 4a^2$

differentiate with respect to x

$\Rightarrow 2y\frac{dy}{dx} = 4a$

$\Rightarrow a = \left(\frac{y}{2}\frac{dy}{dx}\right)$

so, required differential equation is

$y^2 = \left(4 \times \frac{y}{2}\frac{dy}{dx}\right)x + 4\left(\frac{y}{2}\frac{dy}{dx}\right)^2$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 = 0$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

44. Official Ans. by NTA (1)

Sol. Let $y + 1 = Y$

$\therefore \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$

Put $-\frac{1}{Y} = k$

$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$

I.F. $= e^{-\frac{x^2}{2}}$

$\therefore k = (x+c)e^{x^2/2}$

Put $k = -\frac{1}{y+1}$

$\therefore y + 1 = -\frac{1}{(x+c)e^{x^2/2}}$ ...(i)

when x = 2, y = 0, then $c = -2 - \frac{1}{e^2}$

diffentiate equation (i) & put x = 1

we get $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{\left(1+e^2\right)^2}$

45. Official Ans. by NTA (4)

Sol. $xdy - ydx = \sqrt{x^2 - y^2}\,dx$

$\Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x}\sqrt{1 - \frac{y^2}{x^2}}\,dx$

$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$

$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ell n|x| + c$

at x = 1, y = 0 $\Rightarrow$ c = 0

$y = x\sin(\ell nx)$

$A = \int_1^{e^\pi} x\sin(\ell n x)\,dx$

$x = e^t$, $dx = e^t dt \Rightarrow \int_0^\pi e^{2t}\sin(t)\,dt = A$

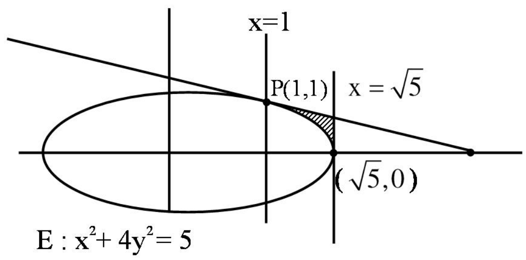
$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^\pi = \frac{1+e^{2\pi}}{5}$

$\alpha = \frac{1}{5}, \beta = \frac{1}{5}$ so $10(\alpha + \beta) = 4$

AREA UNDER THE CURVE

1. Official Ans. by NTA (1)

Sol.



Tangent at P : $x + 4y = 5$

Required Area

$$= \int_{1}^{\sqrt{5}} \left(\frac{5-x}{4} - \frac{\sqrt{5-x^2}}{2} \right) dx$$

$$= \left[\frac{5x}{4} - \frac{x^2}{8} - \frac{x}{4}\sqrt{5-x^2} - \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}} \right]_1^{\sqrt{5}}$$

$$= \frac{5}{4}\sqrt{5} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

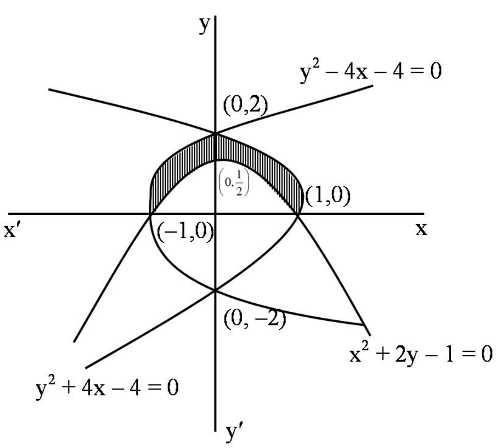
It we assume $\alpha, \beta, \gamma, \in Q$ (Not given in question)

then $\alpha = \frac{5}{4}, \beta = -\frac{5}{4}$ & $\gamma = -\frac{5}{4}$

$|\alpha + \beta + \gamma| = 1.25$

2. Official Ans. by NTA (2)

Sol.



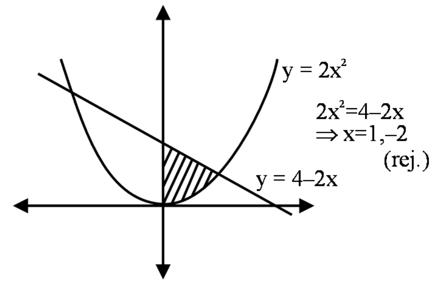
Required Area (shaded)

$$= 2\left[\int_0^2 \left(\frac{4-y^2}{4} \right) dy - \int_0^1 \left(\frac{1-x^2}{2} \right) dx \right]$$

$$= 2\left[\frac{4}{3} - \frac{1}{3} \right] = (2)$$

3. Official Ans. by NTA (4)

Sol.

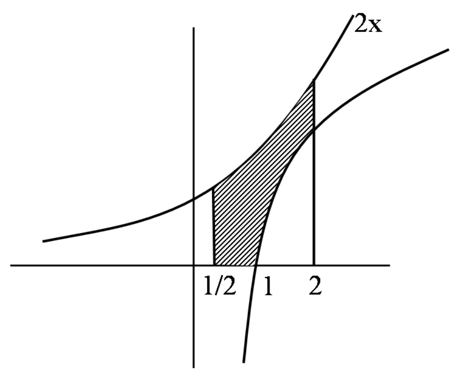


Required area $= \int_0^1 \left(4 - 2x - 2x^2\right) dx = 4x - x^2 - \frac{2x^3}{3}\Big|_0^1$

$= 4 - 1 - \frac{2}{3} = \frac{7}{3}$

4. Official Ans. by NTA (2)

Sol. $R = \left\{ (x, y) : \max\left(0, \log_e x\right) \le y \le 2^x, \frac{1}{2} \le x \le 2 \right\}$



$$\int_{\frac{1}{2}}^{2} 2^x dx - \int_1^2 \ell nx \, dx$$

$$\Rightarrow \left[\frac{2^x}{\ln 2} \right]_{1/2}^{2} - [x \ln x - x]_1^2$$

$$\Rightarrow \frac{(2^2) - 2^{1/2}}{\log_e 2} - (2\ln 2 - 1)$$

$$\Rightarrow \frac{\left(2^2 - \sqrt{2}\right)}{\log_e 2} - 2\ln 2 + 1$$

$\therefore\ \alpha = 2^2 - \sqrt{2},\ \beta = -2,\ \gamma = 1$

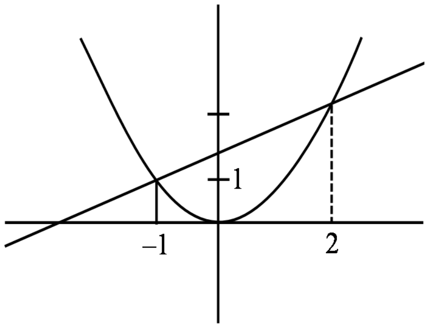
$\Rightarrow (\alpha + \beta + 2\gamma)^2$

$\Rightarrow (2^2 - \sqrt{2} - 2 - 2)^2$

$\Rightarrow (\sqrt{2})^2 = 2$

5. Official Ans. by NTA (3)

Sol.



$y - x = 2$, $x^2 = y$

Now, $x^2 = 2 + x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x + 1)(x - 2) = 0$

Area = $\int_{-1}^{2}(2+x-x^2)$

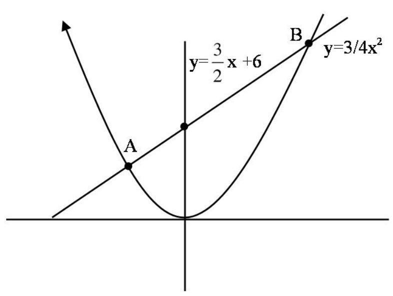
$= \left|2x+\frac{x^2}{2}-\frac{x^3}{3}\right|_{-1}^{2}$

$= \left(4+2-\frac{8}{3}\right)-\left(-2+\frac{1}{2}+\frac{1}{3}\right)$

$= 6 - 3 + 2 - \frac{1}{2} = \frac{9}{2}$

6. Official Ans. by NTA (27)

Sol.



For A & B

$3x^2 = 6x + 24 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

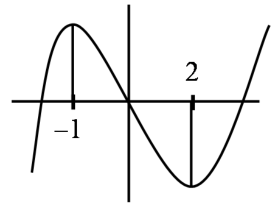
Area = $\int_{-2}^{4}\left(\frac{3}{2}x+6-\frac{3}{4}x^2\right)dx$

$= \left[\frac{3x^2}{4}+6x-\frac{x^3}{4}\right]_{-2}^{4} = 27$

7. Official Ans. by NTA (114)

Sol. $f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1)$

Point = (2,–20) & (–1,7)



$A = \int_{-1}^{0}(2x^3-3x^2-12x)dx + \int_{0}^{2}(12x+3x^2-2x^3)dx$

$A = \left(\frac{x^4}{2}-x^3-6x^2\right)_{-1}^{0} + \left(6x^2+x^3-\frac{x^4}{2}\right)_{0}^{2}$

$4A = 114$

8. Official Ans. by NTA (1)

Sol. $y = 3 \Rightarrow x = 2$

Point is (2,3)

Diff. w.r.t x

$2(y - 2)y' = 1$

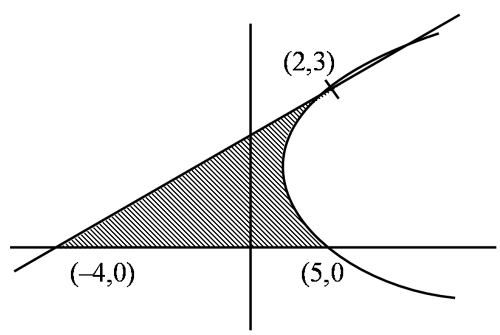
$\Rightarrow y' = \frac{1}{2(y-2)}$

$\Rightarrow y'_{(2,3)} = \frac{1}{2}$

$\Rightarrow \frac{y-3}{x-2} = \frac{1}{2} \Rightarrow x-2y+4=0$

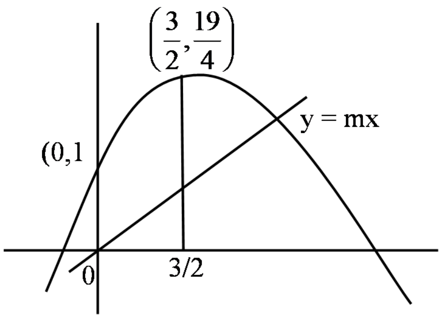
Area $= \int_{0}^{3}\left((y-2)^2+1-(2y-4)\right)dy$

= 9 sq. units



9. Official Ans. by NTA (26)

Sol.



Total area $= \int_0^{3/2} (1+4x-x^2)\,dx$

$= x+2x^2-\left.\frac{x^3}{3}\right|_0^{3/2} = \frac{39}{8}$

& $\frac{39}{16} = \frac{1}{2}.\frac{3}{2}.\frac{3}{2}m$

$\Rightarrow 3m = \frac{13}{2} \Rightarrow 12m = 26$

10. Official Ans. by NTA (1)

Sol. $A = \int_0^{\pi/2}((\sin x+\cos x)-|\cos x-\sin x|)dx$

$A = \int_0^{\pi/2}((\sin x+\cos x)-(\cos x-\sin x))dx$

$+\int_{\pi/4}^{\pi/2}((\sin x+\cos x)-(\sin x-\cos x))dx$

$A = 2\int_0^{\pi/2}\sin x\,dx + 2\int_{\pi/4}^{\pi/2}\cos x\,dx$

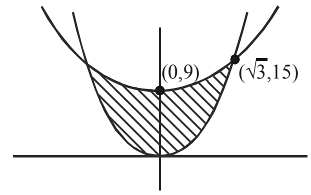
$A = -2\left(\frac{1}{\sqrt{2}}-1\right)+2\left(1-\frac{1}{\sqrt{2}}\right)$

$A = 4 - 2\sqrt{2} = 2\sqrt{2}(\sqrt{2}-1)$

Option (1)

11. Official Ans. by NTA (2)

Sol.



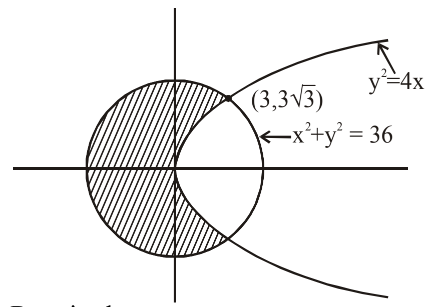
Required area $= 2\int_0^{\sqrt{3}} (2x^2+9-5x^2)\,dx$

$= 2\left[9x-x^3\right]_0^{\sqrt{3}}$

$= 2\left[9\sqrt{3}-3\sqrt{3}\right] = 12\sqrt{3}$

12. Official Ans. by NTA (3)

Sol.



Required area

$= \pi\times(6)^2 - 2\int_0^3\sqrt{9x}\,dx - \int_3^6\sqrt{36-x^2}\,dx$

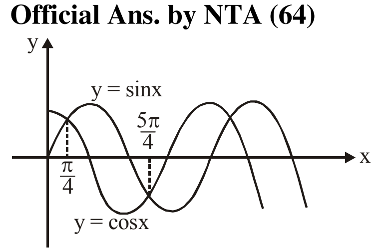
$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36-x^2}+18\sin^{-1}\frac{x}{6}\right)_3^6$

$= 36\pi - 12\sqrt{3} - 2\left(9\pi - 3\pi - \frac{9\sqrt{3}}{2}\right)$

$= 24\pi - 3\sqrt{3}$

13. Official Ans. by NTA (64)

Sol.



$A = \int_{\pi/4}^{5\pi/4}(\sin x-\cos x)\,dx$

$= (-\cos x-\sin x)|_{\pi/4}^{5\pi/4}$

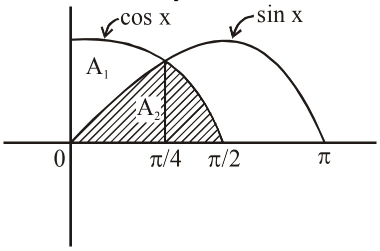
$= \left(-\left(\frac{-1}{\sqrt{2}}\right)-\left(\frac{-1}{\sqrt{2}}\right)\right)-\left(-\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{\sqrt{2}}\right)\right)$

$\Rightarrow A = \frac{2}{\sqrt{2}}+\frac{2}{\sqrt{2}} = 2\sqrt{2}$

$\Rightarrow A^4 = (2\sqrt{2})^4 = 16\times 4 = 64$

14. Official Ans. by NTA (1)

Sol.



$$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\,dx$$

$$A_1 = (\sin x + \cos x)_0^{\pi/4} = \sqrt{2} - 1$$

$$A_2 = \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$$

$$= (-\cos x)_0^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2}$$

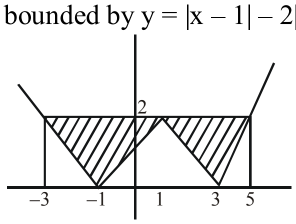
$$A_2 = \sqrt{2}\left(\sqrt{2}-1\right)$$

$A_1 : A_2 = 1 : \sqrt{2}$, $A_1 + A_2 = 1$

15. Official Ans. by NTA (BONUS)

Sol. Remark :

Question is incomplete it should be area bounded by $y = |x - 1| - 2|$ and $y = 2$



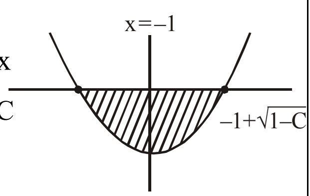
$$\text{Area} = 2\left(\frac{1}{2}.4.2\right)$$

16. Official Ans. by NTA (2)

Sol. $\dfrac{dy}{dx} = 2(x+1)$

$\Rightarrow \int dy = \int 2(x+1)dx$

$\Rightarrow y(x) = x^2 + 2x + C$



$\text{Area} = \dfrac{4\sqrt{8}}{3}$

$-1 + \sqrt{1-C}$

$$\Rightarrow 2\int_{-1}^{-1+\sqrt{1-C}} (-(x+1)^2 - C + 1)dx = \frac{4\sqrt{8}}{3}$$

$$\Rightarrow 2\left[-\frac{(x+1)^3}{3} - Cx + x\right]_{-1}^{-1+\sqrt{1-C}} = \frac{4\sqrt{8}}{3}$$

$$\Rightarrow -(\sqrt{1-C})^3 + 3c - 3C\sqrt{1-C}$$

$$-3 + 3\sqrt{1-C} - 3C + 3 = 2\sqrt{8}$$

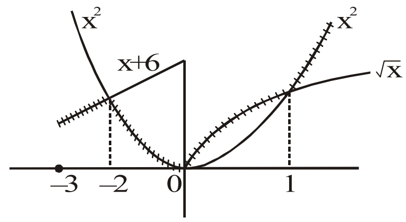
$\Rightarrow C = -1$

$\Rightarrow f(x) = x^2 + 2x - 1,\ f(1) = 2$

17. Official Ans. by NTA (41)

Sol. $f : [-3, 1] \to R$

$$f(x) = \begin{cases} \min\{(x+6), x^2\} & , -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\} & , 0 \le x \le 1 \end{cases}$$



area bounded by y = f(x) and x-axis

$$= \int_{-3}^{-2} (x+6)dx + \int_{-2}^{0} x^2 dx + \int_0^1 \sqrt{x}\,dx$$

$$A = \frac{41}{6}$$

$6A = 41$

18. Official Ans. by NTA (3)

Sol. $4y^2 = x^2(4 - x)(x - 2)$

$$|y| = \frac{|x|}{2}\sqrt{(4-x)(x-2)}$$

$$\Rightarrow y_1 = \frac{x}{2}\sqrt{(4-x)(x-2)}$$

and $y_2 = \dfrac{-x}{2}\sqrt{(4-x)(x-2)}$

$D : x \in [2, 4]$

Required Area

$$= \int_2^4 (y_1 - y_2)\,dx = \int_2^4 x\sqrt{(4-x)(x-2)}\,dx \quad ...(1)$$

Applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

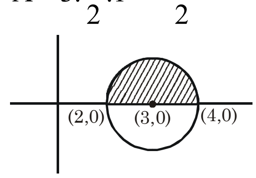
$$\text{Area} = \int_2^4 (6-x)\sqrt{(4-x)(x-2)}\,dx \quad ...(2)$$

(1) + (2)

$$2A = 6\int_2^4 \sqrt{(4-x)(x-2)}\,dx$$

$$A = 3\int_2^4 \sqrt{1-(x-3)^2}\,dx$$

$$A = 3.\frac{\pi}{2}.1^2 = \frac{3\pi}{2}$$



MATRICES

1. Official Ans. by NTA (1)

Sol. $A=\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}, a\in R$

and $P=\dfrac{A+A^T}{2}=\begin{bmatrix}2 & \dfrac{3+a}{2}\\ \dfrac{a+3}{2} & 0\end{bmatrix}$

and $Q=\dfrac{A-A^T}{2}=\begin{bmatrix}0 & \dfrac{3-a}{2}\\ \dfrac{a-3}{2} & 0\end{bmatrix}$

As, det (Q) = 9

$\Rightarrow (a-3)^2=36$

$\Rightarrow a=3\pm 6$

$\therefore \boxed{a=9,-3}$

$\therefore$

$\det.(P)=\begin{vmatrix}2 & \dfrac{3+a}{2}\\ \dfrac{a+3}{2} & 0\end{vmatrix}$

$=0-\dfrac{(a-3)^2}{4}=0$, for $a=-3$

$=0-\dfrac{(a-3)^2}{4}=-\dfrac{1}{4}(12)(12)$, for $a=9$

$\therefore$ Modulus of the sum of all possible values of det. (P) $=|-36|+|0|=36$ Ans.

$\Rightarrow$ Option (1) is correct

2. Official Ans. by NTA (910)

Sol. Let $A=\begin{pmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\end{pmatrix}=I+C$

where $I=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}, C=\begin{pmatrix}0 & -1 & 0\\ 0 & 0 & -1\\ 0 & 0 & 0\end{pmatrix}$

$C^2=\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$,

$C^3=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}=C^4=C^5=......$

$B=7A^{20}-20A^7+2I$

$=7(I+C)^{20}-20(I+C)^7+2I$

$=7(I+20C+{}^{20}C_2C^2)-20(I+7C+{}^7C_2C^2)+2I$

So $b_{13}=7\times{}^{20}C_2-20\times{}^7C_2=\boxed{910}$

3. Official Ans. by NTA (1)

Sol. $|A|=-\dfrac{y}{x}+2\sin x+2$

$\dfrac{dy}{dx}=|A|$

$\dfrac{dy}{dx}=-\dfrac{y}{x}+2\sin x+2$

$\dfrac{dy}{dx}+\dfrac{y}{x}=2\sin x+2$

$I.F.=e^{\int\frac{1}{x}dx}=x$

$\Rightarrow yx=\int x(2\sin x+2)dx$

$xy=x^2-2x\cos x+2\sin x+c$ .....(i)

Now $x=\pi, y=\pi+2$

Use in (i)

$c=0$

Now (i) be comes

$xy=x^2-2x\cos x+2\sin x$

put $x=\pi/2$

$\dfrac{\pi}{2}y=\left(\dfrac{\pi}{2}\right)^2-2.\dfrac{\pi}{2}\cos\dfrac{\pi}{2}+2\sin\dfrac{\pi}{2}$

$\dfrac{\pi}{2}y=\dfrac{\pi^2}{4}+2$

4. Official Ans. by NTA (108)

Sol. $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

$|A|=4$

$\left|3\text{adj}(2A^{-1})\right|=\left|3.2^2\text{adj}(A^{-1})\right|$

$=12^3\left|\text{adj}(A^{-1})\right|=12^3|A^{-1}|^2=\dfrac{12^3}{|A|^2}=\dfrac{12^3}{16}=108$

5. Official Ans. by NTA (3)

Sol. $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$

Let $x=\begin{bmatrix}1\\1\\1\end{bmatrix}$

$AX=\begin{bmatrix}a_{11}+a_{12}+a_{13}\\ a_{21}+a_{22}+a_{23}\\ a_{31}+a_{32}+a_{33}\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$

$\Rightarrow AX = X$

Replace X by AX

$A^2X = AX = X$

Replace X by AX

$A^3X = AX = X$

$$\text{Let } A^3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

$$A^3\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix} x_1+x_2+x_3 \\ y_1+y_2+y_3 \\ z_1+z_2+z_3 \end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

Sum of all the element = 3

6. Official Ans. by NTA (3125)

Sol. $\text{Let matrix } B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & n & i \end{bmatrix}$

$\because \; AB = BA$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix} = \begin{bmatrix} b & a & c \\ e & d & f \\ h & g & i \end{bmatrix}$$

$\Rightarrow d = b,\ e = a,\ f = c,\ g = h$

$$\therefore \text{Matrix } B = \begin{bmatrix} a & b & c \\ b & a & c \\ g & g & i \end{bmatrix}$$

No. of ways of selecting a, b, c, g, i

$= 5 \times 5 \times 5 \times 5 \times 5$

$= 5^5 = 3125$

$\therefore$ No. of Matrices B = 3125

7. Official Ans. by NTA (16)

Sol. $|A| = ad - bc = 15$

where a,b,c,d $\in \{\pm 3, \pm 2, \pm 1, 0\}$

Case I ad = 9 & bc = – 6

For ad possible pairs are (3,3), (–3,–3)

For bc possible pairs are (3,–2), (–3,2), (–2,3),(2,–3)

