MATHEMATICS

LOGARITHM

1. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$, $x > 0$, is

2. The sum of the roots of the equation $x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$, is :

(1) $\log_2 14$ (2) $\log_2 11$

(3) $\log_2 12$ (4) $\log_2 13$

COMPOUND ANGLE

1. The value of $\cot\frac{\pi}{24}$ is :

(1) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$ (2) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

(3) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$ (4) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

2. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x-2y|$ is equal to :

(1) 4 (2) 3 (3) 0 (4) 1

3. Let $\alpha = \max_{x\in\mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$ and $\beta = \min_{x\in\mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c - b$ is equal to :

(1) 42 (2) 47 (3) 43 (4) 50

4. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

is :

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{1}{8\sqrt{2}}$

5. Let $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

(1) $b^2 - a^2 = a^2 + c^2$

(2) b^2, c^2, a^2 are in A.P.

(3) c^2, a^2, b^2 are in A.P.

(4) a^2, b^2, c^2 are in A.P.

6. If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1,$$

$x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is :

(1) $(3, 13\pi/9)$ (2) $(2, 2\pi/3)$

(3) $(2, 8\pi/9)$ (4) $(3, 5\pi/3)$

7. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:

(1) $\frac{1}{\sqrt{7}}$ (2) $2\sqrt{2}-1$

(3) $\sqrt{7}-1$ (4) $\frac{1}{2\sqrt{2}}$

8. If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+.....\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is

(1) $2\sqrt{3}$ (2) $\frac{3}{2}$

(3) $\sqrt{3}$ (4) $\frac{1}{2}$

9. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\text{cosec}^6\alpha$ is equal to :

(1) 350 (2) 500 (3) 400 (4) 250

QUADRATIC EQUATION

1. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to :

(1) 56×3^{25} (2) 56×3^{24}

(3) 52×3^{24} (4) 28×3^{25}

2. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is :

(1) 2 (2) 4 (3) 6 (4) 1

3. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\dfrac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to ________.

4. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is :

(1) 2 (2) 3 (3) 1 (4) 4

5. If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$, then the value of $a^4 + b^4 + c^4$ is equal to _________.

6. Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to

(1) 10 (2) 100 (3) 50 (4) 160

7. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to____ .

8. The sum of all integral values of k ($k \neq 0$) for which the equation $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k}$ in x has no real roots, is ________.

9. Let $\lambda \neq 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\dfrac{\beta\gamma}{\lambda}$ is equal to ________.

10. If $x^2 + 9y^2 - 4x + 3 = 0$, $x, y \in \mathbb{R}$, then x and y respectively lie in the intervals:

(1) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(2) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and [1, 3]

(3) [1, 3] and [1, 3]

(4) [1, 3] and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

11. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _________.

12. The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k+1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is :

(1) $\left(1, \frac{5}{2}\right]$ (2) [2, 3)

(3) $\left[-\frac{1}{2}, 1\right)$ (4) $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

13. cosec18° is a root of the equation :

(1) $x^2 + 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$

(3) $x^2 - 2x + 4 = 0$ (4) $x^2 - 2x - 4 = 0$

14. The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is :

(1) 6 (2) 2 (3) 4 (4) 8

15. The number of the real roots of the equation $(x + 1)^2 + |x - 5| = \dfrac{27}{4}$ is ________.

16. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

17. The integer 'k', for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R, is :

(1) 3 (2) 2 (3) 0 (4) 4

18. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to :

(1) –3 (2) –7 (3) 7 (4) 3

19. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is:

(1) 2 (2) 1 (3) 4 (4) 3

20. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of p_n^2 is ______.

21. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is

22. Let $f : [-1, 1] \to R$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where a, b, c $\in$ R such that $f(-1) = 2$, $f'(-1) = 1$ and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \leq \alpha$, $x \in [-1, 1]$, then the least value of α is equal to ______.

23. The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + ...\infty}}}}$ is equal to

(1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

SEQUENCE & PROGRESSION

1. If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + ...$, where $x > 0$ is 504, then x is equal to

(1) 243 (2) 9 (3) 7 (4) 81

2. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to ________.

3. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to :

(1) 1862 (2) 1842 (3) 1852 (4) 1872

4. The sum of all the elements in the set $\{n \in \{1, 2, ..., 100\}$ | H.C.F. of n and 2040 is 1$\}$ is equal to _______.

5. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is :

(1) 6 (2) 4 (3) 2 (4) 8

6. If the value of $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + ...\text{upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...\text{upto } \infty\right)}$ is l, then l^2 is equal to ________.

7. If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to ________.

8. Let $A = \{n \in \mathbf{N} \mid n^2 \leq n + 10{,}000\}$, $B = \{3k + 1 \mid k \in \mathbf{N}\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to_______ .

9. The sum of the series $\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + ... + \frac{2^{100}}{x^{2^{100}}+1}$ when x = 2 is:

(1) $1 + \frac{2^{101}}{4^{101} - 1}$ (2) $1 + \frac{2^{100}}{4^{101} - 1}$

(3) $1 - \frac{2^{100}}{4^{100} - 1}$ (4) $1 - \frac{2^{101}}{4^{101} - 1}$

10. If the sum of an infinite GP a, ar, ar^2, ar^3,... is 15 and the sum of the squares of its each term is 150, then the sum of $ar^2, ar^4, ar^6, ...$ is :

(1) $\frac{5}{2}$ (2) $\frac{1}{2}$ (3) $\frac{25}{2}$ (4) $\frac{9}{2}$

11. Let $a_1, a_2, \ldots, a_{10}$ be an AP with common difference –3 and $b_1, b_2, \ldots, b_{10}$ be a GP with common ratio 2. Let $c_k = a_k + b_k$, $k = 1,2,\ldots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to _____.

12. If for $x, y \in \mathbf{R}$, $x > 0$,

$y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + \ldots$ upto ∞ terms and $\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y} = \frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to :

(1) $(10^6, 6)$ (2) $(10^4, 6)$

(3) $(10^2, 3)$ (4) $(10^6, 9)$

13. The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$ is :

(1) 1 (2) $\frac{120}{121}$ (3) $\frac{99}{100}$ (4) $\frac{143}{144}$

14. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to :

(1) $7 - 7\sqrt{3}$ (2) $7 + \sqrt{3}$

(3) $7 - \sqrt{3}$ (4) $7 + 3\sqrt{3}$

15. The mean of 10 numbers

$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \ldots$ is _______.

16. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

(1) $\frac{19}{21}$ (2) $\frac{100}{121}$ (3) $\frac{21}{19}$ (4) $\frac{121}{100}$

17. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is ____.

18. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$, then 160 S is equal to ______.

19. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \ldots + (n-1) \cdot 1$, $n \geq 4$. The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to :

(1) $\frac{e-1}{3}$ (2) $\frac{e-2}{6}$ (3) $\frac{e}{3}$ (4) $\frac{e}{6}$

20. Let $a_1, a_2, \ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to :

(1) 57 (2) 72 (3) 48 (4) 36

21. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is :

(1) $\frac{71}{256}$ (2) $\frac{69}{256}$

(3) $-\frac{69}{256}$ (4) $-\frac{71}{256}$

22. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _______.

23. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$ then :

(1) $xy - z = (x + y)z$ (2) $xy + yz + zx = z$

(3) $xyz = 4$ (4) $xy + z = (x + y)z$

24. Let $A_1, A_2, A_3, \ldots\ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ______.

25. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(2) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$

(3) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$

(4) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

26. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to ______.

27. In a increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to

(1) 30 (2) 26 (3) 35 (4) 32

28. The sum of the infinite series $1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots$ is equal to

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

29. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a},\frac{1}{b},6$ be in A.P., where a, b > 0. Then 72(a + b) is equal to ________.

30. Let

$$S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \ldots..$$

up to n-terms, where a > 1. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _______.

31. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ______ .

32. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x+\frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant

$$\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$$ is equal to :

33. If α, β are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots..+ (1)(199)$, then the slope of the line passing through (α,β) and origin is :

(1) 540 (2) 550 (3) 530 (4) 510

34. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\ldots+\frac{1}{(201)^2-1}$ is equal to

(1) $\frac{101}{404}$ (2) $\frac{25}{101}$ (3) $\frac{101}{408}$ (4) $\frac{99}{400}$

35. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:

(1) 1000 (2) 7000 (3) 5000 (4) 3000

36. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to _______.