So total matrix = 2 × 4 = 8

Case II ad = 6 & bc = –9

Similarly total matrix = 2 × 4 = 8

$\Rightarrow$ Total such matrices are = 16

8. Official Ans. by NTA (1)

Sol. $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$\vdots$

$$\therefore P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

9. Official Ans. by NTA (4)

Sol. $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix},\ |A| = 6$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\left.\begin{aligned} \alpha + \beta &= \frac{2}{3} \\ \beta &= -\frac{1}{6} \end{aligned}\right\} \Rightarrow \alpha = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$4(\alpha - \beta) = 4(1) = 4$

10. Official Ans. by NTA (4)

Sol. $C = A^2 - B^2; |C| \neq 0$

$A^5 = B^5$ and $A^3B^2 = A^2B^2$

Now, $A^5 - A^3B^2 = B^5 - A^2B^3$

$\Rightarrow A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$

$\Rightarrow (A^3 + B^3)(A^2 - B^2) = 0$

Post multiplying inverse of $A^2 - B^2$:

$A^3 + B^3 = 0$

11. Official Ans. by NTA (2020)

Sol. $A^n = \begin{bmatrix} 1 & n & \frac{n^2+n}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$

So, required sum

$= 20 \times 3 + 2 \times \left(\frac{20\times 21}{2}\right) + \sum_{r=1}^{20}\left(\frac{r^2+r}{2}\right)$

$= 60 + 420 + 105 + 35 \times 41 = 2020$

12. Official Ans. by NTA (2)

Sol. $AA^T = \begin{pmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$

$AA^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$Q^2 = A^TBA\,A^TBA = A^TBIBA$

$\Rightarrow Q^2 = A^T B^2A$

$Q^3 = A^TB^2AA^TBA \Rightarrow Q^3 = A^TB^3A$

Similarly : $Q^{2021} = A^T B^{2021} A$ ...... (1)

Now $B^2 = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}$

$B^3 = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} \Rightarrow B^3 = \begin{pmatrix} 1 & 0 \\ 3i & 1 \end{pmatrix}$

Similarly $B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore AQ^{2021} A^T = AA^T B^{2021} AA^T = IB^{2021}I$

$\Rightarrow AQ^{2021} A^T = B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore (AQ^{2021} A^T)^{-1} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

13. Official Ans. by NTA (1)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^{2025} - A^{2020} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^6 - A = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

14. Official Ans. by NTA (4)

Sol. adj $(2A) = 2^2$ adj$A$

$\Rightarrow$ adj(adj (2A)) = adj(4 adjA) = 16 adj (adj A)

$= 16\,|A|\,A$

$\Rightarrow$ adj $(32\,|A|\,A) = (32\,|A|)^2$ adj A

$12(32|A|)^2\,|\text{adj } A| = 2^3\,(32|A|)^6\,|\text{adj } A|$

$2^3.2^{30}\,|A|^6\,.\,|A|^2 = 2^{41}$

$|A|^8 = 2^8 \Rightarrow |A| = \pm 2$

$|A|^2 = |A|^2 = 4$

15. Official Ans. by NTA (1)

Sol. Given matrix $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$A^4 + 3\,IA = 2I$

$\Rightarrow A^4 = 2I - 3A$

Also characteristic equation of A is

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 0-\lambda & 2 \\ k & -1-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda + \lambda^2 - 2k = 0$

$\Rightarrow A + A^2 = 2K.I$

$\Rightarrow A^2 = 2KI - A$

$\Rightarrow A^4 = 4K^2I + A^2 - 4AK$

Put $A^2 = 2KI - A$

and $A^4 = 2I - 3A$

$2I - 3A = 4K^2I + 2KI - A - 4AK$

$\Rightarrow I(2 - 2K - 4K^2) = A(2 - 4K)$

$\Rightarrow -2I(2K^2 + K - 1) = 2A(1 - 2K)$

$\Rightarrow -2I(2K - 1)(K + 1) = 2A(1 - 2K)$

$\Rightarrow (2K - 1)(2A) - 2I(2K - 1)(K + 1) = 0$

$\Rightarrow (2K - 1)[2A - 2I(K + 1)] = 0$

$\Rightarrow K = \frac{1}{2}$

16. Official Ans. by NTA (8)

Sol. $(I - A)^3 = I^3 - A^3 - 3A(I - A) = I - A^3$

$\Rightarrow 3A(I - A) = 0$ or $A^2 = A$

$\Rightarrow \begin{bmatrix} a^2 & ab + bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

$\Rightarrow a^2 = a,\ b(a + d - 1) = 0,\ d^2 = d$

If $b \neq 0$, $a + d = 1 \Rightarrow 4$ ways

If $b = 0$, $a = 0, 1$ & $d = 0, 1 \Rightarrow 4$ ways

$\Rightarrow$ Total 8 matrices

17. Official Ans. by NTA (3)

Sol.

$$\begin{bmatrix} \overset{\surd}{a_{11}} & \overset{\surd}{a_{12}} & \overset{\surd}{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$J_{6+i,3} - J_{i+3,3}$; $i \leq j$

$\Rightarrow \int_0^{\frac{1}{2}} \frac{x^{6+i}}{x^3 - 1} - \int_0^{\frac{1}{2}} \frac{x^{i+3}}{x^3 - 1}$

$\Rightarrow \int_0^{1/2} \frac{x^{i+3}(x^3 - 1)}{x^3 - 1}$

$\Rightarrow \frac{x^{3+i+1}}{3 + i + 1} = \left(\frac{x^{4+i}}{4 + i}\right)_0^{1/2}$

$a_{ij} = j_{6+i,3} - j_{i+3,3} = \frac{\left(\frac{1}{2}\right)^{4+i}}{4 + i}$

$a_{11} = \frac{\left(\frac{1}{2}\right)^5}{5} = \frac{1}{5.2^5}$

$a_{12} = \frac{1}{5.2^5}$

$a_{13} = \frac{1}{5.2^5}$

$a_{22} = \frac{1}{6.2^6}$

$a_{23} = \frac{1}{6.2^6}$

$a_{33} = \frac{1}{7.2^7}$

$$A = \begin{bmatrix} \frac{1}{5.2^5} & \frac{1}{5.2^5} & \frac{1}{5.2^5} \\ 0 & \frac{1}{6.2^6} & \frac{1}{6.2^6} \\ 0 & 0 & \frac{1}{7.2^7} \end{bmatrix}$$

$|A| = \frac{1}{5.2^5}\left[\frac{1}{6.2^6} \times \frac{1}{7.2^7}\right]$

$|A| = \frac{1}{210.2^{18}}$

$\left|\text{adj}A^{-1}\right| = \left|A^{-1}\right|^{n-1} = \left|A^{-1}\right|^2 = \frac{1}{(|A|)^2}$

$\Rightarrow (210.2^{18})^2$

$(105)^2 \times 2^{38}$

18. Official Ans. by NTA (3)

Sol. Let $A^T = A$ and $B^T = -B$

$C = A^2B^2 - B^2A^2$

$C^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T$

$= B^2A^2 - A^2B^2$

$C^T = -C$

C is skew symmetric.

So $\det(C) = 0$

so system have infinite solutions.

19. Official Ans. by NTA (540)

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case-I : Seven (1's) and two (0's)

$^9C_2 = 36$

Case-II : One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540

20. Official Ans. by NTA (17)

Sol. $PQ = kI$

$|P|.|Q| = k^3$

$\Rightarrow |P| = 2k \neq 0 \Rightarrow$ P is an invertible matrix

$\because PQ = kI$

$\therefore Q = kP^{-1}I$

$\therefore Q = \frac{\text{adj}.P}{2}$

$\because q_{23} = -\frac{k}{8}$

$\therefore \frac{-(3\alpha+4)}{2} = -\frac{k}{8} \Rightarrow k = 4$

$\therefore |P| = 2k \Rightarrow k = 10 + 6\alpha$ ...(i)

Put value of k in (i).. we get $\alpha = -1$

21. Official Ans. by NTA (4)

Sol. $|A| = 4$

$\Rightarrow |2A| = 2^3 \times 4 = 32$

$\because$ B is obtained by $R_2 \to 2R_2 + 5R_3$

$\Rightarrow |B| = 2 \times 32 = 64$

option (4)

22. Official Ans. by NTA (4)

Sol. $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$ $\qquad AA^T = I_2$

$\Rightarrow \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 = 0 \ \& \ \beta^2 = 1$

$\therefore \alpha^4 + \beta^4 = 1$

23. Official Ans. by NTA (7)

Sol. $A^2 = I$

$\Rightarrow AA' = 1$ (as $A' = A$)

$\Rightarrow$ A is orthogonal

So, $x^2 + y^2 + z^2 = 1$ and $xy + yz + zx = 0$

$\Rightarrow (x + y + z)^2 = 1 + 2 \times 0$

$\Rightarrow x + y + z = 1$

Thus,

$x^3 + y^3 + z^3 = 3 \times 2 + 1 \times (1 - 0)$

$= 7$

24. Official Ans. by NTA (13)

Sol. $a^2 + b^2 = |I_2 + A||I_2 - A|^{-1}$

$= \sec^2\frac{\theta}{2} \times \cos^2\frac{\theta}{2} = 1$

25. Official Ans. by NTA (4)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence

$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}, A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}$

So $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha.2^{19}+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Therefore $\alpha + \beta = 0$ and $2^{20} + 2^{19}\alpha - 2\alpha = 4$

$\Rightarrow \alpha = \frac{4(1-2^{18})}{2(2^{18}-1)} = -2$

hence $\beta = 2$

so $(\beta - \alpha) = 4$

26. Official Ans. by NTA (1)

Sol. $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$, $a, b, c \in I$

$A^2 = \begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} a^2+b^2 & b(a+c) \\ b(a+c) & b^2+c^2 \end{pmatrix}$

Sum of the diagonal entries of

$A^2 = a^2 + 2b^2 + c^2$

Given $a^2 + 2b^2 + c^2 = 1$, $a, b, c \in I$

$b = 0 \ \& \ a^2 + c^2 = 1$

Case–1 : $a = 0 \Rightarrow c = \pm1$ (2-matrices)

Case-2 : $c = 0 \Rightarrow a = \pm1$ (2-matrices)

Total = 4 matrices

27. Official Ans by NTA (1)

Sol. A = XB

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3}a_1 \\ \sqrt{3}a_2 \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$$

$b_1 - b_2 = \sqrt{3}a_1$ ...(1)

$b_1 + kb_2 = \sqrt{3}a_2$ ...(2)

Given, $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$

$(1)^2 + (2)^2$

$(b_1 + b_2)^2 + (b_1 + kb_2)^2 = 3(a_1^2 + a_2^2)$

$a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{(1+k^2)}{3}b_2^2 + \frac{2}{3}b_1b_2(k-1)$

Given, $a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{2}{3}b_2^2$

On comparing we get

$\frac{k^2+1}{3} = \frac{2}{3} \Rightarrow k^2 + 1 = 2$

$\Rightarrow k = \pm 1$ ...(3)

& $\frac{2}{3}(k-1) = 0 \Rightarrow k = 1$ ...(4)

From both we get k = 1

28. Official Ans. by NTA (3)

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$$A^2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^4 = 2^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8 = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow 128\begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$\Rightarrow x - y = \frac{1}{16}$ .....(1)

& $-x + y = \frac{1}{2}$ .....(2)

$\Rightarrow$ From (1) & (2) : No solution.

29. Official Ans. by NTA (766)

Sol. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

diagonal elements of

AA^T, $a^2 + b^2 + c^2$, $d^2 + e^2 + f^2$, $g^2 + b^2 + c^2$

Sum $= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$

a, b, c, d, e, f, g, h, i $\in$ {0, 1, 2, 3}

	Case	**No. of Matrices**
(1)	All – 1s	$\frac{9!}{9!} = 1$
(2)	One $\to$ 3 remaining-0	$\frac{9!}{1!\times 8!} = 9$
(3)	One-2 five-1s three-0s	$\frac{9!}{1!\times 5!\times 3!} = 8\times 63$
(4)	two – 2's one-1 six-0's	$\frac{9!}{2!\times 6!} = 63\times 4$

Total no. of ways = 1 + 9 + 8 × 63 + 63 × 4

$= \boxed{766}$

30. Official Ans. by NTA (1)

Sol. $\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$

$R_2 \to R_1 + R_3 - 2R_2$

$$\Rightarrow \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k-6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$$

$\Rightarrow (k - 6\sqrt{2})(3z - 5x) = 0$

if $3z - 5x = 0 \Rightarrow 3(x + 2d) - 5x = 0$

$\Rightarrow x = 3d$ (Not possible)

$\Rightarrow k = 6\sqrt{2}$ $\Rightarrow k^2 = 72$ **Option (1)**

31. Official Ans. by NTA (2020)

Sol. $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix},\ B=\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

AB = B

$\Rightarrow$ (A – I) B = O

$\Rightarrow$ |A – I | = O, since B $\neq$ O

$$\begin{vmatrix} (a-1) & b \\ c & (d-1) \end{vmatrix}=0$$

ad – bc = 2020

32. Official Ans. by NTA (3)

Sol. $A^2 = \sin^2\alpha\, I$

So, $\left|A^2-\frac{I}{2}\right|=\left(\sin^2\alpha-\frac{1}{2}\right)^2=0$

$\Rightarrow \sin\alpha=\pm\frac{1}{\sqrt{2}}$

33. Official Ans. by NTA (16)

Sol. 2A adj (2A) = |2A|I

$\Rightarrow$ A adj (2A) = –4I ....(i)

Now, E = $|A^4| + |A^{10} – (\text{adj}(2A))^{10}|$

$= (-2)^4 + \frac{|A^{20}-A^{10}(\text{adj } 2A)^{10}|}{|A|^{10}}$

$= 16 + \frac{|A^{20}-(A\,\text{adj}(2A))^{10}|}{|A|^{10}}$

$= 16 + \frac{|A^{20}-2^{10}I|}{2^{10}}$ (from (1))

Now, characteristic roots of A are 2 and –1.

So, characteristic roots of A^{20} are 2^{10} and 1.

Hence, $(A^{20} – 2^{10} I) (A^{20} – I) = 0$

$\Rightarrow |A^{20} – 2^{10}I| = 0$ (as $A^{20} \neq I$)

$\Rightarrow$ E = 16 Ans.

34. Official Ans. by NTA (2)

Sol. $A+2B=\begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}$ ...(1)

$2A-B=\begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$

$\Rightarrow 4A-2B=\begin{pmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{pmatrix}$ ...(2)

$(1)+(2) \Rightarrow 5A=\begin{pmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{pmatrix}$

$A=\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$ and $2A=\begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}$

$\therefore\ B=\begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}-\begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$

$B=\begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$

$\text{tr}(A)=1-1+1=1$
$\text{tr}(B)=-1$

tr(A) = 1 and tr(B) = –1

$\therefore$ tr(A) – tr(B) = 2

35. Official Ans. by NTA (6)

Sol. $P=\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$5I-8P=\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}-\begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix}=\begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$

$P^2=\begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$

$P^3=\begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6=\begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}=P^n$

$\Rightarrow$ n = 6

VECTORS

1. Official Ans. by NTA (4)

Sol. $|\vec{a}| = 3 = a$; $\vec{a}.\vec{c} = c$

Now $|\vec{c}-\vec{a}| = 2\sqrt{2}$

$\Rightarrow c^2 + a^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow c^2 + 9 - 2(c) = 8$

$\Rightarrow c^2 - 2c + 1 = 0 \Rightarrow c = 1 = |\vec{c}|$

Also, $\vec{a}\times\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

Given $(\vec{a}\times\vec{b}) = |\vec{a}\times\vec{b}||\vec{c}|\sin\frac{\pi}{6}$

$= (3)(1)(1/2)$

$= 3/2$

2. Official Ans. by NTA (4)

Sol. $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{c})$

$= 3$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$

$\vec{a}.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}| + |\vec{a}+\vec{b}+\vec{c}|\cos\theta$

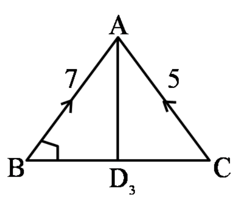
$\Rightarrow 1 = \sqrt{3}\cos\theta$

$\Rightarrow \cos 2\theta = -\frac{1}{3}$

$\Rightarrow 36\cos^2 2\theta = \boxed{4}$

3. Official Ans. by NTA (3)

Sol.



Projection of $\overrightarrow{BA}$

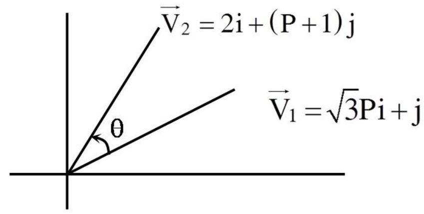
on $\overrightarrow{BC}$ is equal to

$= |\overrightarrow{BA}|\cos\angle ABC$

$= 7\left|\frac{7^2+3^2-5^2}{2\times7\times3}\right| = \frac{11}{2}$

4. Official Ans. by NTA (6)

Sol.



$|\vec{V}_1| = |\vec{V}_2|$

$3P^2 + 1 = 4 + (P+1)^2$

$2P^2 - 2P - 4 = 0 \Rightarrow P^2 - P - 2 = 0$

$P = 2, -1$ (rejected)

$\cos\theta = \frac{\vec{V}_1\cdot\vec{V}_2}{|\vec{V}_1||\vec{V}_2|} = \frac{2\sqrt{3}P + (P+1)}{\sqrt{(P+1)^2+4}\sqrt{3P^2+1}}$

$\cos\theta = \frac{4\sqrt{3}+3}{\sqrt{13}\sqrt{13}} = \frac{4\sqrt{3}+3}{13}$

$\tan\theta = \frac{\sqrt{112-24\sqrt{3}}}{4\sqrt{3}+3} = \frac{6\sqrt{3}-2}{4\sqrt{3}+3} = \frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$

$\Rightarrow \alpha = 6$

5. Official Ans. by NTA (1)

Sol. $\vec{a} = \lambda\vec{b} + \mu\vec{c} = \hat{i}(2\lambda+\mu) + \hat{j}(\lambda-\mu) + \hat{k}(\lambda+\mu)$

$\vec{a}\cdot\vec{d} = 0 = 3(2\lambda+\mu) + 2(\lambda-\mu) + 6(\lambda+\mu)$

$\Rightarrow 14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda$

$\Rightarrow \vec{a} = (0)\hat{i} - 3\lambda\hat{j} + (-\lambda)\hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{10}|\lambda| = \sqrt{10} \Rightarrow |\lambda| = 1$

$\Rightarrow \lambda = 1$ or -1

$[\vec{a}\,\vec{b}\,\vec{c}] = 0$

$[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{b}\,\vec{d}] + [\vec{a}\,\vec{c}\,\vec{d}] = [\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{d}]$

$= \begin{vmatrix} 0 & -3\lambda & \lambda \\ 3 & 0 & 2 \\ 3 & 2 & 6 \end{vmatrix}$

$= 3\lambda(12) + \lambda(6) = 42\lambda = -42$

6. Official Ans. by NTA (4)

Sol. (1) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))$

$=\vec{a}(-\vec{b}\times\vec{c}+\vec{c}\times\vec{b})=-2(\vec{a}\times(\vec{b}\times\vec{c}))$

$=-2(\vec{a}\times\vec{a})=\vec{0}$

(2) Projection of $\vec{a}$ on $\vec{b}\times\vec{c}$

$=\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}=\frac{\vec{a}\cdot\vec{a}}{|\vec{a}|}=|\vec{a}|=2$

(3) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=2[\vec{a}\ \vec{b}\ \vec{c}]=2\vec{a}\cdot(\vec{b}\times\vec{c})$

$=2\vec{a}.\vec{a}=2|\vec{a}|^2=8$

(4) $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow$ $\vec{a},\vec{b},\vec{c}$ are mutually $\perp$ vectors.

$\therefore$ $|\vec{a}\times\vec{b}|=|\vec{c}|\Rightarrow|\vec{a}||\vec{b}|=|\vec{c}|\Rightarrow|\vec{b}|=|\vec{c}|/2$

Also, $|\vec{b}\times\vec{c}|=|\vec{a}|\Rightarrow|\vec{b}||\vec{c}|=2$ $\Rightarrow$ $|\vec{c}|=2$ & $|\vec{b}|=1$

$|3\vec{a}+\vec{b}-2\vec{c}|^2=(3\vec{a}+\vec{b}-2\vec{c})\cdot(3\vec{a}+\vec{b}-2\vec{c})$

$=9|\vec{a}|^2+|\vec{b}|^2+4|\vec{c}|^2$

$=(9\times4)+1+(4\times4)$

$=36+1+16=53$

7. Official Ans. by NTA (1)

Sol. If the vectors are co-planar,

$$\begin{vmatrix} a+b+2 & a+2b+c & -b-c \\ b+1 & 2b & -b \\ b+2 & 2b & 1-b \end{vmatrix}=0$$

Now $R_3\rightarrow R_3-R_2$, $R_1\rightarrow R_1-R_2$

$$\text{So}\begin{vmatrix} a+1 & a+c & -c \\ b+1 & 2b & -b \\ 1 & 0 & 1 \end{vmatrix}=0$$

$=(a+1)2b-(a+c)(2b+1)-c(-2b)$

$=2ab+2b-2ab-a-2bc-c+2bc$

$=2b-a-c=0$

8. Official Ans. by NTA (3)

Sol. $\vec{p}=2\hat{i}+3\hat{j}+\hat{k}$ (Given)

$\vec{q}=\hat{i}+2\hat{j}+\hat{k}$

$$\text{Now }(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$=-2\hat{i}-2\hat{j}-2\hat{k}$

$$\Rightarrow\vec{r}=\pm\sqrt{3}\frac{((\vec{p}+\vec{q})\times(\vec{p}-\vec{q}))}{|(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})|}=\pm\frac{\sqrt{3}(-2\hat{i}-2\hat{j}-2\hat{k})}{\sqrt{2^2+2^2+2^2}}$$

$\vec{r}=\pm(-\hat{i}-\hat{j}-\hat{k})$

According to question

$\vec{r}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

So $|\alpha|=1$, $|\beta|=1$, $|\gamma|=1$

$\Rightarrow|\alpha|+|\beta|+|\gamma|=3$

9. Official Ans. by NTA (4)

Sol. Because vectors are coplanar

$$\text{Hence}\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}=0$$

$\Rightarrow c^2=ab\Rightarrow c=\sqrt{ab}$

10. Official Ans. by NTA (1)

Sol. $|\vec{a}|=2$, $|\vec{b}|=5$

$|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta=\pm8$

$\sin\theta=\pm\frac{4}{5}$

$\therefore$ $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$=10.\left(\pm\frac{3}{5}\right)=\pm6$

$|\vec{a}.\vec{b}|=6$

11. Official Ans. by NTA (60)

Sol. $(\vec{a}+3\vec{b})\perp(7\vec{a}-5\vec{b})$

$(\vec{a}+3\vec{b}).(7\vec{a}-5\vec{b})=0$

$7|\vec{a}|^2-15|\vec{b}|^2+16\vec{a}.\vec{b}=0$ ...(1)

$(\vec{a}-4\vec{b}).(7\vec{a}-2\vec{b})=0$

$7|\vec{a}|^2+8|\vec{b}|^2-30\vec{a}.\vec{b}=0$ ...(2)

from (1) & (2)

$|\vec{a}| = |\vec{b}|$

$\cos\theta = \dfrac{|\vec{b}|}{2|\vec{a}|} \quad \therefore\ \theta = 60°$

12. Official Ans. by NTA (2)

Sol. $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

$\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}+\vec{b}=3\hat{j}+5\hat{k}$; $\vec{a}.\vec{b}=-1+2+6=7$

$((\vec{a}\times((\vec{a}-\vec{b})\times\vec{b}))\times\vec{b})$

$((\vec{a}\times(\vec{a}\times\vec{b}-\vec{b}\times\vec{b}))\times\vec{b})$

$(\vec{a}\times(\vec{a}\times\vec{b}-0))\times\vec{b}$

$(\vec{a}\times(\vec{a}\times\vec{b}))\times\vec{b}$

$((\vec{a}.\vec{b})\vec{a}-(\vec{a}.\vec{a})\vec{b})\times\vec{b}$

$(\vec{a}.\vec{b})\vec{a}\times\vec{b}-(\vec{a}.\vec{a})(\vec{b}\times\vec{b})$

$(\vec{a}.\vec{b})(\vec{a}\times\vec{b})$

$$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix}=-\hat{i}-5\hat{j}+3\hat{k}$$

$\therefore\ 7(-\hat{i}-5\hat{j}+3\hat{k})$

$(\vec{a}+\vec{b})\times(7(-\hat{i}-5\hat{j}+3\hat{k}))$

$7(0\hat{i}+3\hat{j}+5\hat{k})\times(-\hat{i}-5\hat{j}+3\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 5 \\ -1 & -5 & 3 \end{vmatrix}$$

$\Rightarrow 34\hat{i}-(5)\hat{j}+(3\hat{k})$

$\Rightarrow 34\hat{i}-5\hat{j}+3\hat{k}$

$\therefore\ 7(34\hat{i}-5\hat{j}+3\hat{k})$

13. Official Ans. by NTA (2)

Sol. $\vec{a}\times\vec{b}=c$

Take Dot with $\vec{c}$

$(\vec{a}\times\vec{b}).\vec{c}=|\vec{c}|^2=2$

Projection of $\vec{b}$ or $\vec{a}\times\vec{c}=\ell$

$\dfrac{|\vec{b}.(\vec{a}\times\vec{c})|}{|\vec{a}\times\vec{c}|}=\ell$

$\therefore\ \ell=\dfrac{2}{\sqrt{6}}\Rightarrow \ell^2=\dfrac{4}{6}$

$3\ell^2=2$

14. Official Ans. by NTA (2)

Sol. $\vec{a}=(\vec{b}\cdot\vec{c})\vec{b}-(\vec{b}.\vec{b})\vec{c}$

$=1.2\cos\theta\vec{b}-\vec{c}$

$\Rightarrow \vec{a}=2\cos\theta\vec{b}-\vec{c}$

$|\vec{a}|^2=(2\cos\theta)^2+2^2-2.2\cos\theta\vec{b}\cdot\vec{c}$

$\Rightarrow 2=4\cos^2\theta+4-4\cos\theta\cdot 2\cos\theta$

$\Rightarrow -2=-4\cos^2\theta$

$\Rightarrow \cos^2\theta=\dfrac{1}{2}$

$\Rightarrow \sec^2\theta=2$

$\Rightarrow \tan^2\theta=1$

$\Rightarrow \theta=\dfrac{\pi}{4}$

$1+\tan\theta=2.$

15. Official Ans. by NTA (9)

Sol. $\vec{a}=(1,-\alpha,\beta)$

$\vec{b}=(3,\beta,-\alpha)$

$\vec{c}=(-\alpha,-2,1);\ \alpha,\beta\in I$

$\vec{a}.\vec{b}=-1 \Rightarrow 3-\alpha\beta-\alpha\beta=-1$

$\Rightarrow\ \alpha\beta=2$

1	2
2	1
−1	−2
−2	−1

$\vec{b}\cdot\vec{c}=10$

$\Rightarrow\ -3\alpha-2\beta-\alpha=10$

$\Rightarrow\ 2\alpha+\beta+5=0$

$\therefore\ \alpha=-2;\ \beta=-1$

$$[\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$$

$=1(-1+4)-2(3-4)-1(-6+2)$

$=3+2+4=9$

16. Official Ans. by NTA (1)

Sol. $|\vec{a}|=\sqrt{3}; \vec{a}.\vec{c}=3; \ \vec{a}\times\vec{b}=-2\hat{i}+\hat{j}+\hat{k}, \ \vec{a}\times\vec{c}=\vec{b}$

Cross with $\vec{a}$.

$\vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{c})\vec{a}-a^2\vec{c}=\vec{a}\times\vec{b}$

$\Rightarrow 3\vec{a}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow 3\hat{i}+3\hat{j}+3\hat{k}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow \vec{c}=\frac{5\hat{i}}{3}+\frac{2\hat{j}}{3}+\frac{2\hat{k}}{3}$

$\therefore \vec{a}.(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b}).\vec{c}=\frac{-10}{3}+\frac{2}{3}+\frac{2}{3}=-2$

17. Official Ans. by NTA (4)

Sol. $A(\hat{j})$. $B(10\hat{i})$

H $(h\hat{j}+10\hat{k})$

G $(10\hat{i}+h\hat{j}+10\hat{k})$

$\overrightarrow{AG}=10\hat{i}+h\hat{j}+10\hat{k}$

$\overrightarrow{BH}=-10\hat{i}+h\hat{j}+10\hat{k}$

$\cos\theta=\frac{\overrightarrow{AG}\,\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|}$

$\frac{1}{5}=\frac{h^2}{h^2+200}$

$4h^2=200 \Rightarrow h=5\sqrt{2}$

18. Official Ans. by NTA (5)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=1, \ \vec{a}\cdot\vec{b}=12-\lambda$

$(\vec{a}\cdot\vec{b})=|\vec{b}|^2$

$\lambda^2-24\lambda+144=\lambda^2-4\lambda+4+40$

$20\lambda=100 \Rightarrow \lambda=5.$

19. Official Ans. by NTA (90)

Sol. since, $\vec{a}.\vec{b}=0$

$1+15+\alpha\beta=0 \Rightarrow \alpha\beta=-16 \quad \ldots(1)$

Also,

$|\vec{b}\times\vec{c}|^2=75 \Rightarrow (10+\beta^2)14-(5-3\beta)^2=75$

$\Rightarrow 5\beta^2+30\beta+40=0$

$\Rightarrow \beta=-4,-2$

$\Rightarrow \alpha=4,8$

$\Rightarrow |\vec{a}|^2_{max}=(26+\alpha^2)_{max}=90$

20. Official Ans. by NTA (1)

Sol. Equation of planes are

$\vec{r}.(\hat{i}+\hat{j}+\hat{k})-1=0 \Rightarrow x+y+z-1=0$

and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0 \Rightarrow 2x+3y-z+4=0$

equation of planes through line of intersection of these planes is :–

$(x+y+z-1)+\lambda(2x+3y-z+4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$

But this plane is parallel to x–axis whose direction are (1, 0, 0)

$\therefore (1+2\lambda)1+(1+3\lambda)0+(1-\lambda)0=0$

$\lambda=-\frac{1}{2}$

$\therefore$ Required plane is

$0x+\left(1-\frac{3}{2}\right)y+\left(1+\frac{1}{2}\right)z-1+4\left(\frac{-1}{2}\right)=0$

$\Rightarrow \frac{-y}{2}+\frac{3}{2}z-3=0$

$\Rightarrow y-3z+6=0$

$\Rightarrow \boxed{\vec{r}.(\hat{j}-3\hat{k})+6=0}$ Ans.

21. Official Ans. by NTA (3)

Sol. $\left|3\vec{a}+\vec{b}\right|^2=\left|2\vec{a}+3\vec{b}\right|^2$

$\left(3\vec{a}+\vec{b}\right).\left(3\vec{a}+\vec{b}\right)=\left(2\vec{a}+3\vec{b}\right).\left(2\vec{a}+3\vec{b}\right)$

$9\vec{a}.\vec{a}+6\vec{a}.\vec{b}+\vec{b}.\vec{b}=4\vec{a}.\vec{a}+12\vec{a}.\vec{b}+9.\vec{b}.\vec{b}$

$5\left|\vec{a}\right|^2-6\vec{a}.\vec{b}=8\left|\vec{b}\right|^2$

$5(8)^2-6.8.\left|\vec{b}\right|\cos 60^\circ=8\left|\vec{b}\right|^2$ $\left(\because \frac{1}{8}\left|\vec{a}\right|=1 \Rightarrow \left|\vec{a}\right|=8\right)$

$40-3\left|\vec{b}\right|=\left|\vec{b}\right|^2$

$\Rightarrow \left|\vec{b}\right|^2+3\left|\vec{b}\right|-40=0$

$\left|\vec{b}\right|=-8$, $\left|\vec{b}\right|=5$

(rejected)

22. Official Ans. by NTA (3)

Sol. Suppose $\vec{r}=x\vec{a}+y\vec{b}+2\vec{c}$

and $|\vec{a}|=|\vec{b}|=|\vec{c}|=k$

$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$

$\Rightarrow k^2\left(\vec{r}-\vec{b}\right)-k^2x\vec{a}+k^2\left(\vec{r}-\vec{c}\right)-k^2y\vec{b}+$

$k^2\left(\vec{r}-\vec{a}\right)-k^2z\vec{c}=\vec{0}$

$\Rightarrow 3\vec{r}-\left(\vec{a}+\vec{b}+\vec{c}\right)-\vec{r}=\vec{0}$

$\Rightarrow \vec{r}=\frac{\vec{a}+\vec{b}+\vec{c}}{2}$

23. Official Ans. by NTA (1494)

Sol. $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$

$\vec{b}=\hat{i}+2\hat{j}-\hat{k}$

$\vec{c}=3\hat{i}+2\hat{j}-\hat{k}$

$\vec{v}=x\vec{a}+y\vec{b}$ $\vec{v}\left(3\hat{i}+2\hat{j}-k\right)=0$

$\vec{v}\cdot\hat{a}=19$

$\vec{v}=\lambda\vec{c}\times\left(\vec{a}\times\vec{b}\right)$

$\vec{v}=\lambda\left[\left(\vec{c}.\vec{b}\right)\vec{a}-\left(\vec{c}\cdot\vec{a}\right)\vec{b}\right]$

$=\lambda[(3+4+1)\left(2\hat{i}-\hat{j}+2\hat{k}\right)-\left(\frac{6-2-2}{2}\right)\left(\hat{i}+2\hat{j}+\hat{k}\right)$

$=\lambda[16\hat{i}-8\hat{j}+16\hat{k}-2\hat{i}-4\hat{j}+2\hat{k}]$

$\vec{v}=\lambda\left[14\hat{i}-12\hat{j}+18\hat{k}\right]$

$\lambda[14\hat{i}-12\hat{j}+18\hat{k}]\cdot\frac{\left(2\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{4+1+4}}=19$

$\lambda\frac{[28+12+36]}{3}=19$

$\lambda\left(\frac{76}{3}\right)=19$

$4\lambda=3\Rightarrow\lambda=\frac{3}{4}$

$|2v^2|=\left|2\times\frac{3}{4}\left(14\hat{i}-12\hat{j}+18\hat{k}\right)\right|^2$

$\frac{9}{4}\times4\left(7\hat{i}-6\hat{j}+9\hat{k}\right)^2$

$=9(49+36+81)$

$=9(166)$

$=1494$

24. Official Ans. by NTA (3)

Sol. $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$

$\vec{r}.(\hat{i}-2\hat{j})=-2$

point (1, 0, 2)

Eq^n of plane

$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1+\lambda\left\{r.(\hat{i}-2\hat{j})+2\right\}=0$

$\vec{r}.\left\{\hat{i}\left(1+\lambda\right)+\hat{j}\left(1-2\lambda\right)+\hat{k}(1)\right\}-1+2\lambda=0$

Point $\hat{i}+0\hat{j}+2\hat{k}=\vec{r}$

$\therefore(\hat{i}+2\hat{k}).\left\{\hat{i}(1+\lambda)+\hat{j}(1-2\lambda)+\hat{k}(1)\right\}-1+2\lambda=0$

$1+\lambda+2-1+2\lambda=0$

$\lambda=-\frac{2}{3}$

$\therefore \vec{r}.\left[\hat{i}\left(\frac{1}{3}\right)+\hat{j}\left(\frac{7}{3}\right)+\hat{k}\right]=\frac{7}{3}$

$r.\left[\hat{i}+7\hat{j}+3\hat{k}\right]=7$

Ans. 3

25. Official Ans. by NTA (75)

Sol. Let $\vec{c} = \lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$$= \lambda\left(\left(\vec{b}.\vec{b}\right)\vec{a} - \left(\vec{b}.\vec{a}\right)\vec{b}\right)$$

$$= \lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$$

$$= \lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$$

$$\vec{c}.\vec{a} = 7 \Rightarrow 3\lambda+5\lambda+6\lambda = 7$$

$$\lambda = \frac{1}{2}$$

$$\therefore\ 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$$

$$= 2\left(\frac{1}{4}+\frac{49}{4}+25\right) = 25+50 = 75$$

26. Official Ans. by NTA (12)

Sol. $(\vec{r}-\vec{c})\times\vec{a} = 0$

$\Rightarrow\ \vec{r} = \vec{c}+\lambda\vec{a}$

Now, $0 = \vec{b}.\vec{c}+\lambda\vec{a}.\vec{b}$

$$\Rightarrow\ \lambda = \frac{-\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}} = -\frac{2}{-1} = 2$$

So, $\vec{r}\cdot\vec{a} = \vec{a}\cdot\vec{c}+2a^2 = 12$

27. Official Ans. by NTA (2)

Sol. $\vec{a} = \hat{i}+\alpha\hat{j}+3\hat{k}$

$\vec{b} = 3\hat{i}-\alpha\hat{j}+\hat{k}$

area of parallelogram = $|\vec{a}\times\vec{b}| = 8\sqrt{3}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \alpha & 3\\ 3 & -\alpha & 1\end{vmatrix} = \hat{i}(4\alpha)-\hat{j}(-8)+\hat{k}(-4\alpha)$$

$$\therefore\ |\vec{a}\times\vec{b}| = \sqrt{64+32\alpha^2} = 8\sqrt{3}$$

$$\Rightarrow 2+\alpha^2 = 6 \Rightarrow \alpha^2 = 4$$

$$\therefore\ \vec{a}\cdot\vec{b} = 3-\alpha^2+3 = 2$$

28. Official Ans. by NTA (4)

Sol. $\vec{a}_1$ and $\vec{a}_2$ are collinear

so $\dfrac{x}{1} = \dfrac{-1}{y} = \dfrac{1}{z}$

unit vector in direction of

$$x\hat{i}+y\hat{j}+z\hat{k} = \pm\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$$

29. Official Ans. by NTA (4)

Sol. $\vec{a}.\vec{b} = 0$

$$\vec{a}\times\left(\vec{a}\times\vec{b}\right) = \left(\vec{a}.\vec{b}\right)\vec{a}-\left(\vec{a}.\vec{a}\right)\vec{b} = -|\vec{a}|^2\vec{b}$$

Now $\vec{a}\times\left(\vec{a}\times\left(-|\vec{a}|^2\vec{b}\right)\right)$

$$= -|\vec{a}|^2\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)$$

$$= -|\vec{a}|^2\left(-|\vec{a}|^2\vec{b}\right) = |\vec{a}|^4\vec{b}$$

30. Official Ans by NTA (2)

Sol. $\vec{r}\times\vec{a} = \vec{b}\times\vec{r}\ \Rightarrow\ \vec{r}\times(\vec{a}+\vec{b}) = 0$

$\vec{r} = \vec{\lambda}(\vec{a}+\vec{b})\ \Rightarrow\ \vec{r} = \vec{\lambda}(\hat{i}+2\hat{j}-3\hat{k}+2\hat{i}-3\hat{j}+5\hat{k})$

$\vec{r} = \vec{\lambda}(3\hat{i}-\hat{j}+2\hat{k})$ ...(1)

$\vec{r}\cdot(\alpha\hat{i}+2\hat{j}+\hat{k}) = 3$

Put $\vec{r}$ from (1) $\alpha\lambda = 1$ ...(2)

$\vec{r}\cdot(2\hat{i}+5\hat{j}-\alpha\hat{k}) = -1$

Put $\vec{r}$ from (1) $2\lambda\alpha-\lambda = 1$ ...(3)

Solve (2) & (3)

$\alpha = 1,\ \lambda = 1$

$\Rightarrow\ \vec{r} = 3\hat{i}-\hat{j}+2\hat{k}$

$|\vec{r}|^2 = 14\ \&\ \alpha = 1$

$\alpha+|\vec{r}|^2 = 15$

31. Official Ans by NTA (28)

Sol. $\vec{c} = \lambda(\vec{a}\times\vec{b})$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$(\vec{a}\times\vec{b}) = 3\hat{i}-2\hat{j}+\hat{k}$

$\vec{c}\cdot(\hat{i}+\hat{j}+3\hat{k}) = \lambda(3\hat{i}-2\hat{j}+\hat{k}).(\hat{i}+\hat{j}+3\hat{k})$

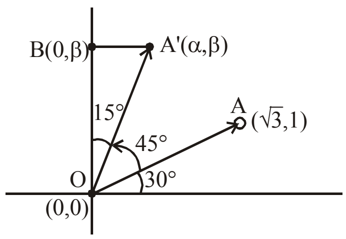
$\Rightarrow \lambda(4)=8 \Rightarrow \lambda=2$

$\vec{c}=2(\vec{a}\times\vec{b})$

$\vec{c}.(\vec{a}\times\vec{b})=2|\vec{a}\times\vec{b}|^2=28$

32. Official Ans. by NTA (1)

Sol.



Area of $\Delta(OA'B) = \frac{1}{2}OA'\cos15° \times OA'\sin15°$

$=\frac{1}{2}(OA')^2\frac{\sin30°}{2}$

$=(3+1)\times\frac{1}{8}=\frac{1}{2}$

33. Official Ans. by NTA (2)

Sol. $\overrightarrow{OP}\perp\overrightarrow{OQ}$

$\Rightarrow -x+2y-3x=0$

$\Rightarrow y=2x$ .....(i)

$|\overrightarrow{PQ}|^2=20$

$\Rightarrow (x+1)^2+(y-2)^2+(1+3x)^2=20$

$\Rightarrow x=1$

$\overrightarrow{OP},\overrightarrow{OQ},\overrightarrow{OR}$ are coplanar.

$$\Rightarrow \begin{vmatrix} x & y & -1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix}=0$$

$\Rightarrow 1(-14-3z)-2(7-9)-1(-z-6)=0$

$\Rightarrow z=-2$

$\therefore x^2+y^2+z^2=1+4+4=9$ **Option (2)**

34. Official Ans. by NTA (486)

Sol. Let $\vec{x}=\lambda\vec{a}+\mu\vec{b}$ (λ and μ are scalars)

$\vec{x}=\hat{i}(2\lambda+\mu)+\hat{j}(2\mu-\lambda)+\hat{k}(\lambda-\mu)$

Since $\vec{x}\cdot(3\hat{i}+2\hat{j}-\hat{k})=0$

$3\lambda+8\mu=0$ .....(1)

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$

$\frac{\vec{x}\cdot\vec{a}}{|\vec{a}|}=\frac{17\sqrt{6}}{2}$

$6\lambda-\mu=51$ .....(2)

From (1) and (2)

$\lambda=8, \mu=-3$

$\vec{x}=13\hat{i}-14\hat{j}+11\hat{k}$

$|\vec{x}|^2=486$

35. Official Ans. by NTA (1)

Sol. $\vec{r}\times\vec{a}-\vec{r}\times\vec{b}=0$

$\Rightarrow \vec{r}\times(\vec{a}-\vec{b})=0$

$\Rightarrow \vec{r}=\lambda(\vec{a}-\vec{b})$

$\Rightarrow \vec{r}=\lambda(-5\hat{i}-4\hat{j}+10\hat{k})$

Also $\vec{r}.(\hat{i}+2\hat{j}+\hat{k})=-3$

$\Rightarrow \lambda(-5-8+10)=-3$

$\lambda=1$

Now $\vec{r}=-5\hat{i}-4\hat{j}+10\hat{k}$

$=\vec{r}.(2\hat{i}-3\hat{j}+\hat{k})$

$=-10+12+10=12$

36. Official Ans. by NTA (2)

Sol. $\vec{a}.\vec{b}=1 \Rightarrow -\alpha\beta-\alpha\beta-3=1$

$\Rightarrow -2\alpha\beta=4 \Rightarrow \boxed{\alpha\beta=-2}$ ....(1)

$\vec{b}.\vec{c}=-3 \Rightarrow -\beta+2\alpha+1=-3$

$\boxed{\beta-2\alpha=4}$ ....(2)

Solving (1) & (2), $(\alpha,\beta) = (-1, 2)$

$$\frac{1}{3}[\vec{a}\ \vec{b}\ \vec{c}] = \frac{1}{3}\begin{vmatrix} \alpha & \beta & 3 \\ -\beta & -\alpha & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

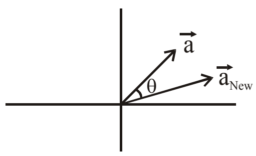
$$= \frac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{3}\begin{vmatrix} 0 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = \frac{1}{3}[2(4-1)] = 2$$

37. Official Ans. by NTA (4)

Sol. $\vec{a}_{Old} = 3p\hat{i} + \hat{j}$

$\vec{a}_{New} = (p+1)\hat{i} + \sqrt{10}\hat{j}$



$\Rightarrow |\vec{a}_{Old}| = |\vec{a}_{New}|$

$\Rightarrow ap^2 + 1 = p^2 + 2p + 1 + 10$

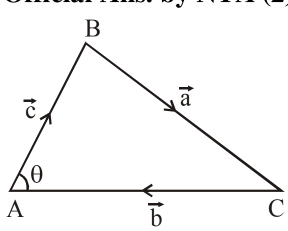
$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0 \rightarrow p = \frac{5}{4}, -1$

38. Official Ans. by NTA (2)

Sol.