TRIGONOMETRIC EQUATION

1. The number of solutions of $\sin^7x + \cos^7x = 1$, $x \in [0, 4\pi]$ is equal to

(1) 11 (2) 7 (3) 5 (4) 9

2. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to :

(1) 8π (2) 11π (3) 12π (4) 9π

3. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to:

(1) 23 (2) –27 (3) –23 (4) 27

4. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :

(1) $-\frac{11\pi}{30}$ (2) $\frac{\pi}{10}$ (3) $-\frac{7\pi}{30}$ (4) $-\frac{\pi}{15}$

5. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to ________.

6. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \le x \le \frac{\pi}{4}$ is :

(1) 3 (2) 1 (3) 0 (4) 2

7. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

(1) $\left(0,\frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0,\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) $\left(0,\frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4) $\left(0,\frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

8. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1+\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

9. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is

10. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

11. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10}n - 1)$, $n > 0$, then the value of n is equal to :

(1) 20 (2) 12 (3) 9 (4) 16

12. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

(1) 3 (2) 4 (3) 8 (4) 2

13. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

SOLUTION OF TRIANGLE

1. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

(1) $10+6\sqrt{2}$ (2) $8+2\sqrt{2}$

(3) $6+8\sqrt{3}$ (4) $4+2\sqrt{3}$

2. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

(1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{5}-1}{2}$

(3) $\frac{\sqrt{2}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

3. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _______.

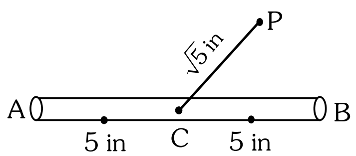
HEIGHT & DISTANCE

1. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

(1) $8(2+2\sqrt{3}+\sqrt{2})$ (2) $8(\sqrt{6}+\sqrt{2}+2)$

(3) $8(\sqrt{2}+2+\sqrt{3})$ (4) $8(\sqrt{6}-\sqrt{2}+2)$

2. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC=\sqrt{5}$ inches and $\angle PCB=\tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :



(1) $\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}(1)$

(3) $\tan^{-1}\left(\frac{4}{3}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

3. Two poles, AB of length a metres and CD of length a + b (b ≠ a) metres are erected at the same horizontal level with bases at B and D. If BD = x and $\tan\angle ACB = \frac{1}{2}$, then:

(1) $x^2 + 2(a + 2b)x - b(a + b) = 0$

(2) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(3) $x^2 - 2ax + b(a + b) = 0$

(4) $x^2 - 2ax + a(a + b) = 0$

4. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

(1) $12\sqrt{15}$ (2) $12\sqrt{10}$

(3) $8\sqrt{10}$ (4) $6\sqrt{10}$

5. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

(1) 10 (2) $10\sqrt{3}$

(3) $10(\sqrt{3}+1)$ (4) $10(\sqrt{3}-1)$

6. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

(1) $1800\sqrt{3}$ m (2) $3600\sqrt{3}$ m

(3) $2400\sqrt{3}$ m (4) $1200\sqrt{3}$ m

7. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

(1) $20\sqrt{3}$ (2) $25\sqrt{3}$

(3) 30 (4) 25

8. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle to ΔABC is 2, then the height of the pole is equal to :

(1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

DETERMINANT

1. Let a, b, c, d be in arithmetic progression with common difference λ. If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$,

then value of λ^2 is equal to ___________.

2. The value of $k \in \mathbf{R}$, for which the following system of linear equations

$3x - y + 4z = 3,$

$x + 2y - 3z = -2,$

$6x + 5y + kz = -3,$

has infinitely many solutions, is :

(1) 3 (2) – 5 (3) 5 (4) –3

3. The values of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solution, are :

(1) $\lambda = 3, \mu = 5$ (2) $\lambda = 3, \mu \neq 10$

(3) $\lambda \neq 2, \mu = 10$ (4) $\lambda = 2, \mu \neq 10$

4. The values of a and b, for which the system of equations

$2x + 3y + 6z = 8$

$x + 2y + az = 5$

$3x + 5y + 9z = b$

has no solution, are :

(1) $a = 3, b \neq 13$ (2) $a \neq 3, b \neq 13$

(3) $a \neq 3, b = 3$ (4) $a = 3, b = 13$

5. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is:

(1) 4 (2) 1 (3) 2 (4) 3

6. Let

$$f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi]$$

Then the maximum value of f(x) is equal to _________.

7. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$(1 + \cos^2\theta)x + \sin^2\theta\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4 \sin 3\theta)\, z = 0$

has a non–trivial solution, then the value of θ is :

(1) $\frac{4\pi}{9}$ (2) $\frac{7\pi}{18}$ (3) $\frac{\pi}{18}$ (4) $\frac{5\pi}{18}$

8. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to _________.

9. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval:

(1) [68, 69) (2) [62, 63)

(3) [65, 66) (4) [60, 61)

10. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is:

(1) **R** (2) $(-\infty, -9) \cup (-9, \infty)$

(3) $[-9, -8)$ (4) $(-\infty, -9) \cup [-8, \infty)$

11. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a = -\frac{1}{3}, b \neq \frac{7}{3}$ (2) $a \neq \frac{1}{3}, b = \frac{7}{3}$

(3) $a \neq -\frac{1}{3}, b = \frac{7}{3}$ (4) $a = \frac{1}{3}, b \neq \frac{7}{3}$

12. If $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9}$, r = 1, 2, 3,…, $i = \sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

(1) $a_2a_6 - a_4a_8$ (2) a_9

(3) $a_1a_9 - a_3a_7$ (4) a_5

13. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos \gamma)y + (\cos \beta)z = 0$

$(\cos \gamma)x + y + (\cos \alpha)z = 0$

$(\cos \beta)x + (\cos \alpha)y + z = 0$

has :

(1) no solution

(2) infinitely many solution

(3) exactly two solutions

(4) a unique solution

14. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in \mathbf{R}$ for which the system is inconsistent and S_2 be the set of all $a \in \mathbf{R}$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

(1) $n(S_1) = 2, n(S_2) = 2$

(2) $n(S_1) = 1, n(S_2) = 0$

(3) $n(S_1) = 2, n(S_2) = 0$

(4) $n(S_1) = 0, n(S_2) = 2$

15. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3, m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3, m \neq \frac{4}{5}$ (4) $k = 3, m \neq \frac{4}{5}$

16. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to ________.

17. The following system of linear equations

$2x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(2) has infinitely many solutions

(3) does not have any solution

(4) has a unique solution

18. Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c,$

where a, b and c are real constants. Then the system of equations :

(1) has a unique solution when $5a = 2b + c$

(2) has infinite number of solutions when $5a = 2b + c$

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

19. For the system of linear equations :

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R,$

consider the following statements :

(A) The system has unique solution if $k \neq 2$, $k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct ?

(1) (C) and (D) only (2) (B) and (E) only

(3) (A) and (E) only (4) (A) and (D) only

20. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) (a + 2) (a + 3) (a + 4)

(2) –2

(3) (a + 1) (a + 2) (a + 3)

(4) 0

21. The maximum value of

$$f(x)=\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R \text{ is:}$$

(1) $\sqrt{7}$ (2) $\frac{3}{4}$ (3) $\sqrt{5}$ (4) 5

22. The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution if k is equal to :

(1) 0 (2) 1

(3) –1 (4) –2

23. The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi),$$

are

(1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$

(3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{7\pi}{12}, \frac{11\pi}{12}$

24. Let α,β,γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, (a,b,c ∈ R and a,b ≠ 0). If the system of equations (in, u,v,w) given by αu + βv + γw = 0, βu + γv + αw = 0; γu + αv + βw = 0 has non-trivial solution, then the value of $\frac{a^2}{b}$ is

(1) 5 (2) 3

(3) 1 (4) 0

25. Let the system of linear equations

4x + λy + 2z = 0

2x – y + z = 0

μx + 2y + 3z = 0, λ, μ ∈ R.

has a non-trivial solution. Then which of the following is true ?

(1) μ = 6, λ ∈ R (2) λ = 2, μ ∈ R

(3) λ = 3, μ ∈ R (4) μ = –6, λ ∈ R

STRAIGHT LINE

1. Consider a triangle having vertices A(–2, 3), B(1, 9) and C(3, 8). If a line L passing through the circum-centre of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is ________.