$|\vec{a}| = 8, |\vec{b}| = 7, |\vec{c}| = 10$

$$\cos\theta = \frac{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{c}|} = \frac{17}{28}$$

Projection of $\vec{c}$ on $\vec{b}$

$= |\vec{c}|\cos\theta$

$= 10 \times \frac{17}{28}$

$= \frac{85}{14}$

39. Official Ans. by NTA (2)

Sol. $|\vec{a}| = |\vec{b}|$, $|\vec{a}\times\vec{b}| = |\vec{a}|$, $\vec{a} \perp \vec{b}$

$|\vec{a}\times\vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90° = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$

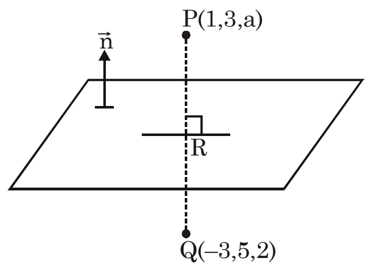
$\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a} = \hat{i}$, $\vec{b} = \hat{j} \Rightarrow \vec{a}\times\vec{b} = \hat{k}$

$$\cos\theta = \frac{(\hat{i}+\hat{j}+\hat{k}).\hat{i}}{\sqrt{3}\ \sqrt{1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

40. Official Ans. by NTA (1)

Sol.



plane $= 2x - y + z = b$

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore\ -2 - 4 + \frac{a+2}{2} = b$

$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10$ ...(i)

<PQ> = <4, –2, a – 2>

$\therefore\ \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$

$\Rightarrow a - 2 = 2 \Rightarrow a = 4, b = -3$

$\therefore |a + b| = 1$

3D

1. Official Ans. by NTA (81)

Sol. Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$\Rightarrow 3x - z - 2 = 0$

$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \parallel$ to $3x - z - 2 = 0$

$\Rightarrow \boxed{3\alpha - 8 = 0}$ .... (1)

$\vec{a} \perp i + 2\hat{j} + 3\hat{k}$

$\Rightarrow \alpha + 2\beta + 38 = 0$ .... (2)

$\vec{a}.(\hat{i}+\hat{j}+2\hat{k}) = 0$

$\Rightarrow \alpha + \beta + 28 = 2$ .... (3)

on solving 1, 2 & 3

$\alpha = 1, \quad \beta = -5, \quad 8 = 3$

So $(\alpha - \beta + 8) = \boxed{81}$

2. Official Ans. by NTA (6)

Sol. If $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$

then shortest distance between two lines is

$$L = \frac{(\vec{a}-\vec{c}).(\vec{b}\times\vec{d})}{|b \times d|}$$

$$\therefore \vec{a}-\vec{c} = ((\alpha+4)\hat{i}+2\hat{j}+3\hat{k})$$

$$\frac{\vec{b}\times\vec{d}}{|b\times d|} = \frac{(2\hat{i}+2\hat{j}+\hat{k})}{3}$$

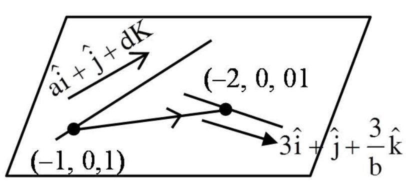
$$\therefore ((\alpha+4)\hat{i}+2\hat{j}+3\hat{k}).\frac{(2\hat{i}+2\hat{j}+\hat{k})}{3} = 9$$

or $\alpha = 6$

3. Official Ans. by NTA (1)

Sol. $\frac{x+1}{a} = y = \frac{z-1}{a}$

$$\frac{x+2}{3} = y = \frac{z}{3/b}$$



lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b} - a\right) - 1(a-3) = 0$$

$$a - \frac{3}{b} - a + 3 = 0$$

$$b = 1, a \in R - \{0\}$$

4. Official Ans. by NTA (4)

Sol. Plane p is $\perp^r$ to line

$$\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$$

& passes through pt. (2, 3) equation of plane p

$2(x-2) + 1(y-3) + 1(z+1) = 0$

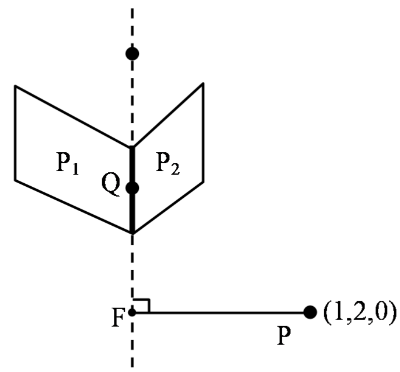
$2x + y + z - 6 = 0$

pt (1,2,2) satisfies above equation

5. Official Ans. by NTA (2)

Sol. $P_1 : x - y + 2z = 2$

$P_2 = 2x + y - 3 = 2$



Let line of Intersection of planes P_1 and P_2 cuts xy plane in point Q.

$\Rightarrow$ z–coordinate of point Q is zero

$$\Rightarrow \left.\begin{matrix} x - y = 2 \\ \text{and } 2x + y = 2 \end{matrix}\right\} \Rightarrow x = \frac{4}{3}, y = \frac{-2}{3}$$

$$\Rightarrow Q\left(\frac{4}{3}, \frac{-2}{3}, 0\right)$$

Vector parallel to the line of intersection

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} + 5\hat{j} + 3\hat{k}$$

Equation of Line of intersection

$$\frac{x - \frac{4}{3}}{-1} = \frac{y + \frac{2}{3}}{5} = \frac{z-0}{3} = \lambda(\text{say})$$

Let coordinates of foot of perpendicular be

$$F\left(-\lambda + \frac{4}{3}, 5\lambda - \frac{2}{3}, 3\lambda\right)$$

$$\overrightarrow{PF} = \left(-\lambda + \frac{1}{3}\right)\hat{i} + \left(5\lambda - \frac{8}{3}\right)\hat{j} + (3\lambda)\hat{k}$$

$$\overrightarrow{PF} \cdot \vec{a} = 0$$

$\Rightarrow \lambda - \frac{1}{3} + 25\lambda \frac{-40}{3} + 9\lambda = 0$

$\Rightarrow 35\lambda = \frac{41}{3} \Rightarrow \boxed{\lambda = \frac{41}{105}}$

Now, $\alpha = -\lambda + \frac{4}{3}, \beta = 5\lambda - \frac{2}{3}, \gamma = 3\lambda$

$\Rightarrow \alpha + \beta + \gamma = 7\lambda + \frac{2}{3}$

$= 7\left(\frac{41}{105}\right) + \frac{2}{3}$

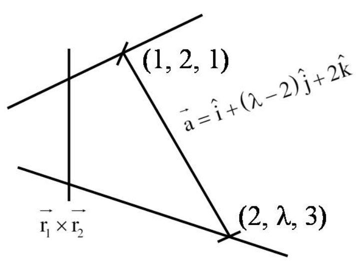
$= \frac{51}{15}$

$\Rightarrow 35(\alpha + \beta + \gamma) = \frac{51}{15} \times 35 = 119$

6. Official Ans. by NTA (1)

Sol. $L_1: \frac{(x-1)}{2} = \frac{(y-2)}{1} = \frac{(z-1)}{3}$ $\quad \vec{r_1} = 2\hat{i} + \hat{j} + 3\hat{k}$

$L_2: \frac{(x-2)}{1} = \frac{y-\lambda}{2} = \frac{z-3}{4}$ $\quad \vec{r_2} = \hat{i} + 2\hat{j} + 4\hat{k}$



Shortest distance = Projection of $\vec{a}$ on $\vec{r_1} \times \vec{r_2}$

$= \frac{|\vec{a}.(\vec{r_1} \times \vec{r_2})|}{|\vec{r_1} \times \vec{r_2}|}$

$|\vec{a} \cdot (\vec{r_1} \times \vec{r_2})| = \begin{vmatrix} 1 & \lambda-2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = |14 - 5\lambda|$

$|\vec{r_1} \times \vec{r_2}| = \sqrt{38}$

$\therefore \frac{1}{\sqrt{38}} = \frac{|14-5\lambda|}{\sqrt{38}}$

$\Rightarrow |14 - 5\lambda| = 1$

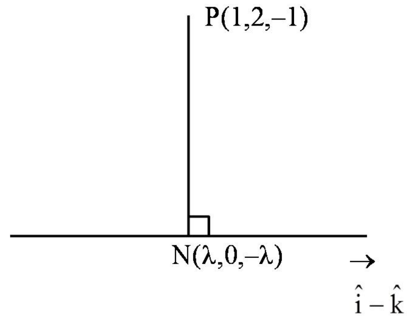
$\Rightarrow 14 - 5\lambda = 1$ or $14 - 5\lambda = -1$

$\Rightarrow \lambda = \frac{13}{5}$ or 3

$\therefore$ Integral value of $\lambda = 3$.

7. Official Ans. by NTA (3)

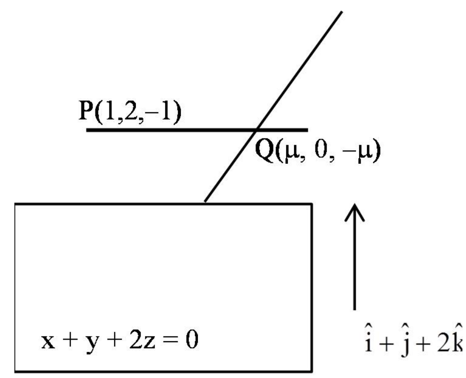
Sol.



$\overrightarrow{PN}.(\hat{i} - \hat{k}) = 0$

$\Rightarrow N(1, 0, -1)$

Now,



$\overrightarrow{PQ}.(\hat{i} + \hat{j} + 2\hat{k}) = 0$

$\Rightarrow \mu = -1$

$\Rightarrow Q(-1, 0, 1)$

$\overrightarrow{PN} = 2\hat{j}$ and $\overrightarrow{PQ} = 2\hat{i} + 2\hat{j} - 2\hat{k}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

8. Official Ans. by NTA (1)

Sol. $\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$

$(k + 1)[2 - 6] - 4[1 - 9] + 6[2 - 6] = 0$

$k = 1$

9. Official Ans. by NTA (4)

Sol. Normal of req. plane $(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} - \hat{j} - \hat{k})$

$= -2\hat{i} + \hat{j} - 3\hat{k}$

Equation of plane

$-2(x + 1) + 1(y - 0) - 3(z + 2) = 0$

$-2x + y - 3z - 8 = 0$

$2x - y + 3z + 8 = 0$

$a + b + c = 4$

10. Official Ans. by NTA (5)

Sol. For infinite solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$\Delta = 3(2+\alpha) = 0$

$\Rightarrow \alpha = -2$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$1(1+2\beta) - 2(1+4) - (\beta - 2) = 0$

$\beta - 7 = 0$

$\beta = 7$

$\therefore\ \alpha + \beta = 5$ Ans.

11. Official Ans. by NTA (3)

Sol. $\overrightarrow{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$

$\overrightarrow{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$

$$\overrightarrow{BA} \times \vec{\ell} = \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix}$$

$a\hat{i} + b\hat{j} + c\hat{k} = -14\hat{i} - \hat{j}(14) + \hat{k}(-14)$

$a = 1, b = 1, c = 1$

Plane is $(x-2) + (y-3) + (z+2) = 0$

$x + y + z - 3 = 0$

$d = \sqrt{3} \Rightarrow d^2 = 3$

12. Official Ans. by NTA (4)

Sol. First line is $(\phi + \alpha, 2\phi + 1, 3\phi + 1)$

and second line is $(q\beta + 4, 3q + 6, 3q + 7)$.

For intersection $\phi + \alpha = q\beta + 4$ ...(i)

$2\phi + 1 = 3q + 6$ ...(i)

$3\phi + 1 = 3q + 7$ ...(iii)

for (ii) & (iii) $\phi = 1, q = -1$

So, from (i) $\alpha + \beta = 3$

Now, point of intersection is $(\alpha + 1, 3, 4)$

It lies on the plane.

Hence, $\alpha = 5$ & $\beta = -2$

13. Official Ans. by NTA (7)

Sol. $\overrightarrow{QR} : -\dfrac{x-3}{1} = \dfrac{y+4}{-1} = \dfrac{z+5}{-6} = r$

$\Rightarrow (x,y,z) \equiv (r+3, -r-4, -6r-5)$

Now, satisfying it in the given plane.

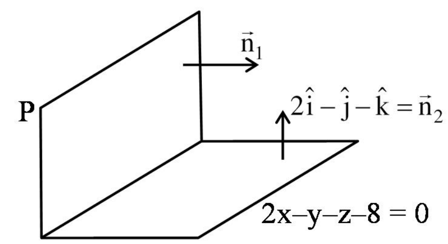
We get $r = -2$.

so, required point of intersection is T(1,–2,7).

Hence, PT = 7.

14. Official Ans. by NTA (2)

Sol. Equation of plane P can be assumed as



$P : x + 2y + 3z + 1 + \lambda(x - y - z - 6) = 0$

$\Rightarrow P : (1+\lambda)x + (2-\lambda)y + (3-\lambda)z + 1 - 6\lambda = 0$

$\Rightarrow \vec{n}_1 = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (3-\lambda)\hat{k}$

$\therefore \vec{n}_1 . \vec{n}_2 = 0$

$\Rightarrow 2(1+\lambda) - (2-\lambda) - (3-\lambda) = 0$

$\Rightarrow 2 + 2\lambda - 2 + \lambda - 3 + \lambda = 0 \Rightarrow \lambda = \dfrac{3}{4}$

$\Rightarrow P : \dfrac{7x}{4} + \dfrac{5}{4}y + \dfrac{9z}{4} - \dfrac{14}{4} = 0$

$\Rightarrow 7x + 5y + 9z = 14$

(0, 1, 1) lies on P

15. Official Ans. by NTA (26)

Sol. $L_1 : \dfrac{x-1}{2} = \dfrac{y-3}{1} = \dfrac{z-4}{2}$

for foot of $\perp$ r of (1, 3, 4) on $x - 2y - z - 3 = 0$

$(1+t) - 2(3-2t) - (4-t) - 3 = 0$

$\Rightarrow t = 2$

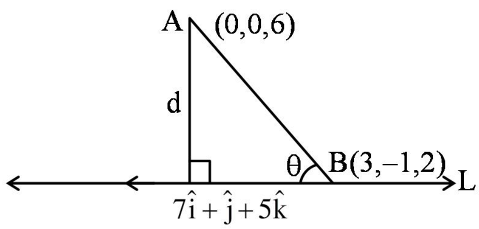
So foot of $\perp$ r $\triangleq$ (3, –1, 2)

& point of intersection of L_1 with plane

is (–11, –3, –8)

dr's of L is <14, 2, 10>

$\cong$ <7, 1, 5>



$$d = AB\sin\theta = \left|\frac{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & -4\\ 7 & 1 & 5\end{vmatrix}}{\sqrt{7^2+1^2+5^2}}\right|$$

$$\Rightarrow d^2 = \frac{1^2+(43)^2+(10)^2}{49+1+25} = 26$$

16. Official Ans. by NTA (3)

Sol. $(x+y+4z-16)+\lambda(-x+y+z-6)=0$

Passes through (1,2,3)

$-1+\lambda(-2) \Rightarrow \lambda = -\frac{1}{2}$

$2(x+y+4z-16)-(-x+y+z-6)=0$

$3x + y + 7z - 26 = 0$

17. Official Ans. by NTA (96)

Sol. Containing the line $\begin{vmatrix} x+1 & y-1 & z-3\\ 6 & 7 & 8\\ 3 & 5 & 7\end{vmatrix} = 0$

$9(x + 1) - 18(y - 1) + 9(z - 3) = 0$

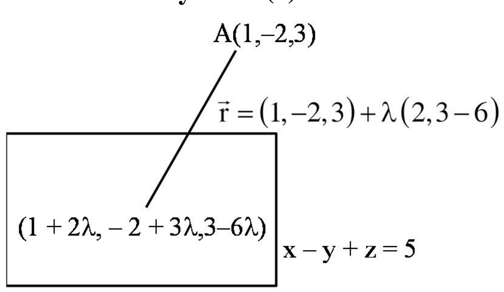
$x - 2y + z = 0$

$PQ = \left|\frac{7+4+13}{\sqrt{6}}\right| = 4\sqrt{6}$

$PQ^2 = 96$

18. Official Ans. by NTA (4)

Sol.



$(1+2\lambda)+2-3\lambda+3-6\lambda=5$

$\Rightarrow 6-7\lambda = 5 \Rightarrow \lambda = \frac{1}{7}$

so, $P = \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$

$$AP = \sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$AP = \sqrt{\left(\frac{4}{49}\right)+\frac{9}{49}+\frac{36}{49}} = 1$$

19. Official Ans. by NTA (4)

Sol. Required equation of plane

$P_1 + \lambda P_2 = 0$

$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$

Given that its dist. From origin is $\frac{2}{\sqrt{21}}$

Thus $\frac{|4\lambda-1|}{\sqrt{(2\lambda+1)^2+(\lambda-1)^2+(-3\lambda-1)^2}} = \frac{\sqrt{2}}{\sqrt{21}}$

$\Rightarrow 21(4\lambda-1)^2 = 2(14\lambda^2+8\lambda+3)$

$\Rightarrow 336\lambda^2 - 168\lambda + 21 = 28\lambda^2 + 16\lambda + 6$

$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$

$\Rightarrow 308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$

$\Rightarrow (2\lambda-1)(154\lambda-15) = 0$

$\Rightarrow \lambda = \frac{1}{2} \text{ or } \frac{15}{154}$

for $\lambda = \frac{1}{2}$ reqd. plane is

$4x - y - 5z + 2 = 0$

20. Official Ans. by NTA (1)

Sol. $n = 2(\ell+m)$

$\ell m + n(\ell+m) = 0$

$\ell m + 2(\ell+m)^2 = 0$

$2\ell^2 + 2m^2 + 5m\ell = 0$

$2\left(\frac{\ell}{m}\right)^2 + 2 + 5\left(\frac{\ell}{m}\right) = 0.$

$2t^2 + 5t + 2 = 0$

$(t + 2)(2t + 1) = 0$

$\Rightarrow t = -2; -\frac{1}{2}$

(i) $\frac{\ell}{m} = -2$

$\frac{n}{m} = -2$

$(-2m, m, -2m)$

(–2, 1, –2)

(ii) $\frac{\ell}{m} = -\frac{1}{2}$

$n = -2\ell$

$(\ell, -2\ell, -2\ell)$

(1, –2, –2)

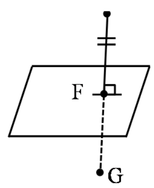
$$\cos\theta = \frac{-2-2+4}{\sqrt{9}\ \sqrt{9}} = 0 \Rightarrow 0 = \frac{\pi}{2}$$

21. Official Ans. by NTA (72)

Sol. Since R (3,5,γ) lies on the plane $2x - y + z + 3 = 0$.

Therefore, $6 - 5 + \gamma + 3 = 0$

$\Rightarrow \gamma = -4$



Now,

dr's of line QS

are 2, –1,1

equation of line QS is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \text{ (say)}$$

$\Rightarrow F(2\lambda + 1, -\lambda + 3, \lambda + 4)$

F lies in the plane

$\Rightarrow 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$

$\Rightarrow 4\lambda + 2 + \lambda - 3 + \lambda + 7 = 0$

$\Rightarrow 6\lambda + 6 = 0 \Rightarrow \lambda = -1.$

$\Rightarrow F(-1,4,3)$

Since, F is mid-point of QS.

Therefore, co-ordinated of S are (–3,5,2).

So, $SR = \sqrt{36+0+36} = \sqrt{72}$

$SR^2 = 72.$

22. Official Ans. by NTA (1)

Sol. Equation of plane is

$3x - 2y + 4z - 7 + \lambda(x + 5y - 2z + 9) = 0$

$(3 + \lambda)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$

passing through (1, 4, –3)

$\Rightarrow 3 + \lambda + 20\lambda - 8 - 12 + 6\lambda + 9\lambda - 7 = 0$

$\Rightarrow \lambda = \frac{2}{3}$

$\Rightarrow$ equation of plane is

$-11x - 4y - 8z + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = -23$

23. Official Ans. by NTA (61)

Sol. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$

$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 6\lambda - 1$

for point of intersection of line & plane

$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

$7\lambda = 7 \Rightarrow \lambda = 1$

point : (3, 5, 5)

$(\text{distance})^2 = (3 + 1)^2 + (5 + 1)^2 + (5 - 2)^2$

$= 16 + 36 + 9 = 61$

24. Official Ans. by NTA (4)

Sol. $P_1 : 2x + 3y + 2z = 0$

$\Rightarrow \vec{n}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$P_2 : x - 2y + z = 0$

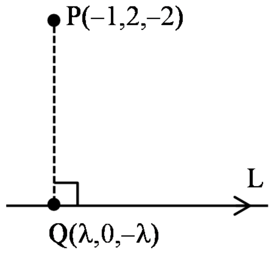
$\Rightarrow \vec{n}_2 = \hat{i} - 2\hat{j} + k$

Direction vector of line L which is line of intersection of P_1 & P_2

$\vec{r} = \vec{n}_1 \times \vec{n}_2 = 7\hat{i} - 7\hat{k}$

DR's of L are (1, 0, –1)

$\Rightarrow$ Equation of L : $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1} = \lambda$



DR's of $\overrightarrow{PQ} = (\lambda + 1, -2, 2 - \lambda)$

$\because \overrightarrow{PQ} \perp \vec{r}$

$\Rightarrow (\lambda + 1)(1) + (-2)(0) + (2 - \lambda)(-1) = 0$

$\Rightarrow \lambda = \frac{1}{2} \Rightarrow Q\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$

$\Rightarrow PQ = \frac{\sqrt{34}}{2}$

25. Official Ans. by NTA (7)

Sol. Point (2, 2, –2) also lies on given plane

So $2 + 3 \times 2 - 2(-2) + \beta = 0$

$\Rightarrow 2 + 6 + 4 + \beta = 0 \Rightarrow \beta = -12$

Also $\alpha \times 1 - 5 \times 3 + 2 \times -2 = 0$

$\Rightarrow \alpha - 15 - 4 = 0 \Rightarrow \alpha = 19$

$\therefore \alpha + \beta = 19 - 12 = 7$

26. Official Ans. by NTA (2)

Sol. $P_1 : x - 2y - 2z + 1 = 0$

$P_2 : 2x - 3y - 6z + 1 = 0$

$$\left|\frac{x-2y-2z+1}{\sqrt{1+4+4}}\right| = \left|\frac{2x-3y-6z+1}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\frac{x-2y-2z+1}{3} = \pm\frac{2x-3y-6z+1}{7}$$

Since $a_1a_2 + b_1b_2 + c_1c_2 = 20 > 0$

$\therefore$ Negative sign will give acute bisector

$7x - 14y - 14z + 7 = -[6x - 9y - 18z + 3]$

$\Rightarrow 13x - 23y - 32z + 10 = 0$

$\left(-2, 0, -\frac{1}{2}\right)$ satisfy it $\therefore$ Ans (2)

27. Official Ans. by NTA (3)

Sol. $3y - 2z - 1 = 0 = 3x - z + 4$

$3y - 2z - 1 = 0$ D.R′s $\Rightarrow$ (0, 3, –2)

$3x - z + 4 = 0$ D.R′s $\Rightarrow$ (3, –1, 0)

Let DR′s of given line are a, b, c

Now $3b - 2c = 0$ & $3a - c = 0$

$\therefore 6a = 3b = 2c$

$a : b : c = 3 : 6 : 9$

Any pt on line

$3K - 1, 6K + 1, 9K + 1$

Now $3(3K - 1) + 6(6K + 1)1 + 9(9K + 1) = 0$

$\Rightarrow K = \frac{1}{3}$

Point on line $\Rightarrow$ (0, 3, 4)

Given point (2, –1, 6)

$\Rightarrow$ Distance $= \sqrt{4+16+4} = 2\sqrt{6}$

Option (3)

28. Official Ans. by NTA (1)

Sol. P(9, 6, 9)

$$\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$$

$Q = (20, b, -a - 9)$

$$\frac{\frac{20+a}{2}-3}{7} = \frac{\frac{b+6}{2}-2}{5} = \frac{-\frac{9}{2}-1}{-9}$$

$$\frac{14+9}{14} = \frac{b+2}{10} = \frac{a+2}{18}$$

$\Rightarrow a = -56$ and $b = -32$

$\Rightarrow |a + b| = 88$

29. Official Ans. by NTA (1)

Sol. $$\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z-0}{-\frac{1}{2}}$$

$$\frac{x-0}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$$

Shortest distance $= \frac{(a_2 - a_1).(b_1 \times b_2)}{|b_1 \times b_2|}$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(\frac{1}{2}+\frac{1}{2}\right) - \hat{j}\left(1+\frac{1}{2}\right) + \hat{k}\left(1-\frac{1}{2}\right)$$

$$= \hat{i} - \frac{3}{2}\hat{j} + \frac{\hat{k}}{2} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{2}$$

$$\frac{b_1 \times b_2}{|b_1 \times b_2|} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{14}}$$

$$\frac{(a_2-a_1).(b_1\times b_2)}{|b_1 \times b_2|} = \left(-\lambda\hat{i} + \left(-2\lambda+\frac{1}{2}\right) + \lambda\hat{k}\right)\left(\frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{14}}\right)$$

$$= \left|\frac{-2\lambda+6\lambda-\frac{3}{2}+\lambda}{\sqrt{14}}\right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\left|5\lambda - \frac{3}{2}\right| = \frac{7}{2}$$

$$5\lambda = \frac{3}{2} \pm \frac{7}{2}$$

$$5\lambda = 5, -2$$

$$\lambda = 1, -\frac{2}{5}$$

30. Official Ans. by NTA (3)

Sol. Normal vector :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} + 17\hat{k}$$

So drs of normal to the required plane is

<11, 1, 17>

plane passes through (1, 2, –3)

So eq^n of plane :

$$11(x-1) + 1(y-2) + 17(z+3) = 0$$

$$\Rightarrow 11x + y + 17z + 38 = 0$$

31. Official Ans. by NTA (4)

Sol. Let $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = t$

$\Rightarrow x = 3 + t, y = 2t + 4, z = 2t + 5$

for point of intersection with $x + y + z = 17$

$3 + t + 2t + 4 + 2t + 5 = 17$

$\Rightarrow 5t = 5 \Rightarrow t = 1$

$\Rightarrow$ point of intersection is (4, 6, 7)

distance between (1, 1, 9) and (4, 6, 7)

is $\sqrt{9+25+4} = \sqrt{38}$

32. Official Ans. by NTA (4)

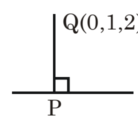
Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = r$

$\Rightarrow P(x, y, z) = (2r + 1, 3r - 1, -2r + 1)$

Since, $\overrightarrow{QP} \perp (2\hat{i}+3\hat{j}-2\hat{k})$

$\Rightarrow 4r + 2 + 9r - 6 + 4r + 2 = 0$

$\Rightarrow r = \frac{2}{17}$



$\Rightarrow P\left(\frac{21}{17}, \frac{-11}{17}, \frac{13}{17}\right)$

$\Rightarrow \overrightarrow{PQ} = \frac{21\hat{i}-28\hat{j}-21\hat{k}}{17}$

So, $\overrightarrow{QP} : \frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

33. Official Ans. by NTA (3)

Sol. $n = \ell + m$

Now, $\ell^2 + m^2 = n^2 = (\ell + m)^2$

$\Rightarrow 2\ell m = 0$

If $\ell = 0 \Rightarrow m = n = \pm\frac{1}{\sqrt{2}}$

And, If $m = 0 \Rightarrow n = \ell = \pm\frac{1}{\sqrt{2}}$

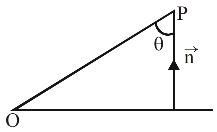
So, direction cosines of two lines are

$\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

Thus, $\cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

34. Official Ans. by NTA (3)

Sol.



Normal to plane $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -2\\ 0 & 1 & 1\end{vmatrix}$

$=3\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{OP}=2\hat{i}-\hat{j}+\hat{k}$

$\cos\theta=\frac{6+1+1}{\sqrt{6}\sqrt{11}}=\frac{8}{\sqrt{66}}\Rightarrow\sin\theta=\sqrt{\frac{2}{66}}$

$\therefore$ Projection of $\overrightarrow{OP}$ on plane $=\left|\overrightarrow{OP}\right|\sin\theta$

$=\sqrt{\frac{2}{11}}$

option (3)

35. Official Ans. by NTA (44)

Sol. $\ell_1:\vec{r}=(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}$

$\ell_2:\vec{r}=(3+2s)\hat{i}+(3+2s)\hat{j}+(4+s)\hat{k}$

DR of $\ell_1\equiv(1, 2, 2)$

DR of $\ell_2\equiv(2, 2, 1)$

DR of ℓ (line $\perp$ to ℓ_1 & ℓ_2)

$=(-2, 3, -2)$

$\therefore\ \ell:\ \vec{r}=-2\mu\hat{i}+3\mu\hat{j}-2\mu\hat{k}$

for intersection of ℓ & ℓ_1

$3+t=-2\mu$

$-1+2t=3\mu$

$4+2t=-2\mu$

$\Rightarrow t=-1$ & $\lambda=-1$

$\therefore$ Point of intersection $P\equiv(2, -3, 2)$

Let point on ℓ_2 be Q $(3+2s, 3+2s, 2+s)$

Given $PQ=\sqrt{17}$ $\Rightarrow(PQ)^2=17$

$\Rightarrow(2s+1)^2+(6+2s)^2+(s)^2=17$

$\Rightarrow 9s^2+28s+20=0$

$\Rightarrow s=-2,\ -\frac{10}{9}$

$s\neq-2$ as point lies on 1st octant.

$\therefore\ a=3+2\left(-\frac{10}{9}\right)=\frac{7}{9}$

$b=3+2\left(-\frac{10}{9}\right)=\frac{7}{9}$

$c=2+\left(-\frac{10}{9}\right)=\frac{8}{9}$

$\therefore\ 18(a+b+c)=18\left(\frac{22}{9}\right)=44$

36. Official Ans. by NTA (4)

Sol. $x+2y+z=6$

$(y+2z=4)\times 2$

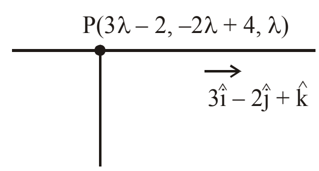
$x-3z=-2\ \Rightarrow x=3z-2\Rightarrow y=4-2z$

$\frac{x+2}{3}=z$ $\qquad\frac{y-4}{-2}=z$

$\Rightarrow$ line of intersection of two planes is

$\frac{x+2}{3}=\frac{y-4}{-2}=z=\lambda$ (Let)

$\because$ AP $\perp^{ar}$ to line



$\therefore\overline{AP}.\left(3\hat{i}-2\hat{j}+\hat{k}\right)=0$

$(3\lambda-5).3+(-2\lambda+2)(-2)+(\lambda-1).1=0$

$9\lambda-15+4\lambda-4+\lambda-1=0$

$14\lambda=20$

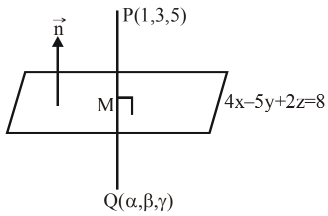
$\lambda=\frac{10}{7}\Rightarrow P\left(\frac{16}{7},\frac{8}{7},\frac{10}{7}\right)$

$\Rightarrow\alpha+\beta+\gamma=\frac{16+8+10}{7}=\frac{34}{7}$

$21(\alpha+\beta+\gamma)=102$

37. Official Ans. by NTA (1)

Sol.



Point Q is image of point P w.r.to plane, M is mid point of P and Q, lies in plane

$$M\left(\frac{1+\alpha}{2},\frac{3+\beta}{2},\frac{5+\gamma}{2}\right)$$

$4x - 5y + 2z = 8$

$$4\left(\frac{1+\alpha}{2}\right)-5\left(\frac{3+\beta}{2}\right)+2\left(\frac{5+\gamma}{2}\right)=8 \quad ..(1)$$

Also PQ perpendicualr to the plane

$\Rightarrow \overrightarrow{PQ} \parallel \vec{n}$

$$\frac{\alpha-1}{4}=\frac{\beta-3}{-5}=\frac{\gamma-5}{2}=k \text{ (let)}$$

$$\left.\begin{array}{l}\alpha=1+4k\\ \beta=3-5k\\ \gamma=5+2k\end{array}\right\} \quad ...(2)$$

use (2) in (1)

$$2(1+4k)-5\left(\frac{6-5k}{2}\right)+(10+2k)=8$$

$$k=\frac{2}{5}$$

from (2) $\alpha=\frac{13}{5},\beta=1,\gamma=\frac{29}{5}$

$5(\alpha + \beta + \gamma) = 13 + 5 + 29 = 47$

38. Official Ans. by NTA (2)

Sol. $P_1 : x + 5y + 7z = 3,$

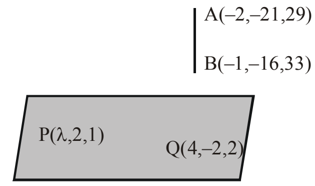
$P_2 : x - 3y - z = 5$

$P_3 : x + 5y + 7z = \frac{5}{2}$

so P_1 and P_3 are parallel.

39. Official Ans. by NTA (8)

Sol.



$\overrightarrow{AB}.\overrightarrow{PQ}=0$

$\Rightarrow \left(\hat{i}+5\hat{j}-6\hat{k}\right).\left((4-\lambda)\hat{i}-4\hat{j}+\hat{k}\right)=0$

$\Rightarrow 4-\lambda-20-6=0$

$\Rightarrow \lambda=-22$

$\Rightarrow \left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4=4+8-4=8$

40. Official Ans. by NTA (3)

Sol. A(1, 5, 35), B(7, 5, 5), C(1, λ, 7), D(2λ, 1, 2)

$\overline{AB}=6\hat{i}-30\hat{k}$, $\overline{BC}=-6\hat{i}(\lambda-5)\hat{j}+2\hat{k}$,

$\overline{CD}=(2\lambda-1)\hat{i}+(1-\lambda)\hat{j}-5\hat{k}$

Points are coplanar

$$\Rightarrow 0=\begin{vmatrix}6 & 0 & -30\\ -6 & \lambda-5 & 2\\ 2\lambda-1 & 1-\lambda & -5\end{vmatrix}$$

$= 6(-5\lambda + 25 - 2 + 2\lambda)$

$-30(-6 + 6\lambda - (2\lambda^2 - \lambda - 10\lambda + 5))$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 11\lambda - 5 - 6 + 6\lambda)$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 17\lambda - 11)$

$= 6(-3\lambda + 23 + 10\lambda^2 - 85\lambda + 55)$

$= 6(10\lambda^2 - 88\lambda + 78) = 12(5\lambda^2 - 44\lambda + 39)$

$\Rightarrow 0 = 12(5\lambda^2 - 44\lambda + 39)$

$\lambda_1+\lambda_2=\frac{44}{5}$

41. Official Ans by NTA (2)

Sol. Plane passing through (42, 0, 0), (0, 42, 0), (0, 0, 42)

From intercept from, equation of plane is

$x + y + z = 42$

$\Rightarrow (x - 11) + (y - 19) + (z - 12) = 0$

let $a = x - 11, b = y - 19, c = z - 12$

$a + b + c = 0$

Now, given expression is

$$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$$

$$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$$

If $a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3\ abc$

$\Rightarrow 3$

42. Official Ans by NTA (2)

Sol. (3,5,7) satisfy the line L_1

$$\frac{3-a}{\ell}=\frac{5-2}{3}=\frac{7-b}{4}$$

$$\frac{3-a}{\ell}=1 \quad \& \quad \frac{7-b}{4}=1$$

$a+\ell=3$ ...(1) & $b = 3$ ...(2)

$\vec{v}_1 = <4,3,8> - <3,5,7>$

$\vec{v}_1 = <1,-2,1>$

$\vec{v}_2 = <\ell,3,4>$

$\vec{v}_1.\vec{v}_2 = 0 \Rightarrow \ell-6+4=0 \Rightarrow \ell=2$

$a+\ell=3 \Rightarrow a=1$

$$L_1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

$$L_2 : \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

A = <1, 2, 3>

B = <2, 4, 5>

$\overrightarrow{AB} = <1,2,2>$

$\vec{p}=2\hat{i}+3\hat{j}+4\hat{k}$

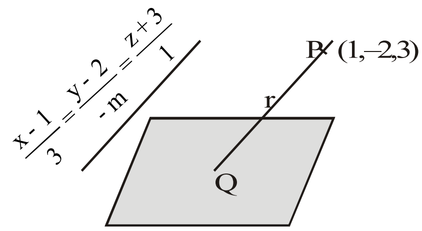
$\vec{q}=3\hat{i}+4\hat{j}+5\hat{k}$

$\vec{p}\times\vec{q}=-\hat{i}+2\hat{j}-\hat{k}$

Shortest distance $=\left|\frac{\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = \frac{1}{\sqrt{6}}$

43. Official Ans by NTA (2)

Sol.



$$\text{DC of line} \equiv \left(\frac{3}{\sqrt{m^2+10}},\frac{-m}{\sqrt{m^2+10}},\frac{1}{\sqrt{m^2+10}}\right)$$

$$Q \equiv \left(1+\frac{3r}{\sqrt{m^2+10}},-2+\frac{-mr}{\sqrt{m^2+10}},3+\frac{r}{\sqrt{m^2+10}}\right)$$

Q lies on $x + 2y - 3z + 10 = 0$

$$1+\frac{3r}{\sqrt{m^2+10}}-4-\frac{2mr}{\sqrt{m^2+10}}-9-\frac{3r}{\sqrt{m^2+10}}+10=0$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(3-2m-3)=2$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(-2m)=2$$

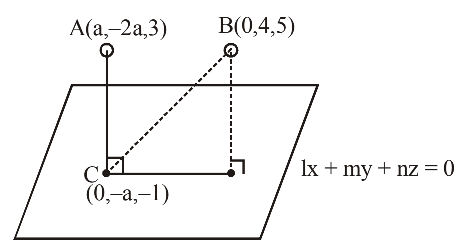
$r^2m^2 = m^2 + 10$

$\frac{7}{2}m^2=m^2+10 \Rightarrow \frac{5}{2}m^2=10 \Rightarrow m^2 = 4$

$|m| = 2$

44. Official Ans. by NTA (4)

Sol.



C lies on plane $\Rightarrow -ma - n = 0 \Rightarrow \frac{m}{n}=-\frac{1}{a}$ ....(1)

$\overrightarrow{CA} \parallel l\hat{i}+m\hat{j}+n\hat{k}$

$$\frac{a-0}{l}=\frac{-a}{m}=\frac{4}{n}\Rightarrow\frac{m}{n}=-\frac{a}{4} \quad ....(2)$$

From (1) & (2)

$$-\frac{1}{a}=\frac{-a}{4}\Rightarrow a^2=4\Rightarrow a=2 \quad (\text{since } a>0)$$

From (2) $\frac{m}{n}=\frac{-1}{2}$

Let $m=-t \Rightarrow n=2t$

$\frac{2}{l}=\frac{-2}{-t}\Rightarrow l=t$

So plane : t(x – y + 2z) = 0

$BD=\frac{6}{\sqrt{6}}=\sqrt{6} \qquad C\cong(0,-2,-1)$

$CD=\sqrt{BC^2-BD^2}$

$=\sqrt{(0^2+6^2+6^2)-\left(\sqrt{6}\right)^2}$

$=\sqrt{66}$

45. Official Ans. by NTA (2)

Sol. P(3, –1, 2)

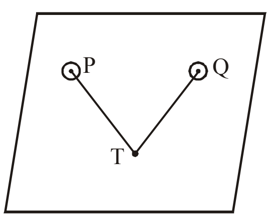
Q(1, 2, –4)

$\overrightarrow{PR}\parallel 4\hat{i}-\hat{j}+2\hat{k}$

$\overrightarrow{QS}\parallel -2\hat{i}+\hat{j}-2\hat{k}$

dr's of normal to the plane containing P, T & Q will be proportional to :

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & 2\\ -2 & 1 & -2\end{vmatrix}$$



$\therefore \quad \frac{\ell}{0}=\frac{m}{4}=\frac{n}{2}$

For point, T : $\overrightarrow{PT}=\frac{x-3}{4}=\frac{y+1}{-1}=\frac{z-2}{2}=\lambda$

$\overrightarrow{QT}=\frac{x-1}{-2}=\frac{y-1}{1}=\frac{z+4}{-2}=\mu$

T : (4λ + 3, –λ –1, 2λ + 2)

$\cong(2\mu+1,\mu+2,-2\mu-4)$

$4\lambda+3=-2\mu+1 \Rightarrow 2\lambda+\mu=-1$

$\lambda+\mu=-3 \Rightarrow \lambda=2$

& $\mu=-5 \qquad \lambda+\mu=-3 \Rightarrow \lambda=2$

So point T : (11, –3, 6)

$\overrightarrow{OA}=\left(11\hat{i}-3\hat{j}+6\hat{k}\right)\pm\left(\frac{2\hat{j}+\hat{k}}{\sqrt{5}}\right)\sqrt{5}$

$\overrightarrow{OA}=\left(11\hat{i}-3\hat{j}+6\hat{k}\right)\pm\left(2\hat{j}+\hat{k}\right)$

$\overrightarrow{OA}=11\hat{i}-\hat{j}+7\hat{k}$

or

$9\hat{i}-5\hat{j}+5\hat{k}$

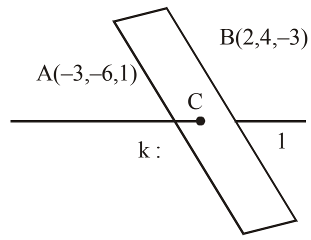
$|\overrightarrow{OA}|=\sqrt{121+1+49}=\sqrt{171}$

or

$\sqrt{81+25+25}=\sqrt{131}$

46. Official Ans. by NTA (3)

Sol.



Point C is

$\left(\frac{2k-3}{k+1},\frac{4k-6}{k+1},\frac{-3k+1}{k+1}\right)$

$\frac{x-1}{-1}=\frac{y+4}{2}=\frac{z+2}{3}$

Plane $lx+my+nz=0$

$l(-1)+m(2)+n(3)=0$

$-l+2m+3n=0$ ....(1)

It also satisfy point (1, –4, –2)

$l-4m-2n=0$ ....(2)

Solving (1) and (2)

2m + 3n = 4m + 2n

n = 2m

$l-4m-4m=0$

$l=8m$

$\frac{l}{8}=\frac{m}{1}=\frac{n}{2}$

$l:m:n=8:1:2$

Plane is 8x + y + 2z = 0

It will satisfy point C

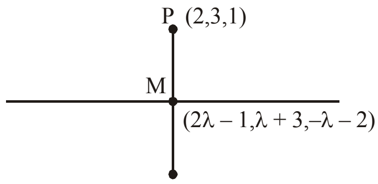
$8\left(\frac{2k-3}{k+1}\right)+\left(\frac{4k-6}{k+1}\right)+2\left(\frac{-3k+1}{k+1}\right)=0$

16k – 24 + 4k – 6 – 6k + 2 = 0

14k = 28 $\quad\therefore$ k = 2

47. Official Ans. by NTA (2)

Sol. Line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$



$\overrightarrow{PM}=(2\lambda-3,\lambda,-\lambda-3)$

$\overrightarrow{PM}\perp\left(2\hat{i}+\hat{j}-\hat{k}\right)$

$4\lambda-6+\lambda+\lambda+3=0 \quad \Rightarrow \lambda=\frac{1}{2}$

$\therefore\ M\equiv\left(0,\frac{7}{2},\frac{-5}{2}\right)$

$\therefore$ Reflection (–2, 4, –6)

Plane : $\begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix}=0$

$\Rightarrow (x-2)(-10+3)-(y-1)(15-4)+(z+1)(-1)=0$

$\Rightarrow -7x+14-11y+11-z-1=0$

$\Rightarrow 7x+11y+z=24$

$\therefore\ \alpha=7, \beta=11, \gamma=1$

$\alpha+\beta+\gamma=19$ **Option (2)**

48. Official Ans. by NTA (0)

Sol. Let point P is (α, β, γ)

$$\left(\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right)^2+\left(\frac{\ell\alpha-n\gamma}{\sqrt{\ell^2+n^2}}\right)^2+\left(\frac{\alpha-2\beta+\gamma}{\sqrt{6}}\right)^2=9$$

Locus is

$$\frac{(x+y+z)^2}{3}+\frac{(\ell x-nz)^2}{\ell^2+n^2}+\frac{(x-2y+z)^2}{6}=9$$

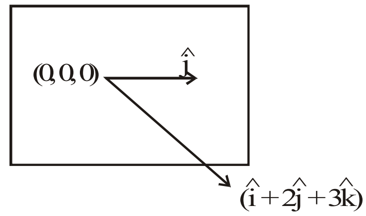
$$x^2\left(\frac{1}{2}+\frac{\ell^2}{\ell^2+n^2}\right)+y^2+z^2\left(\frac{1}{2}+\frac{n^2}{\ell^2+n^2}\right)+2zx\left(\frac{1}{2}-\frac{\ell n}{\ell^2+n^2}\right)-9=0$$

Since its given that $x^2+y^2+z^2=9$

After solving $\ell=n$

49. Official Ans. by NTA (4)

Sol.



$\vec{n}=\hat{j}\times(\hat{i}+2\hat{j}+3\hat{k})$

$=-3\hat{i}+0\hat{j}+\hat{k}$

So,(-3) (x – 1) + 0 (y – 2) + (1) (z – 3) = 0

$\Rightarrow -3x+z=0$

Option 4

Alternate :

Required plane is

$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix}=0$

$\Rightarrow 3x-z=0$

50. Official Ans. by NTA (4)

Sol. Required plane is

$p_1+\lambda p_2=(2+3\lambda)x-(7+5\lambda)y$

$+(4+4\lambda)z-3+11\lambda=0$;

which is satisfied by (–2, 1, 3).

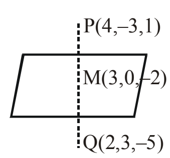
Hence, $\lambda=\frac{1}{6}$

Thus, plane is $15x-47y+28z-7=0$

So, $2a+b+c-7=4$

51. Official Ans. by NTA (28)

Sol.



Plane is $1(x-3)-3(y-0)+3(z+2)=0$

$x-3y+3z+3=0$

$(a^2+b^2+c^2+d^2)_{min}=28$

52. Official Ans. by NTA (4)

Sol. Let plane is $x - 2y + 2z + \lambda = 0$

distance from (1,2,3) = 1

$$\Rightarrow \frac{|\lambda+3|}{5} = 1 \Rightarrow \lambda = 0, -6$$

$\Rightarrow a = 1, b = -2, c = 2, d = -6$ or 0

$b - d = 4$ or -2, $c - a = 1$

$\Rightarrow k = 4$ or -2

53. Official Ans. by NTA (38)

Sol. Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Now $(1, -1, \alpha)$ lies on it so

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0 \Rightarrow 5\alpha + 38 = 0 \Rightarrow |5\alpha| = 38$$

COMPLEX NUMBER

1. Official Ans. by NTA (3)

ALLEN Ans. (2)

Sol. As $|z\omega| = 1$

$\Rightarrow$ If $|z| = r$, then $|\omega| = \frac{1}{r}$

Let $\arg(z) = \theta$

$\therefore \arg(\omega) = \left(\theta - \frac{3\pi}{2}\right)$

So, $z = re^{i\theta}$

$\Rightarrow \bar{z} = re^{i(-\theta)}$

$$\omega = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)}$$

Now, consider

$$\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1+3e^{i\left(-\frac{3\pi}{2}\right)}} = \left(\frac{1-2i}{1+3i}\right)$$

$$= \frac{(1-2i)(1-3i)}{(1+3i)(1-3i)} = -\frac{1}{2}(1+i)$$

$\therefore$ prin arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

$= \text{prin arg}\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

$= \left(-\frac{1}{2}(1+i)\right)$

$= -\left(\pi - \frac{\pi}{4}\right) = \frac{-3\pi}{4}$

So, option (2) is correct.