2. Let the equation of the pair of lines, y = px and y = qx, can be written as (y – px) (y – qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is :

(1) $x^2 - 3xy + y^2 = 0$ (2) $x^2 + 4xy - y^2 = 0$

(3) $x^2 + 3xy - y^2 = 0$ (4) $x^2 - 3xy - y^2 = 0$

3. A ray of light through (2,1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be :

(1) 11x + 7y + 8 = 0 or 11x + 7y – 15 = 0

(2) 11x – 7y – 8 = 0 or 11x + 7y + 15 = 0

(3) 2x – 7y + 29 = 0 or 2x – 7y – 7 = 0

(4) 2x – 7y – 39 = 0 or 2x – 7y – 7 = 0

4. The point P (a,b) undergoes the following three transformations successively :

(a) reflection about the line $y = x$.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a + b$ is equal to :

(1) 13 (2) 9 (3) 5 (4) 7

5. Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point :

(1) (1,2) (2) (2,2) (3) (2,1) (4) (1,3)

6. Let ABC be a triangle with A(–3, 1) and $\angle ACB = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan\theta$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $\frac{4}{3}$ (4) 2

7. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is :

(1) $3x^2 - 2y - 6 = 0$ (2) $3x^2 + 2y - 6 = 0$

(3) $2x^2 + 3y - 9 = 0$ (4) $2x^2 - 3y + 9 = 0$

8. Let A(a, 0), B(b, 2b +1) and C(0, b), $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

(1) $\frac{-2b}{b+1}$ (2) $\frac{2b}{b+1}$ (3) $\frac{2b^2}{b+1}$ (4) $\frac{-2b^2}{b+1}$

9. If p and q are the lengths of the perpendiculars from the origin on the lines,

$x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ and

$x \sin \alpha + y \cos\alpha = k \sin 2\alpha$

respectively, then k^2 is equal to :

(1) $4p^2 + q^2$ (2) $2p^2 + q^2$

(3) $p^2 + 2q^2$ (4) $p^2 + 4q^2$

10. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

(1) $\frac{4}{\sqrt{5}}$ (2) $\frac{16}{\sqrt{5}}$ (3) $\frac{8}{\sqrt{5}}$ (4) $\frac{12}{\sqrt{5}}$

11. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is _______.

12. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + y^2 = 7$ (2) $y^2 = \frac{1}{6\sqrt{3}}x$

(3) $2x^2 - 18y^2 = 9$ (4) $x^2 + 9y^2 = 9$

13. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (–5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ________.

14. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1,1), (2, 2) and (4, 4) respectively. Then which of these stones is / are on the path of the man ?

(1) A only (2) C only

(3) All the three (4) B only

15. The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on :

(1) $(x-2)^2 + (y-2)^2 = 12$

(2) $(x-4)^2 + (y+2)^2 = 16$

(3) $(x-4)^2 + (y-4)^2 = 8$

(4) $(x-2)^2 + (y-4)^2 = 4$

16. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

(1) Right angled triangle

(2) Equilateral triangle

(3) Isosceles triangle

(4) None of the above

17. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

(1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

18. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin.If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to :

19. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is :

(1) (–1, 0) (2) (–2, –2)

(3) (0, 2) (4) (1, 4)

20. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \leq 100$ and $4x + 3y \leq 75$ for $x \geq 0$ and $y \geq 0$ is _______ .

21. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

(1) 1 (2) 2 (3) 3 (4) 0

22. The equation of one of the straight lines which passes through the point (1,3) and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line $y + 1 = 3\sqrt{2}$, x is

(1) $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

(2) $5\sqrt{2}x + 4y - \left(15 + 4\sqrt{2}\right) = 0$

(3) $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

(4) $4\sqrt{2}x - 5y - \left(5 + 4\sqrt{2}\right) = 0$

23. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$ is equal to :

(1) $\frac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$ (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

CIRCLE

1. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (– 4,1) and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then a + b is equal to :