2. Official Ans. by NTA (1)

Sol. $z = \frac{1}{1-\cos\theta + 2i\sin\theta}$

$$= \frac{2\sin^2\frac{\theta}{2} - 2i\sin\theta}{(1-\cos\theta)^2 + 4\sin^2\theta}$$

$$= \frac{\sin\frac{\theta}{2} - 2i\cos\frac{\theta}{2}}{4\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)}$$

$$\text{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$$

$$\sin\frac{2\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$$

$$1 - \cos^2\frac{\theta}{2} + 4\cos\frac{\theta}{2} = \frac{5}{2}$$

$$3\cos^2\frac{\theta}{2} = \frac{3}{2}$$

$$\cos^2\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

$\theta \in (0, \pi)$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta - [-\cos\theta]_0^{\frac{\pi}{2}}$$

$= -(0 - 1)$

$= 1$

3. Official Ans. by NTA (2)

Sol. $z^2 + 3\bar{z} = 0$

Put $z = x + iy$

$\Rightarrow x^2 - y^2 + 2ixy + 3(x - iy) = 0$

$\Rightarrow (x^2 - y^2 + 3x) + i(2xy - 3y) = 0 + i0$

$\therefore\ x^2 - y^2 + 3x = 0$ .....(1)

$2xy - 3y = 0$ .....(2)

$x = \frac{3}{2}, y = 0$

Put $x = \frac{3}{2}$ in equation (1)

$\frac{9}{4} - y^2 + \frac{9}{2} = 0$

$y^2 = \frac{27}{4} \Rightarrow y = \pm\frac{3\sqrt{3}}{2}$

$\therefore\ (x, y) = \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right), \left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$

Put $y = 0 \Rightarrow x^2 - 0 + 3x = 0$

$x = 0, -3$

$\therefore\ (x, y) = (0, 0), (-3, 0)$

$\therefore$ No of solutions = n = 4

$$\sum_{K=0}^{\infty}\left(\frac{1}{n^k}\right) = \sum_{K=0}^{\infty}\left(\frac{1}{4^k}\right)$$

$$= \frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

4. Official Ans. by NTA (11)

Sol. Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ & $A = \begin{pmatrix} o & i \\ 1 & 0 \end{pmatrix}^n$

$\Rightarrow AX = IX$

$\Rightarrow A = I$

$\Rightarrow \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n = I$

$\Rightarrow A^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ n is multiple of 8

So number of 2 digit numbers in the set

S = 11 (16, 24, 32, ...,96)

5. Official Ans. by NTA (1)

Sol. Equation of circle is $(x^2 - y^2) + 2y^2 + 2x = 0$

$x^2 + y^2 + 2x = 0$

Centre : (–1, 0)

Parabola : $x^2 - 6x - y + 13 = 0$

$(x - 3)^2 = y - 4$

Vertex : (3, 4)

Equation of line $\equiv y - 0 = \frac{4 - 0}{3 + 1}(x + 1)$

$y = x + 1$

y-intercept = 1

6. Official Ans. by NTA (1)

Sol. $S_1 : |z - 3 - 2i|^2 = 8$

$|z - 3 - 2i| = 2\sqrt{2}$

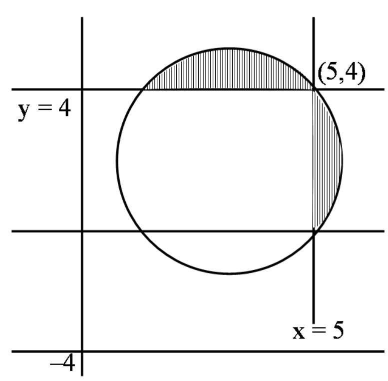
$(x - 3)^2 + (y - 2)^2 = (2\sqrt{2})^2$

$S_2 : x \geq 5$

$S_3 : |z - \bar{z}| \geq 8$

$|2iy| \geq 8$

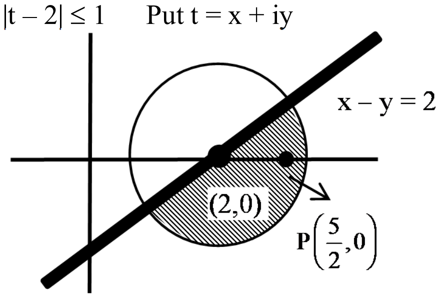
$2|y| \geq 8\ \therefore\ y \geq 4, y \leq -4$



$n(S_1 \cap S_2 \cap S_3) = 1$

7. Official Ans. by NTA (4)

Sol. $|t-2| \le 1$ Put $t = x + iy$



$(x-2)^2 + y^2 \le 1$

Also, $t(1+i) + \bar{t}(1-i) \ge 4$

Gives $x - y \ge 2$

Let point on circle be $A(2 + \cos\theta, \sin\theta)$

$\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$(AP)^2 = \left(2 + \cos\theta - \frac{5}{2}\right)^2 + \sin^2\theta$

$= \cos^2\theta - \cos\theta + \frac{1}{4} + \sin^2\theta$

$= \frac{5}{4} - \cos\theta$

For $(AP)^2$ maximum $\theta = -\frac{3\pi}{4}$

$(AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}+4}{4\sqrt{2}}$

8. Official Ans. by NTA (1)

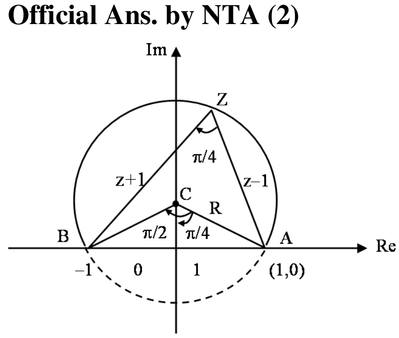
Sol. $\text{Re}(z) = \frac{3 - 6\cos^2\theta}{1 + 9\cos^2\theta} = 0$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $\sin^2 3\theta + \cos^2\theta = 1$.

9. Official Ans. by NTA (2)

Sol.



In ΔOAC

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{AC}$

$\Rightarrow AC = \sqrt{2}$

Also, $\tan\frac{\pi}{4} = \frac{OA}{OC} = \frac{1}{OC}$

$\Rightarrow OC = 1$

$\therefore$ centre (0, 1) ; Radius = $\sqrt{2}$

10. Official Ans. by NTA (13)

Sol. $Z = \frac{1-\sqrt{3}i}{2} = e^{-i\frac{\pi}{3}}$

$z^r + \frac{1}{z^r} = 2\cos\left(-\frac{\pi}{3}\right)r = 2\cos\frac{r\pi}{3}$

$\Rightarrow 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3 = 8\left(\cos^3\frac{r\pi}{3}\right) = 2\left(\cos r\pi + 3\cos\frac{r\pi}{3}\right)$

$\Rightarrow 21 + \left(z + \frac{1}{2}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \ldots\ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$

$= 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3$

$= 21 + \sum_{r=1}^{21}\left(2\cos r\pi + 6\cos\frac{r\pi}{3}\right)$

$= 21 - 2 - 6$

$= 13$

11. Official Ans. by NTA (1)

Sol. $\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p + iq)$

$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}(p + iq)$

$p + iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$p = -1,\ q = \sqrt{3}$

$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0.$

12. Official Ans. by NTA (6)

Sol. $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}}$

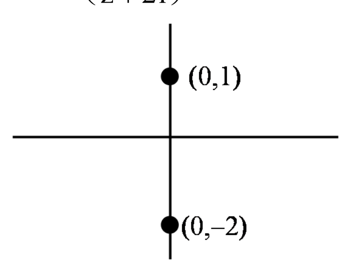
$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{2^{\frac{n+2}{2}} i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$

This is positive integer for n = 6

13. Official Ans. by NTA (4)

Sol. Given $\frac{z-i}{z+2i} \in R$

Then $\arg\left(\frac{z-i}{z+2i}\right)$ is 0 or Π



$\Rightarrow$ S is straight line in complex

14. Official Ans. by NTA (6)

Sol. $|z-3| = \text{Re}(z)$

let $Z = x = iy$

$\Rightarrow (x-3)^2 + y^2 = x^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2$

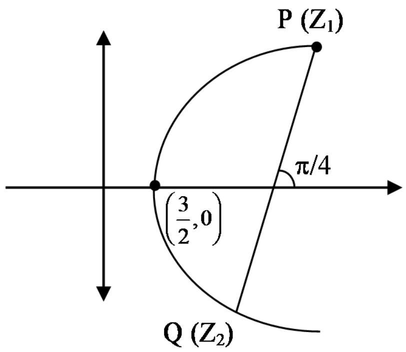
$\Rightarrow y^2 = 6x - 9$

$\Rightarrow y^2 = 6\left(x - \frac{3}{2}\right)$

$\Rightarrow z_1$ and z_2 lie on the parabola mentioned in eq.(1)

$\arg(z_1 - z_2) = \frac{\pi}{4}$

$\Rightarrow$ Slope of PQ = 1.



Let $P\left(\frac{3}{2}+\frac{3}{2}t_1^2, 3t_1\right)$ and $Q\left(\frac{3}{2}+\frac{3}{2}t_2^2, 3t_2\right)$

Slope of PQ = $\dfrac{3(t_2 - t_1)}{\frac{3}{2}(t_1^2 - t_1^2)} = 1$

$\Rightarrow \dfrac{2}{t_2 + t_2} = 1$

$\Rightarrow t_2 + t_1 = 2$

$\text{Im}(z_1 + z_2) = 3t_1 + 3t_2 = 3(t_1 + t_2) = 3\ (2)$

Ans. 6.00

Aliter :

Let $z_1 = x_1 + iy_1$; $z_2 = x_2 + iy_2$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$\therefore\ \arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ ______(1)

$|z_1 - 3| = \text{Re}(z_1) \Rightarrow (x_1 - 3)^2 + y_1^2 = x_1^2$ __ (2)

$|z_2 - 3| = \text{Re}(z_2) \Rightarrow (x_2 - 3)^2 + y_2^2 = x_2^2$ __(2)

sub (2) & (3)

$(x_1 - 3)^2 - (x_2 - 3)^2 + y_1^2 - y_2^2 = x_1^2 - x_2^2$

$(x_1 - x_2)(x_1 + x_2 - 6) + (y_1 - y_2)(y_1 + y_2)$

$= (x_1 - x_2)(x_1 + x_2)$

$x_1 + x_2 - 6 + y_1 + y_2 = x_1 + x_2 \Rightarrow y_1 + y_2 = 6.$

15. Official Ans. by NTA (98)

Sol. Let $z = x + iy$

$\arg\left(\dfrac{x-2+iy}{x+2+iy}\right) = \dfrac{\pi}{4}$

$\arg(x - 2 + iy) - \arg(x + 2 + iy) = \dfrac{\pi}{4}$

$\tan^{-1}\left(\dfrac{y}{x-2}\right) - \tan^{-1}\left(\dfrac{y}{x+2}\right) = \dfrac{\pi}{4}$

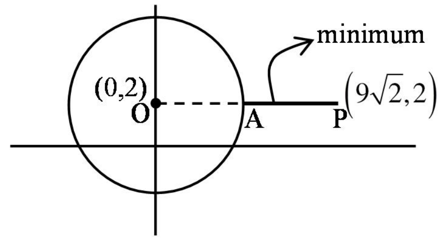
$\dfrac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \left(\frac{y}{x-2}\right).\left(\frac{y}{x+2}\right)} = \tan\dfrac{\pi}{4} = 1$

$\dfrac{xy + 2y - xy + 2y}{x^2 - 4 + y^2} = 1$

$4y = x^2 - 4 + y^2$

$x^2 + y^2 - 4y - 4 = 0$

locus is a circle with center (0, 2) & radius = $2\sqrt{2}$



min. value = $(AP)^2 = (OP - OA)^2$

$= \left(9\sqrt{2} - 2\sqrt{2}\right)^2$

$= \left(7\sqrt{2}\right)^2 = 98$

16. Official Ans. by NTA (4)

Sol. $\dfrac{z-i}{z-1}$ is purely Imaginary number

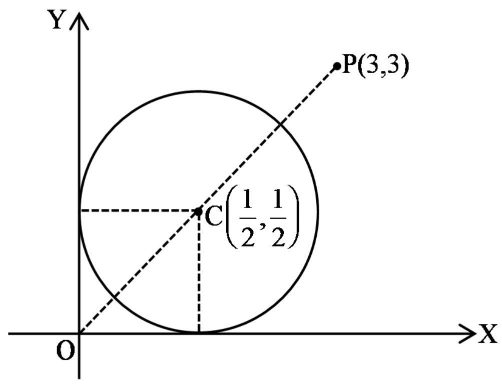
Let $z = x + iy$

$\therefore\ \dfrac{x + i(y-1)}{(x-1)+i(y)} \times \dfrac{(x-1)-iy}{(x-1)-iy}$

$\Rightarrow \dfrac{x(x-1) + y(y-1) + i(-y-x+1)}{(x-1)^2 + y^2}$ is purely Imaginary number

$\Rightarrow x(x-1) + y(y-1) = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$



$\therefore\ |z-(3+3i)|_{min} = |PC| - \frac{1}{\sqrt{2}}$

$= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2}$

17. Official Ans. by NTA (5)

Sol. $|z-2-2i| \le 1$

$|x+iy-2-2i| \le 1$

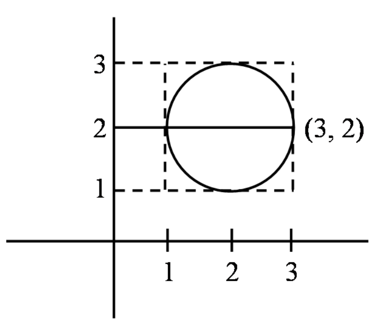
$|(x-2)+i(y-2)| \le 1$

$(x-2)^2+(y-2)^2 \le 1$

$|3iz+6|_{max}$ at a + ib

$|3i|\ \left|z+\frac{6}{3i}\right|$

$3|z-2i|_{max}$



From Figure maximum distance at 3 + 2i

a + ib = 3 + 2i = a + b = 3 + 2 = 5 **Ans.**

18. Official Ans. by NTA (310)

Sol. $K = \frac{1}{2^9}\left[\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^{24}} + \frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^{24}}\right]$

$K = \frac{1}{512}\left[\frac{\left(e^{i\frac{2\pi}{3}}\right)^{21}}{\left(e^{-\frac{i\pi}{4}}\right)^{24}} + \frac{\left(e^{\frac{i\pi}{3}}\right)^{21}}{\left(e^{\frac{i\pi}{4}}\right)^{24}}\right]$

$K = \frac{1}{512}\left[e^{i(14\pi+6\pi)} + e^{i(7\pi-6\pi)}\right]$

$K = \frac{1}{512}\left[e^{20\pi i} + e^{\pi i}\right]$

$K = \frac{1}{512}[1+(-1)] = 0$

$n = [|k|] = 0$

$\sum_{j=0}^{5}(j+5)^2 - \sum_{j=0}^{5}(j+5)$

$\sum_{j=0}^{5}(j^2+25+10j-j-5)$

$\sum_{j=0}^{5}(j^2+9j+20)$

$\sum_{j=0}^{5} j^2 + 9\sum_{j=0}^{5} j + 20\sum_{j=0}^{5} 1$

$\frac{5\times6\times11}{6} + 9\left(\frac{5\times6}{2}\right) + 20\times6$

= 55 + 135 + 120

= 310

19. Official Ans. by NTA (10)

Sol. Put $z = x + iy$

$x + iy + \alpha|x + iy - 1| + 2i = 0$

$\Rightarrow x + \alpha\sqrt{(x-1)^2 + y^2} + i(y+2) = 0 + 0i$

$\Rightarrow y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$\Rightarrow y = -2$ and $\alpha^2 = \dfrac{x^2}{x^2 - 2x + 5}$

Now $\dfrac{x^2}{x^2-2x+5} \in \left[0, \dfrac{5}{4}\right]$

$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right] \Rightarrow \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$

$\therefore p = -\dfrac{\sqrt{5}}{2}; q = \dfrac{\sqrt{5}}{2}$

$\Rightarrow 4(p^2 + q^2) = 4\left(\dfrac{5}{4} + \dfrac{5}{4}\right) = 10$

20. Official Ans. by NTA (4)

Sol. (i) $(2 - i)z = (2 + i)\bar{z}$

$\boxed{y = \dfrac{x}{2}}$

(ii) $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

$\boxed{x + 2y = 2}$

(iii) $iz + \bar{z} + 1 + i = 0$

Eq^n of tangent $\boxed{x - y + 1 = 0}$

Solving (i) and (ii)

$x = 1, y = \dfrac{1}{2}$

Now, $p = r \Rightarrow \left|\dfrac{1 - \frac{1}{2} + 1}{\sqrt{2}}\right| = r$

$\Rightarrow r = \dfrac{3}{2\sqrt{2}}$

21. Official Ans. by NTA (48)

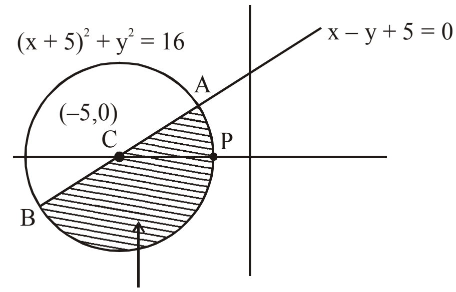
Sol. $|z + 5| \le 4$

$(x + 5)^2 + y^2 \le 16$ …(1)

$z(1+i) + \bar{z}(1-i) \ge -10$

$(z+\bar{z}) + i(z-\bar{z}) \ge -10$

$x - y + 5 \ge 0$ …(2)



Region bounded by inequalities (1) & (2)

$|z + 1|^2 = |z - (-1)|^2$

Let P(–1, 0)

$\boxed{|z+1|^2_{Max.} = PB^2}$ (where B is in 3rd quadrant)

for point of intersection

$\left.\begin{matrix}(x+5)^2 + y^2 = 16 \\ x - y + 5 = 0\end{matrix}\right\rangle y = \pm 2\sqrt{2}$

$A\left(2\sqrt{2} - 5, 2\sqrt{2}\right)$ $B\left(-2\sqrt{2} - 5, -2\sqrt{2}\right)$

$PB^2 = \left(+2\sqrt{2} + 4\right)^2 + \left(2\sqrt{2}\right)^2$

$|z+1|^2 = 8 + 16 + 16\sqrt{2} + 8$

$\alpha + \beta\sqrt{2} = 32 + 16\sqrt{2}$

$\alpha = 32, \beta = 16 \Rightarrow \alpha + \beta = 48$

22. Official Ans. by NTA (3)

Sol. $x^3 - 2x^2 + 2x - 1 = 0$

$x = 1$ satisfying the equation

$\therefore$ $x - 1$ is factor of

$x^3 - 2x^2 + 2x - 1$

$= (x - 1)(x^2 - x + 1) = 0$

$x = 1, \dfrac{1 + i\sqrt{3}}{2}, \dfrac{1 - i\sqrt{3}}{2}$

$x = 1, -\omega^2, -\omega$

sum of 162th power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + (\omega)^{324} + (\omega)^{162}$

$= 1 + 1 + 1 = 3$

$n + m = 45$

23. Official Ans by NTA (1)

Sol. $\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\ell n\, 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq \log_{\sqrt{2}}(16)$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$

$\Rightarrow \frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$

$\Rightarrow (|z| + 3)(|z| - 1) \geq 3(|z| + 1)$

$|z|^2 + 2|z| - 3 \geq 3|z| + 3$

$\Rightarrow |z|^2 + |z| - 6 \geq 0$

$\Rightarrow (|z| - 3)(|z| + 2) \geq 0 \Rightarrow |z| - 3 \geq 0$

$\Rightarrow |z| \geq 3 \quad \Rightarrow |z|_{min} = 3$

24. Official Ans. by NTA (2)

Sol. $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$

$\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$

$2|z| + 22 \geq (|z| - 1)^2$

$2|z| + 22 \geq |z|^2 + 1 - 2|z|$

$|z|^2 - 4|z| - 21 \leq 0$

$\Rightarrow \quad |z| \leq 7$

$\therefore$ Largest value of $|z|$ is 7

25. Official Ans. by NTA (36)

Sol. Let $M = (P^{-1}AP - I)^2$

$= (P^{-1}AP)^2 - 2P^{-1}AP + I$

$= P^{-1}A^2P - 2P^{-1}AP + I$

$PM = A^2P - 2AP + P$

$= (A^2 - 2A.I + I^2)P$

$\Rightarrow \quad \text{Det}(PM) = \text{Det}((A - I)^2 \times P)$

$\Rightarrow \quad \text{DetP.DetM} = \text{Det}(A - I)^2 \times \text{Det}(P)$

$\Rightarrow \quad \text{Det } M = (\text{Det}(A - I))^2$

Now $A - I = \begin{bmatrix} 1 & 7 & w^2 \\ -1 & -w-1 & 1 \\ 0 & -w & -w \end{bmatrix}$

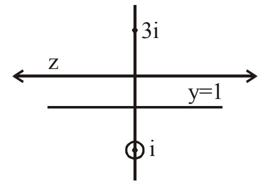
$\text{Det}(A - I) = (w^2 + w + w) + 7(-w) + w^3 = -6w$

$\text{Det}((A - I))^2 = 36w^2$

$\Rightarrow \quad \alpha = 36$

26. Official Ans. by NTA (4)

Sol. $\omega = z\bar{z} - 2z + 2$



$\left|\frac{z+i}{z-3i}\right| = 1$

$\Rightarrow \quad |z+i| = |z-3i|$

$\Rightarrow \quad z = x + i, \; x \in \mathbb{R}$

$\omega = (x + i)(x - i) - 2(x + i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$\text{Re}(\omega) = x^2 - 2x + 3$

For min $(\text{Re}(\omega))$, $x = 1$

$\Rightarrow \quad \omega = 2 - 2i = 2(1 - i) = 2\sqrt{2}\, e^{-i\frac{\pi}{4}}$

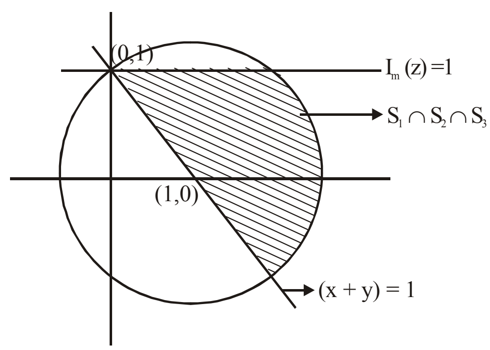
$\omega^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$

For real & minimum value of n,

$n = 4$

27. Official Ans. by NTA (3)

Sol. For $|z-1| \leq \sqrt{2}$, z lies on and inside the circle of radius $\sqrt{2}$ units and centre (1, 0).



For S_2

Let $z = x + iy$

Now, $(1 - i)(z) = (1 - i)(x + iy)$

$\text{Re}((1 - i)z) = x + y$

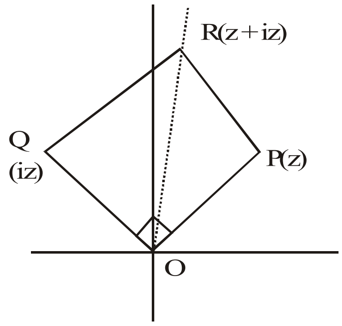
$\Rightarrow x + y \geq 1$

$\Rightarrow S_1 \cap S_2 \cap S_3$ has infinity many elements

Ans. (3)

28. Official Ans. by NTA (2)

Sol.



$A = \frac{1}{2}|z|\,|iz|$

$= \frac{|z|^2}{2}$

29. Official Ans. by NTA (2)

Sol. $az\bar{z} + \alpha\bar{z} + \bar{\alpha}z + d = 0 \rightarrow$ Circle

centre $= \frac{-\alpha}{a}$ $\quad 2 = \sqrt{\frac{\alpha\bar{\alpha}}{a^2} - \frac{d}{a}} = \sqrt{\frac{\alpha\bar{\alpha} - ad}{a^2}}$

So $|\alpha|^2 - ad > 0$ & $a \in R - \{0\}$

30. Official Ans. by NTA (6)

Sol. If $0, z, z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1^2 + z_2^2 = 0\ (z_1 + z_2) + z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$

$\Rightarrow a^2 = 3 \times 12$

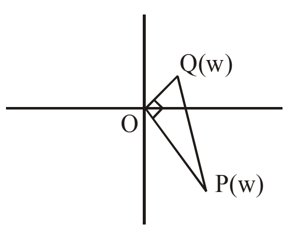
$\Rightarrow |a| = 6$

31. Official Ans. by NTA (2)

Sol. $w = 1 - \sqrt{3}.i \Rightarrow |w| = 2$

Now, $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and $amp(z) = \frac{\pi}{2} + amp(w)$



$\Rightarrow$ Area of triangle $= \frac{1}{2}.OP.OQ$

$= \frac{1}{2}.2.\frac{1}{2} = \frac{1}{2}$

32. Official Ans. by NTA (0)

Sol. $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(1)

Now P(x) is divisible by $x^2 + x + 1$

$\Rightarrow P(x) = Q(x)(x^2 + x + 1)$

$P(w) = 0 = P(w^2)$ where w, w^2 are non-real cube roots of units

$P(x) = f(x^3) + xg(x^3)$

$P(w) = f(w^3) + wg(w^3) = 0$

$f(1) + wg(1) = 2$ ...(2)

$P(w^2) = f(w^6) + w^2g(w^6) = 0$

$f(1) + w^2g(1) = 0$ ...(3)

(2) + (3)

$\Rightarrow 2f(1) + (w + w^2)g(1) = 0$

$2f(1) = g(1)$ ...(4)

(2) – (3)

$\Rightarrow (w - w^2)g(1) = 0$

$g(1) = 0 = f(1)$ from (4)

from (1) $P(1) = f(1) + g(1) = 0$

PROBABILITY

1. Official Ans. by NTA (2)

Sol. AAEIIMNNOTX

--------M----------

Total words with M at fourth Place $= \frac{10!}{2!2!2!}$

Total words $= \frac{11!}{2!2!2!}$

Required probability $= \frac{10!}{11!} = \frac{1}{11}$

2. Official Ans. by NTA (2)

Sol. $D < 0$

$\Rightarrow 4(a + 4)^2 - 4(-5a + 64) < 0$

$\Rightarrow a^2 + 16 + 8a + 5a - 64 < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\Rightarrow (a + 16)(a - 3) < 0$

$\Rightarrow a \in (-16, 3)$

$\therefore$ Possible a : $\{-5, -4, ..., 3\}$

$\therefore$ Required probability $= \frac{8}{36}$

$= \frac{2}{9}$

3. Official Ans. by NTA (2)

Sol. $P(\bar{A}\cap B)+P(A\cap\bar{B})=1-k$

$P(\bar{A}\cap C)+P(A\cap\bar{C})=1-2k$

$P(\bar{B}\cap C)+P(B\cap\bar{C})=1-k$

$P(A\cap B\cap C)=k^2$

$P(A)+P(B)-2P(A\cap B)=1-k$ .....(i)

$P(B)+P(C)-2P(B\cap C)=1-k$ .....(ii)

$P(C)+P(A)-2P(A\cap C)=1-2k$ .....(iii)

(1) + (2) + (3)

$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)$

$-P(C\cap A)=\frac{-4k+3}{2}$

So

$P(A\cup B\cup C)=\frac{-4k+3}{2}+k^2$

$P(A\cup B\cup C)=\frac{2k^2-4k+3}{2}$

$=\frac{2(k-1)^2+1}{2}$

$P(A\cup B\cup C)>\frac{1}{2}$

4. Official Ans. by NTA (4)

Sol. $A=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $|A| = ad - bc$

Total case = 6^4

For non-singular matrix $|A|\neq 0 \Rightarrow ad-bc\neq 0$

$\Rightarrow ad \neq bc$

And a, b, c, d are all different numbers in the set {1, 2, 3, 4, 5, 6}

Now for ad = bc

(i) $6\times 1 = 2\times 3$

$\left.\begin{array}{l}\Rightarrow a=6,\ b=2,\ c=3,\ d=1 \\ \text{or } a=1,\ b=2,\ c=3,\ d=6 \\ \vdots \end{array}\right\}$ 8 such cases

(ii) $6\times 2 = 3\times 4$

$\left.\begin{array}{l}\Rightarrow a=6,\ b=3,\ c=4,\ d=2 \\ \text{or } a=2,\ b=3,\ c=4,\ d=6 \\ \vdots \end{array}\right\}$ 8 such cases

favourable cases

$= {}^6C_4\lfloor 4 - 16$

required probability

$=\frac{{}^6C_4\lfloor 4-16}{6^4}=\frac{43}{162}$

5. Official Ans. by NTA (1)

Sol. required probability = $\frac{{}^9C_3.3^6}{4^9}$

$=\frac{{}^9C_3}{27}.\left(\frac{3}{4}\right)^9$

$=\frac{28}{9}.\left(\frac{3}{4}\right)^9 \Rightarrow k=\frac{28}{9}$

Which satisfies $|x-3|<1$

6. Official Ans. by NTA (2)

Sol. mean = $\Sigma x_ip_i = \sum_{r=0}^{\infty} r.\frac{1}{3^r}=\frac{3}{4}$

p(x is even) = $\frac{1}{3^2}+\frac{1}{3^4}+...\infty$

$=\frac{\frac{1}{9}}{1-\frac{1}{9}}=\frac{1/9}{8/9}=\frac{1}{8}$

7. Official Ans. by NTA (4)

Sol. P(Head) = $\frac{1}{2}$

1 – P(All tail) ≥ 0.9

$1-\left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{10}$

$\Rightarrow n_{min} = 4$

8. Official Ans. by NTA (3)

Sol. Total number of cases = ${}^{90}C_1 = 90$

Now, $2^n - 2 = (3-1)^n - 2$

${}^nC_0 3^n - {}^nC_1.3^{n-1}+...+(-1)^{n-1}.{}^nC_{n-1}3+(-1)^n.{}^nC_n - 2$

$3\left(3^{n-1}-n3^{n-2}+...+(-1)^{n-1}.n\right)+(-1)^n-2$

$(2^n - 2)$ is multiply of 3 only when n is odd

Req. Probability $=\frac{45}{90}=\frac{1}{2}$

9. Official Ans. by NTA (1)

Sol. $P(E) < \frac{1}{2}$

$\Rightarrow \sum_{r=n}^{8} {}^8C_r \left(\frac{1}{2}\right)^{8-r} \left(\frac{1}{2}\right)^{r} < \frac{1}{2}$

$\Rightarrow \sum_{r=n}^{8} {}^8C_r \left(\frac{1}{2}\right)^{8} < \frac{1}{2}$

$\Rightarrow {}^8C_n + {}^8C_{n+1} + ... + {}^8C_8 < 128$

$\Rightarrow 256 - \left({}^8C_0 + {}^8C_1 + ... + {}^8C_{n-1}\right) < 128$

$\Rightarrow {}^8C_0 + {}^8C_1 + ... + {}^8C_{n-1} > 128$

$\Rightarrow n - 1 \ge 4$

$\Rightarrow n \ge 5$

10. Official Ans. by NTA (4)

Sol. P(Exactly one of A or B)

$= P\left(A \cap \bar{B}\right) + P\left(\bar{A} \cap B\right) = \frac{5}{9}$

$= P(A)P(\bar{B}) + P(\bar{A})P(B) = \frac{5}{9}$

$\Rightarrow P(A)(1-P(B)) + (1-P(A))P(B) = \frac{5}{9}$

$\Rightarrow p(1 - 2p) + (1 - p)\,2p = \frac{5}{9}$

$\Rightarrow 36p^2 - 27p + 5 = 0$

$\Rightarrow p = \frac{1}{3}$ or $\frac{5}{12}$

$p_{max} = \frac{5}{12}$

11. Official Ans. by NTA (4)

Sol. $P(x \ge 5 \mid x > 2) = \frac{P(x \ge 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 . \frac{1}{6} + \left(\frac{5}{6}\right)^5 . \frac{1}{6} + ... + \infty}{\left(\frac{5}{6}\right)^2 . \frac{1}{6} + \left(\frac{5}{6}\right)^3 . \frac{1}{6} + ... + \infty}$$

$$\frac{\dfrac{\left(\frac{5}{6}\right)^4 . \frac{1}{6}}{1 - \frac{5}{6}}}{\dfrac{\left(\frac{5}{6}\right)^2 . \frac{1}{6}}{1 - \frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

12. Official Ans. by NTA (2)

Sol. $D \ne 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} \ne 0 \Rightarrow \lambda \ne 5$

For no solution $D = 0 \Rightarrow \lambda = 5$

$D_1 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} \ne 0 \Rightarrow \mu \ne 3$

$p = \frac{5}{6}$

$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

Option (2)

13. Official Ans. by NTA (2)

Sol. Probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces

$= 2\left[\left(\frac{1}{6} - x\right)\left(\frac{1}{6} + x\right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}\right]$

$\Rightarrow 2\left[\left(\frac{1}{6} - x\right)\left(\frac{1}{6} + x\right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}\right] = \frac{13}{96}$

(given)

$x = \frac{1}{8}$

14. Official Ans. by NTA (3)

Sol. C – I '0' Head

T T T $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{1}{64}$

C – II '1' head

H T T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C – III '2' Head

H H T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C–IV '3' Heads

H H H $\left(\frac{1}{8}\right)\left(\frac{1}{8}\right) = \frac{1}{64}$

Total probability = $\frac{5}{16}$.

15. Official Ans. by NTA (28)

Sol. I_1 = first unit is functioning

I_2 = second unit is functioning

$P(I_1) = 0.9$, $P(I_2) = 0.8$

$P(\bar{I}_1) = 0.1$, $P(\bar{I}_2) = 0.2$

$$P = \frac{0.8 \times 0.1}{0.1 \times 0.2 + 0.9 \times 0.2 + 0.1 \times 0.8} = \frac{8}{28}$$

$$98P = \frac{8}{28} \times 98 = 28$$

16. Official Ans. by NTA (1)

Sol. g(3) = 2g(1) can be defined in 3 ways

number of onto functions in this condition = 3 × 4!

Total number of onto functions = 6!

Required probability $= \frac{3 \times 4!}{6!} = \frac{1}{10}$

17. Official Ans. by NTA (2)

Sol. Total ways of choosing square = $^{64}C_2$

$$= \frac{64 \times 63}{2 \times 1} = 32 \times 63$$

ways of choosing two squares having common side = 2 (7 × 8) = 112

Required probability $= \frac{112}{32 \times 63} = \frac{16}{32 \times 9} = \frac{1}{18}$

Ans. (2)

18. Official Ans. by NTA (781)

Sol.

x	–2	–1	3	4	6
P(X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

$\bar{X} = 2.3$

$-a + 6b = \frac{9}{10}$ .... (1)

$\sum P_i = \frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$

$a + b = \frac{4}{15}$ .... (2)

From equation (1) and (2)

$a = \frac{1}{10}$, $b = \frac{1}{6}$

$\sigma^2 = \Sigma p_i x_i^2 - (\bar{X})^2$

$\frac{1}{5}(4) + a(1) + \frac{1}{3}(9) + \frac{1}{5}(16) + b(36) - (2.3)^2$

$= \frac{4}{5} + a + 3 + \frac{16}{5} + 36b - (2.3)^2$

$= 4 + a + 3 + 36b - (2.3)^2$

$= 7 + a + 36b - (2.3)^2$

$= 7 + \frac{1}{10} + 6 - (2.3)^2$

$= 13 + \frac{1}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{131}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{1310 - (23)^2}{100}$

$= \frac{1310 - 529}{100}$

$\sigma^2 = \frac{781}{100}$

$100\sigma^2 = 781$

19. Official Ans. by NTA (3)

Sol. Total subsets = $2^5 = 32$

Probability $= \frac{^5C_2 \times 3^3}{32 \times 32} = \frac{10 \times 27}{12^{10}} = \frac{135}{2^9}$

20. Official Ans. by NTA (4)

Sol. $^nC_2\left(\frac{1}{2}\right)^n = {^nC_3}\left(\frac{1}{2}\right)^n \Rightarrow {^nC_2} = {^nC_3}$

$\Rightarrow n = 5$

Probability of getting an odd number for odd number of times is

$$^5C_1\left(\frac{1}{2}\right)^5 + {^5C_3}\left(\frac{1}{2}\right)^5 + {^5C_5}\left(\frac{1}{2}\right)^5 = \frac{1}{2^5}(5 + 10 + 1)$$

$$= \frac{1}{2}$$

21. Official Ans. by NTA (6)

Sol. Let $P(B_1) = p_1$, $P(B_2) = p_2$, $P(B_3) = p_3$

given that $p_1(1-p_2)(1-p_3) = \alpha$ .....(i)

$p_2(1-p_1)(1-p_3) = \beta$ .....(ii)

$p_3(1-p_1)(1-p_2) = \gamma$ .....(iii)

and $(1-p_1)(1-p_2)(1-p_3) = p$ .....(iv)

$$\Rightarrow \frac{p_1}{1-p_1} = \frac{\alpha}{p}, \frac{p_2}{1-p_2} = \frac{\beta}{p} \ \& \ \frac{p_3}{1-p_3} = \frac{\gamma}{p}$$

Also $\beta = \dfrac{\alpha p}{\alpha+2p} = \dfrac{3\gamma p}{p-2\gamma}$

$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma$

$\Rightarrow \alpha p - 6p\gamma = 5\alpha\gamma$

$$\Rightarrow \frac{p_1}{1-p_1} - \frac{6p_3}{1-p_3} = \frac{5p_1p_3}{(1-p_1)(1-p_3)}$$

$\Rightarrow p_1 - 6p_3 = 0$

$\Rightarrow \dfrac{p_1}{p_3} = 6$

22. Official Ans. by NTA (3)

Sol. Required probability $= \left(\dfrac{2}{3}\times\dfrac{3}{4}\right)^3 = \dfrac{1}{8}$

23. Official Ans. by NTA (2)

Sol. $ax^2 + bx + c = 0$

For equal roots D = 0

$\Rightarrow b^2 = 4ac$

Case I : ac = 1

(a, b, c) = (1, 2, 1)

Case II : ac = 4

(a, b, c) = (1, 4, 4)

or (4, 4, 1)

or (2, 4, 2)

Case III : ac = 9

(a, b, c) = (3, 6, 3)

Required probability $= \dfrac{5}{216}$

24. Official Ans. by NTA (3)

Sol. Consider following events

A : Person chosen is a smoker and non vegetarian.

B : Person chosen is a smoker and vegetarian.

C : Person chosen is a non-smoker and vegetarian.

E : Person chosen has a chest disorder

Given

$$P(A) = \frac{160}{400} \quad P(B) = \frac{100}{400} \quad P(C) = \frac{140}{400}$$

$$P\left(\frac{E}{A}\right) = \frac{35}{100} \quad P\left(\frac{E}{B}\right) = \frac{20}{100} \quad P\left(\frac{E}{C}\right) = \frac{10}{100}$$

To find

$$P\left(\frac{A}{E}\right) = \frac{P(A)P\left(\frac{E}{A}\right)}{P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right) + P(C).P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{160}{400}\times\frac{35}{100}}{\frac{160}{400}\times\frac{35}{100} + \frac{100}{400}\times\frac{20}{100} + \frac{140}{400}\times\frac{10}{100}}$$

$= \dfrac{28}{45}$ option (3)

25. Official Ans. by NTA (3)

Sol. n(s) = n(when 7 appears on thousands place)

+ n(7 does not appear on thousands place)

$= 9 \times 9 \times 9 + 8 \times 9 \times 9 \times 3$

$= 33 \times 9 \times 9$

n(E) = n(last digit 7 & 7 appears once)

+ n(last digit 2 when 7 appears once)

$= 8 \times 9 \times 9 + (9 \times 9 + 8 \times 9 \times 2)$

$\therefore \ P(E) = \dfrac{8\times9\times9+9\times25}{33\times9\times9} = \dfrac{97}{297}$

26. Official Ans. by NTA (3)

Sol. Digits = 3, 3, 4, 4, 4, 5, 5

Total 7 digit numbers $= \dfrac{7!}{2!\,2!\,3!}$

Number of 7 digit number divisible by 2

$\Rightarrow$ last digit = 4

						4

3 , 3 , 4 , 4 , 5 , 5

Now 7 digit numbers which are divisible by $2 = \dfrac{6!}{2!\,2!\,2!}$

Required probability $= \dfrac{\dfrac{6!}{2!\,2!\,2!}}{\dfrac{7!}{3!\,2!\,2!}} = \dfrac{3}{7}$

27. Official Ans. by NTA (1)

Sol. Let the coin be tossed n-times

$P(H) = P(T) = \dfrac{1}{2}$

$P(7 \text{ heads}) = {}^nC_7\left(\dfrac{1}{2}\right)^{n-7}\left(\dfrac{1}{2}\right)^7 = \dfrac{{}^nC_7}{2^n}$

$P(9 \text{ heads}) = {}^nC_9\left(\dfrac{1}{2}\right)^{n-9}\left(\dfrac{1}{2}\right)^9 = \dfrac{{}^nC_9}{2^n}$

P(7 heads) = P(9 heads)

${}^nC_7 = {}^nC_9 \Rightarrow n = 16$

$P(2 \text{ heads}) = {}^{16}C_2\left(\dfrac{1}{2}\right)^{14}\left(\dfrac{1}{2}\right)^2 = \dfrac{15\times 8}{2^{16}}$

$P(2 \text{ heads}) = \dfrac{15}{2^{13}}$

28. Official Ans by NTA (2)

Sol. Total cases :

$\underline{6}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}$

$n(s) = 6\cdot 6!$

Favourable cases :

Number divisible by 3 ≡

Sum of digits must be divisible by 3

Case-I

1, 2, 3, 4, 5, 6

Number of ways = 6!

Case-II

0, 1, 2, 4, 5, 6

Number of ways = 5·5!

Case-III

0, 1, 2, 3, 4, 5

Number of ways = 5·5!

n(favourable) = 6! + 2·5·5!

$P = \dfrac{6! + 2\cdot 5\cdot 5!}{6\cdot 6!} = \dfrac{4}{9}$

29. Official Ans. by NTA (3)

Sol. E_1 : Event denotes spade is missing

$$P(E_1) = \frac{1}{4};\ P(\bar{E}_1) = \frac{3}{4}$$

A : Event drawn two cards are spade

$$P(A) = \frac{\dfrac{1}{4}\times\left(\dfrac{{}^{12}C_2}{{}^{51}C_2}\right) + \dfrac{3}{4}\times\left(\dfrac{{}^{13}C_2}{{}^{51}C_2}\right) + \dfrac{3}{4}\times\left(\dfrac{{}^{13}C_2}{{}^{51}C_2}\right)}{\dfrac{1}{4}\times\left(\dfrac{{}^{12}C_2}{{}^{51}C_2}\right) + \dfrac{3}{4}\times\left(\dfrac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$$= \frac{39}{50}$$

30. Official Ans. by NTA (4)

Sol. $\underset{\text{odd place}}{1}\quad \underset{\text{even place}}{0}\quad \underset{\text{odd place}}{0}\quad \underset{\text{even place}}{1}$

or $\underset{\text{even place}}{1}\quad \underset{\text{odd place}}{0}\quad \underset{\text{even place}}{0}\quad \underset{\text{odd place}}{1}$

$\Rightarrow \left(\dfrac{1}{2}\cdot\dfrac{1}{3}\cdot\dfrac{1}{2}\cdot\dfrac{2}{3}\right) + \left(\dfrac{2}{2}\cdot\dfrac{1}{2}\cdot\dfrac{1}{3}\cdot\dfrac{1}{2}\right)$

$\Rightarrow \dfrac{1}{9}$

31. Official Ans. by NTA (6)

Sol. Let $P(E_1) = P_1$; $P(E_2) = P_2$; $P(E_3) = P_3$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = \alpha = P_1(1-P_2)(1-P_3)$......(1)

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = \beta = (1-P_1)P_2(1-P_3)$......(2)

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = \gamma = (1-P_1)(1-P_2)P_3$......(3)

$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P = (1-P_1)(1-P_2)(1-P_3)$......(4)

Given that, $(\alpha - 2\beta)P = \alpha\beta$

$\Rightarrow (P_1(1-P_2)(1-P_3) - 2(1-P_1)P_2(1-P_3))P$

$= P_1P_2(1-P_1)(1-P_2)(1-P_3)^2$

$\Rightarrow (P_1(1-P_2) - 2(1-P_1)P_2) = P_1P_2$

$\Rightarrow (P_1 - P_1P_2 - 2P_2 + 2P_1P_2) = P_1P_2$

$\Rightarrow P_1 = 2P_2$ ......(1)

and similarly, $(\beta - 3\gamma)P = 2B\gamma$

$P_2 = 3P_3$ ......(2)

So, $P_1 = 6P_3 \Rightarrow \boxed{\dfrac{P_1}{P_3} = 6}$

32. Official Ans. by NTA (1)

Sol. $P(X = 1) = {}^5C_1.p.q^4 = 0.4096$

$P(X = 2) = {}^5C_2.p^2.q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

Now

$P(X = 3) = {}^5C_3.\left(\frac{1}{5}\right)^3.\left(\frac{4}{5}\right)^2 = \frac{10\times 16}{125\times 25} = \frac{32}{625}$

STATISTICS

1. Official Ans. by NTA (1)

Sol. Let other two numbers be a, (21 – a)

Now,

$$10.25 = \frac{\left(4+16+25+49+a^2+(21-a)^2\right)}{6} - (6.5)^2$$

(Using formula for variance)

$\Rightarrow 6(10.25) + 6(6.5)^2 = 94 + a^2 + (21 - a)^2$

$\Rightarrow a^2 + (21 - a)^2 = 221$

$\therefore$ a = 10 and (21 – a) = 21 – 10 = 11

So, remaining two observations are 10, 11.

$\Rightarrow$ Option (1) is correct.

2. Official Ans. by NTA (4)

Sol. $10 = \frac{7+10+11+15+a+b}{6}$

$\Rightarrow a + b = 17$ .....(i)

$$\frac{20}{3} = \frac{7^2+10^2+11^2+15^2+a^2+b^2}{6} - 10^2$$

$a^2 + b^2 = 145$ ......(ii)

Solve (i) and (ii) a = 9, b = 8 or a = 8, b = 9

$|a - b| = 1$

3. Official Ans. by NTA (4)

Sol.

Class	**Frequency**	**x_i**	**$f_i\,x_i$**
0-6	a	3	3a
6-12	b	9	9b
12-18	12	15	180
18-24	9	21	189
24-30	5	27	135
	N=(26+a+b)		**(504+3a+9b)**

$$\text{Mean} = \frac{3a+9b+180+189+135}{a+b+26} = \frac{309}{22}$$

$\Rightarrow 66a + 198b + 11088 = 309a + 309b + 8034$

$\Rightarrow 243a + 111b = 3054$

$\Rightarrow \boxed{81a+37b=1018} \rightarrow (1)$

$$\text{Now, Median} = 12 + \frac{\frac{a+b+26}{2}-(a+b)}{12}\times 6 = 14$$

$\Rightarrow \frac{13}{2} - \left(\frac{a+b}{4}\right) = 2$

$\Rightarrow \frac{a+b}{4} = \frac{9}{2}$

$\Rightarrow \boxed{a+b=18} \rightarrow (2)$

From equation (1) & (2)

a = 8, b = 10

$\therefore (a - b)^2 = (8 - 10)^2$

4. Official Ans. by NTA (164)

Sol. $\because$ Sum of frequencies = 584

$\Rightarrow \alpha + \beta = 390$

Now, Median is at $\frac{584}{2} = 292^{th}$

$\because$ Median = 45 (lies in class 40 – 50)

$\Rightarrow \alpha + 110 + 54 + 15 = 292$

$\Rightarrow \alpha = 113, \beta = 277$

$\Rightarrow |\alpha - \beta| = 164$

5. Official Ans. by NTA (3)

Sol. $n_1 = 100$ $\quad m = 250$

$\bar{X}_1 = 15$ $\quad \bar{X} = 15.6$

$V_1(x) = 9$ $\quad Var(x) = 13.44$

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}\left(\bar{x}_1 - \bar{x}_2\right)^2$$

$n_2 = 150$, $\bar{x}_2 = 16$, $V_2(x) = \sigma_2$

$$13.44 = \frac{100\times 9 + 150\times\sigma_2^2}{250} + \frac{100\times 150}{(250)^2}\times 1$$

$\Rightarrow \sigma_2^2 = 16 \Rightarrow \sigma_2 = 4$

6. Official Ans. by NTA (4)

Sol. Mean $= \frac{6+10+7+13+a+12+b+12}{8} = 9$

$60 + a + b = 72$

$a + b = 12 \quad \ldots(1)$

variance $= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{37}{4}$

$\Sigma x_i^2 = 6^2 + 10^2 + 7^2 + 13^2 + a^2 + b^2 + 12^2 + 12^2$

$= a^2 + b^2 + 642$

$\frac{a^2+b^2+642}{8} - (9)^2 = \frac{37}{4}$

$\frac{a^2+b^2}{8} + \frac{321}{4} - 81 = \frac{37}{4}$

$\frac{a^2+b^2}{8} = 81 + \frac{37}{4} - \frac{321}{4}$

$\frac{a^2+b^2}{8} = 81 - 71$

$\therefore a^2 + b^2 = 80 \quad \ldots(2)$

From (1) $a^2 + b^2 + 2ab = 144$

$80 + 2ab = 144 \quad \therefore 2ab = 64$

$(a - b)^2 = a^2 + b^2 - 2ab = 80 - 64 = 16$

7. Official Ans. by NTA (3)

Sol. Given $32 + 8\alpha + 9\beta = (8 + \alpha + \beta) \times 6$

$\Rightarrow 2\alpha + 3\beta = 16 \quad \ldots(i)$

Also, $4 \times 16 + 4 \times \alpha + 9\beta = (8 + \alpha + \beta) \times 6.8$

$\Rightarrow 640 + 40\alpha + 90\beta = 544 + 68\alpha + 68\beta$

$\Rightarrow 28\alpha - 22\beta = 96$

$\Rightarrow 14\alpha - 11\beta = 48 \quad \ldots(ii)$

from (i) & (ii)

$\alpha = 5$ & $\beta = 2$

so, new mean $= \frac{32+35+18}{15} = \frac{85}{15} = \frac{17}{3}$

8. Official Ans. by NTA (4)

Sol. Given :

Mean $(\bar{x}) = \frac{\Sigma x_i}{20} = 10$

or $\Sigma x_i = 200$ (incorrect)

or $200 - 25 + 35 = 210 = \Sigma x_i$ (Correct)

Now correct $\bar{x} = \frac{210}{20} = 10.5$

again given S.D = 2.5 (σ)

$\sigma^2 = \frac{\Sigma x_i^2}{20} - (10)^2 = (2.5)^2$

or $\Sigma x_i^2 = 2125$ (incorrect)

or $\Sigma x_i^2 = 2125 - 25^2 + 35^2$

$= 2725$ (Correct)

$\therefore$ correct $\sigma^2 = \frac{2725}{20} - (10.5)^2$

$\underline{\sigma}^2 = 26$

or $\sigma = 26$

$\therefore \underline{\alpha} = 10.5, \beta = 26$

9. Official Ans. by NTA (12)

Sol. $5 = \frac{3+7+x+y}{4} \Rightarrow x+y=10$

$\text{Var}(x) = 10 = \frac{3^2+7^2+x^2+y^2}{4} - 25$

$140 = 49 + 9 + x^2 + y^2$

$x^2 + y^2 = 82$

$x + y = 10$

$\Rightarrow (x,y) = (9,1)$

Four numbers are 21,9,10,8

Mean $= \frac{48}{4} = 12$

10. Official Ans. by NTA (13)

Sol. $\frac{n^2-1}{12} = 14 \Rightarrow n = 13$

11. Official Ans. by NTA (30)

Sol. $\sum P(X) = 1 \Rightarrow k + 2k + 2k + 3k + k = 1$

$\Rightarrow k = \frac{1}{9}$

Now, $p = P\left(\frac{kX<4}{X<3}\right) = \frac{P(X=2)}{P(X<3)} = \frac{\frac{2k}{9k}}{\frac{k}{9k}+\frac{2k}{9k}} = \frac{2}{3}$

$\Rightarrow p = \frac{2}{3}$

Now, $5p = \lambda k$

$\Rightarrow (5)\left(\frac{2}{3}\right) = \lambda(1/9)$

$\Rightarrow \lambda = 30$

12. Official Ans. by NTA (25)

Sol. $\sigma_b^2 = 2$ (variance of boys) n_1 = no. of boys

$\bar{x}_b = 12$ n_2 = no. of girls

$\sigma_g^2 = 2$

$$\bar{x}_g = \frac{50\times15-12\times\sigma_b}{30} = \frac{750-12\times20}{30} = 17 = \mu$$

variance of combined series

$$\sigma^2 = \frac{n_1\sigma_b^2+n_2\sigma_g^2}{n_1+n_2} + \frac{n_1\cdot n_2}{(n_1+n_2)^2}(\bar{x}_b-\bar{x}_g)^2$$

$$\sigma^2 = \frac{20\times2+30\times2}{20+30} + \frac{20\times30}{(20+30)^2}(12-17)^2$$

$\sigma^2 = 8.$

$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$

13. Official Ans. by NTA (3)

Sol. Let 8, 16, x_1, x_2, x_3, x_4, x_5 be the observations.

Now $\dfrac{x_1+x_2+...+x_5+14}{7} = 8$

$\Rightarrow \sum_{i=1}^{5} x_i = 42$ ...(1)

Also $\dfrac{x_1^2+x_2^2+...x_5^2+8^2+6^2}{7} - 64 = 16$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 560 - 100 = 460$ ...(2)

So variance of $x_1, x_2, \ldots, x_5$

$$= \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{2300-1764}{25} = \frac{536}{25}$$

14. Official Ans. by NTA (11)

Sol. $\sigma^2 = \dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2$

$$= \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right)^2 < 10$$

$90 + 10k^2 - 81 - k^2 - 18k < 1000$

$9k^2 - 18k - 991 < 0$

$k^2 - 2k < \dfrac{991}{9}$

$(k-1)^2 < \dfrac{1000}{9}$

$\dfrac{-10\sqrt{10}}{3} < k-1 < \dfrac{10\sqrt{10}}{3}$

$k < \dfrac{10\sqrt{10}}{3} + 1$

$k \le 11$

Maximum value of k is 11.

15. Official Ans. by NTA (4)

Sol. $\sum_{i=1}^{18}(x_i-\alpha) = 36, \sum_{i=1}^{18}(x_i-\beta)^2 = 90$

$\Rightarrow \sum_{i=1}^{18} x_i = 18(\alpha+2), \sum_{i=1}^{18} x_i^2 - 2\beta\sum_{i=1}^{18} x_i + 18\beta^2 = 90$

Hence $\sum x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$

Given $\dfrac{\sum x_i^2}{18} - \left(\dfrac{\sum x_i}{18}\right)^2 = 1$

$\Rightarrow 90 - 18\beta^2 + 36\beta(\alpha+2) - 18(\alpha+2)^2 = 18$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4\alpha - 4 = 1$

$\Rightarrow (\alpha-\beta)^2 + 4(\alpha-\beta) = 0 \Rightarrow |\alpha-\beta| = 0$ or 4

As α and β are distinct $|\alpha-\beta| = 4$

16. Official Ans by NTA (5)

Sol. $\sigma^2 = \dfrac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2} + \dfrac{n_1n_2}{(n_1+n_2)}(\bar{x}_1-\bar{x}_2)^2$

$n_1 = 10$, $n_2 = n$, $\sigma_1^2 = 2$, $\sigma_2^2 = 1$

$\bar{x}_1 = 2$, $\bar{x}_2 = 3$, $\sigma^2 = \dfrac{17}{9}$

$\dfrac{17}{9} = \dfrac{10\times2+n}{n+10} + \dfrac{10n}{(n+10)^2}(3-2)^2$

$\Rightarrow \dfrac{17}{9} = \dfrac{(n+20)(n+10)+10n}{(n+10)^2}$

$\Rightarrow 17n^2 + 1700 + 340n = 90n + 9(n^2+30n+200)$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$2n^2 - 5n - 25 = 0$

$\Rightarrow (2n+5)(n-5) = 0 \Rightarrow n = \dfrac{-5}{2}, 5$

$\dfrac{-5}{2}$ ↓ (Rejected)

Hence n = 5

17. Official Ans. by NTA (4)

Sol. For a, b, c

mean $= \dfrac{a+b+c}{3} (= \bar{x})$

$b = a + c$

$\Rightarrow \bar{x} = \dfrac{2b}{3}$ ......(1)

S.D. $(a+2, b+2, c+2)$ = S.D. $(a, b, c) = d$

$\Rightarrow d^2 = \dfrac{a^2+b^2+c^2}{3} - (\bar{x})^2$

$\Rightarrow d^2 = \dfrac{a^2+b^2+c^2}{3} - \dfrac{4b^2}{9}$

$\Rightarrow 9d^2 = 3(a^2+b^2+c^2) - 4b^2$

$\Rightarrow b^2 = 3(a^2+c^2) - 9d^2$

18. Official Ans. by NTA (68)

Sol. Let number be $a_1, a_2, a_3, \ldots\ldots a_{2n}, b_1, b_2, b_3 \ldots. b_n$

$$\sigma^2 = \frac{\sum a^2 + \sum b^2}{3n} - (5)^2$$

$$\Rightarrow \sum a^2 + \sum b^2 = 87n$$

Now, distribution becomes

$a_1 + 1,\ a_2 + 1,\ a_3 + 1, \ldots\ldots. a_{2n} + 1,\ b_1 - 1,\ b_2 - 1 \ldots.. b_n - 1$

Variance

$$= \frac{\sum(a+1)^2 + \sum(b-1)^2}{3n} - \left(\frac{12n+2n+3n-n}{3n}\right)^2$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n}$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n} - \left(\frac{16}{3}\right)^2$$

$$= \frac{87n + 3n + 2(12n) - 2(3n)}{3n} - \left(\frac{16}{3}\right)^2$$

$$\Rightarrow k = \frac{108}{3} - \left(\frac{16}{5}\right)^2$$

$\Rightarrow 9k = 3(108) - (16)^2 = 324 - 256 = 68$

Ans. 68.00

19. Official Ans. by NTA (35)

Sol. $\frac{\sum x_i}{25} = 40\ \&\ \frac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow 1000 - 60 + N = 975$

$\Rightarrow N = 35$ years

20. Official Ans. by NTA (1)

Sol. Let observations are denoted by x_i for $1 \le i < 2n$

$$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a+a+\ldots+a)-(a+a+\ldots+a)}{2n}$$

$\Rightarrow \bar{x} = 0$

and $\sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2+a^2+\ldots+a^2}{2n} - 0 = a^2$

$\Rightarrow \sigma_x = a$

Now, adding a constant b then $\bar{y} = \bar{x} + b = 5$

$\Rightarrow b = 5$

and $\sigma_y = \sigma_x$ (No change in S.D.) $\Rightarrow a = 20$

$\Rightarrow a^2 + b^2 = 425$

MATHEMATICAL REASONING

1. Official Ans. by NTA (2)

Sol.

p	q	~p	~q	p∧~q	q∨~p	(p∧~q) ⇒(q∨~p)	p⇒q
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
T	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T

$\therefore\ (p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$\equiv p \Rightarrow q$

So, option (2) is correct.

2. Official Ans. by NTA (2)

Sol. Truth Table

P	q	P→q
T	T	T
T	F	F
F	T	T
F	F	T

3. Official Ans. by NTA (4)

Sol. (1) $(p \to q) \vee (\sim q \to p)$

$= (\sim p \vee q) \vee (q \vee p)$

$= (\sim p \vee p) \vee q$

$= t \vee q = t$

(2) $(q \to p) \vee (\sim q \to p)$

$= (\sim q \vee p) \vee (q \vee p)$

$= (\sim q \vee q) \vee p$

$= t \vee p = t$

(3) $(p \to \sim q) \vee (\sim q \to p)$

$= (\sim p \vee \sim q) \vee (q \vee p)$

$= (\sim p \vee p) \vee (\sim q \vee q)$

$= t \vee t = t$

(4) $(\sim q \to q) \vee (\sim q \to p)$

$= (p \vee q) \vee (q \vee p)$

$= (p \vee p) \vee (q \vee p)$

$= p \vee q$

Which is not a tautology.

4. Official Ans. by NTA (4)

Sol. $(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$ $\{p \to q \equiv \sim p \vee q\}$

$\equiv (\sim p \vee q) \wedge (\sim p \vee \sim q)$ {commutative property}

$\equiv \sim p \vee (q \wedge \sim q)$ {distributive property}

$\equiv \sim p$

5. Official Ans. by NTA (3)

Sol. p : weather is food

q : ground is not wet

$\sim (p \wedge q) \equiv \sim p \vee \sim q$

≡ weather is not good or ground is wet

6. Official Ans. by NTA (4)

Sol. Using Truth Table

P	Q	$P \vee Q$	$\sim P$	$(P \vee Q) \wedge P$	$(P \vee Q) \wedge \sim P \to Q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

P	Q	$\sim Q$	$P \wedge \sim Q$	$P \to Q$	$\sim (P \to Q)$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	F	F	T	F
F	F	T	F	T	F

$\sim (P \to Q)$	$P \wedge \sim Q$	$\sim (P \to Q) \Leftrightarrow P \wedge \sim Q$
F	F	T
T	T	T
F	F	T
F	F	T

7. Official Ans. by NTA (1)

Sol. P : for all $M > 0$, there exists $x \in S$ such that $x \geq M$.

$\sim P$: there exists $M > 0$, for all $x \in S$

Such that $x < m$

Negation of 'there exsits' is 'for all'.

8. Official Ans. by NTA (3)

Sol.

p	q	r	$\underbrace{p \vee q}_{a}$	$\underbrace{q \to r}_{b}$	$a \wedge b$	$\sim r$	$\underbrace{a \wedge b \wedge (\sim r)}_{c}$	$\underbrace{p \wedge q}_{d}$	$c \to d$
T	F	T	T	T	T	F	F	F	T
F	F	T	F	T	F	F	F	F	T
T	F	F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	F	F	T

9. Official Ans. by NTA (3)

Sol. $S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 = q \vee (\sim p \vee p) = qvt = t =$ tautology

$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$

= fallacy

10. Official Ans. by NTA (1)

Sol. $\big(p \wedge (p \to q) \wedge (q \to r)\big) \to r$

$\equiv \big(p \wedge (\sim p \vee q) \vee (\sim q \vee r)\big) \to r$

$\equiv \big((p \wedge q) \wedge (\sim p \vee r)\big) \to r$

$\equiv (p \wedge q \wedge r) \to r$

$\equiv \sim (p \wedge q \wedge r) \vee r$

$\equiv (\sim p) \vee (\sim q) \vee (\sim r) \vee r$

$\Rightarrow$ tautology

11. Official Ans. by NTA (1)

Sol. $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge p) \wedge (p \wedge q)$

$\Rightarrow [\sim (p \wedge q) \vee (p \wedge q)] \wedge (\sim (p \wedge q) \vee$

$(r \wedge p)) \Rightarrow t \wedge [\sim (p \wedge q) \vee (r \wedge p)]$

$\Rightarrow \sim (p \wedge q) \vee (r \wedge p)$

$\Rightarrow (p \wedge q) \Rightarrow (r \wedge p)$

Aliter :

given statement says

" if p and q both happen then

p and q and r will happen"

it Simply implies

" If p and q both happen then

'r' too will happen "

i.e.

" if p and q both happen then r and p too will happen

i.e.

$(p \wedge q) \Rightarrow (r \wedge p)$

12. Official Ans. by NTA (3)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$ is tautology

p	q	$\sim q$	$p \wedge \sim q$	$p \vee q$	$(p \wedge \sim q) \rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

13. Official Ans. by NTA (1)

Sol. $\because \sim(A \Rightarrow B) = A \wedge \sim B$

$\therefore \sim((p \vee r) \Rightarrow (q \vee r))$

$= (p \vee r) \wedge (\sim q \wedge \sim r)$

$= ((p \vee r) \wedge (\sim r)) \wedge (\sim q)$

$= p \wedge (\sim r) \wedge (\sim q)$

14. Official Ans. by NTA (4)

Sol.

p	q	$\sim p$	$\sim q$	$p - q$	$\sim(p \rightarrow q)$	$q \rightarrow p$	$\sim(q \rightarrow p)$
T	T	F	F	T	F	T	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	F	T	F

$p \wedge \sim q$	$\sim p \rightarrow \sim q$	$p \rightarrow \sim q$	$\sim(p \rightarrow \sim q)$
F	T	F	T
T	T	T	F
F	F	T	F
F	T	T	F

$p \wedge \sim q \equiv \sim (p \rightarrow q)$

Option (4)

15. Official Ans. by NTA (2)

Sol. (A)

p	q	$\sim q$	$p \rightarrow q$	$\sim p$	$(\sim q \wedge (p \rightarrow q))$	
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

(B)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Both are tautologies

16. Official Ans. by NTA (4)

Sol. $\left(A \wedge (A \rightarrow B)\right) \rightarrow B$

$= \left(A \wedge (\sim A \vee B)\right) \rightarrow B$

$= \left((A \wedge \sim A) \vee (A \wedge B)\right) \rightarrow B$

$= (A \wedge B) \rightarrow B$

$= \sim (A \wedge B) \vee B$

$= (\sim A \vee \sim B) \vee B$

$= T$

17. Official Ans. by NTA (3)

Sol. Constrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

$\Rightarrow$ If you will not earn money, you will not work. option (3)

18. Official Ans. by NTA (3)

Sol. $F_1 : (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$

$F_2 : (A \vee B) \vee (B \rightarrow \sim A)$

$F_1 : \{(A \wedge \sim B) \vee \sim A\} \vee [(A \vee B) \wedge \sim C]$

$: \{(A \vee \sim A) \wedge (\sim A \vee \sim B)\} \vee [(A \vee B) \wedge \sim C]$

$: \{t \wedge (\sim A \vee \sim B)\} \vee [(A \vee B) \wedge \sim C]$

$: (\sim A \vee \sim B) \vee [(A \vee B) \wedge \sim C]$

$: \underbrace{[(\sim A \vee \sim B) \vee (A \vee B)]}_{t} \wedge [(\sim A \vee \sim B) \wedge \sim C]$

$F_1 : (\sim A \vee \sim B) \wedge \sim C \neq t$ (tautology)

$F_2 : (A \vee B) \vee (\sim B \vee \sim A) = t$ (tautology)

19. Official Ans. by NTA (2)

Sol. $\sim(\sim p \wedge (p \vee q))$

$p \vee (\sim p \wedge \sim q)$

$\underbrace{(p \vee \sim p)}_{t} \wedge (p \vee \sim q)$

$p \vee \sim q$

20. Official Ans. by NTA (4)

Sol. $A \rightarrow (B \rightarrow A)$

$\equiv A \rightarrow (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A)$

$\equiv (\sim A \vee A) \vee \sim B$

$\equiv T \vee \sim B \equiv T$

$\therefore T \vee B = T$

$\equiv (\sim A \vee A) \vee B$

$\equiv \sim A \vee (A \vee B)$

$\equiv A \rightarrow (A \vee B)$

21. Official Ans. by NTA (4)

Sol.

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \to (p \to q)$ is tautology

22. Official Ans. by NTA (1)

Sol. Option (1)

$(p \wedge q) \longrightarrow (p \to q)$

$= \sim(p \wedge q) \vee (\sim p \vee q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee (\sim q \vee q)$

$= \sim p \vee t$

$= t$

Option (2)

$(p \wedge q) \wedge (p \vee q) = (p \wedge q)$ (Not a tautology)

Option (3)

$(p \wedge q) \vee (p \to q)$

$= (p \wedge q) \vee (\sim p \vee q)$

$= \sim p \vee q$ (Not a tautology)

Option (4)

$= (p \wedge q) \wedge (\sim p \vee q)$

$= p \wedge q$ (Not a tautology)

Option (1)

23. Official Ans. by NTA (1)

Sol. $\because p \to q \equiv \sim p \vee q$

So, $* \equiv \vee$

Thus, $p * (\sim q) \equiv p \vee (\sim q)$

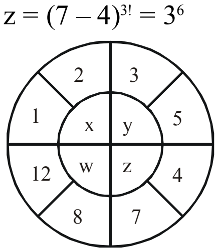
$\equiv q \to p$

24. Official Ans. by NTA (4 or 16 or 64)

Sol. $x = (2-1)^{1!} = 1$

$w = (12-8)^{4!} = 4^{24}$

$z = (7-4)^{3!} = 3^6$



hence $y = (5-3)^{2!} = 2^2$

25. Official Ans. by NTA (2)

Sol. LHS of all the options are some i.e.

$((P \to Q) \wedge \sim Q)$

$\equiv (\sim P \vee Q) \wedge \sim Q$

$\equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)$

$\equiv \sim P \wedge \sim Q$

(A) $(\sim P \wedge \sim Q) \to Q$

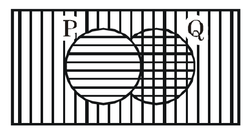
$\equiv \sim(\sim P \wedge \sim Q) \vee Q$

$\equiv (P \vee Q) \vee Q \neq$ tautology

(B) $(\sim P \wedge \sim Q) \to \sim P$

$\equiv \sim(\sim P \wedge \sim Q) \vee \sim P$

$\equiv (P \vee Q) \vee \sim P$



$\Rightarrow$ Tautology

(C) $(\sim P \wedge \sim Q) \to P$

$\equiv (P \vee Q) \vee P \neq$ Tautology

(D) $(\sim P \wedge \sim Q) \to (P \wedge Q)$

$\equiv (P \vee Q) \vee (P \wedge Q) \neq$ Tautology

Aliter :

P	Q	$P \vee Q$	$P \vee Q$	$\sim P$	$(P \vee Q) \vee \sim P$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T