(1) 3 (2) 11 (3) 5 (4) 7

2. Let the circle $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

(1) $\frac{25}{9} < C < \frac{13}{3}$ (2) $100 < C < 165$

(3) $81 < C < 156$ (4) $100 < C < 156$

3. Two tangents are drawn from the point P(–1, 1) to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to:

(1) 2 (2) $(3\sqrt{2}+2)$

(3) 4 (4) $3(\sqrt{2}-1)$

4. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P, Q} is equal to

(1) $\{(4,0),(0,6)\}$

(2) $\{(2+2\sqrt{2}, 3-\sqrt{5}), (2-2\sqrt{2}, 3+\sqrt{5})\}$

(3) $\{(2+2\sqrt{2}, 3+\sqrt{5}), (2-2\sqrt{2}, 3-\sqrt{5})\}$

(4) $\{(-1,5),(5,1)\}$

5. Let

$A = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid 2x^2 + 2y^2 - 2x - 2y = 1\}$,

$B = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid 4x^2 + 4y^2 - 16y + 7 = 0\}$ and

$C = \{(x,y) \in \mathbf{R}\times\mathbf{R} \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\}$.

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to

(1) $\frac{3+\sqrt{10}}{2}$ (2) $\frac{2+\sqrt{10}}{2}$

(3) $\frac{3+2\sqrt{5}}{2}$ (4) $1+\sqrt{5}$

6. Consider a circle C which touches the y-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to :

(1) $\sqrt{53}$ (2) 9 (3) 8 (4) $\sqrt{82}$

7. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1) (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to ________.

8. A circle C touches the line x = 2y at the point (2,1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

(1) $7\sqrt{5}$ (2) 15

(3) $\sqrt{285}$ (4) $4\sqrt{15}$

9. Let the equation $x^2 + y^2 + px + (1 - p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer} is ________.

10. Let $\mathbb{Z}$ be the set of all integers,

$A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \le 4\}$,

$B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \le 4\}$ and

$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y-2)^2 \le 4\}$

If the total number of relation from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is :

(1) 16 (2) 25 (3) 49 (4) 9

11. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is 4x + 3y = 10, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \ne C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to ________.

12. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is ________.

13. Let B be the centre of the circle $x^2+y^2-2x+4y+1=0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8.\left(\frac{\text{area }\Delta APQ}{\text{area }\Delta BPQ}\right)$ is equal to _____.

14. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5, 7) is A, then 24A is equal to_____.

15. If one of the diameters of the circle $x^2+y^2-2x-6y+6=0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is ______.

16. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2+y^2=1$ is a circle of radius r, then r is equal to :

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

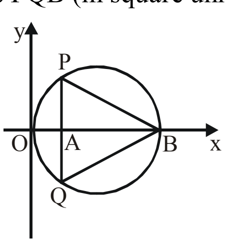
17. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points P, A and B lie on :

(1) a straight line (2) a hyperbola

(3) an ellipse (4) a parabola

18. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $(4,-2\sqrt{2})$, and given that $a-2\sqrt{2}\,b=3$, then (a^2+b^2+ab) is equal to ______.

19. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ ⊥ OB. Then, the area of the triangle PQB (in square units) is



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

20. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2+y^2+ax+2ay+c=0$, $(a<0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x+2y=0$, is euqal to :

(1) $\sqrt{11}$ (2) $\sqrt{7}$ (3) $\sqrt{6}$ (4) $\sqrt{10}$

21. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha+\sqrt{3}\beta$, where α, β are integers, then α + β is equal to_______.

22. Two tangents are drawn from a point P to the circle $x^2+y^2-2x-4y+4=0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right)\in(0,\pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

(1) 11 : 4 (2) 9 : 4 (3) 3 :1 (4) 2 : 1

23. Let the tangent to the circle $x^2+y^2=25$ at the point R(3, 4) meet x-axis and y-axis at point P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to

(1) $\frac{529}{64}$ (2) $\frac{125}{72}$ (3) $\frac{625}{72}$ (4) $\frac{585}{66}$

24. The line $2x-y+1=0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x-2y=4$. Then, the radius of the circle is:

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$ (3) $5\sqrt{4}$ (4) $4\sqrt{5}$

25. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles.

(2) Both circles' centres lie inside region of one another.

(3) Both circles pass through the centre of each other.

(4) Circles have two intersection points.

26. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 24x - 10y + 160 = 0$ is ______ .

27. Choose the correct statement about two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) circles have same centre

(2) circles have no meeting point

(3) circles have only one meeting point

(4) circles have two meeting points

28. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

(1) Rhombus (2) Square

(3) Rectangle (4) Parallelogram

29. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(1) $\left(0, \pm\sqrt{3}\right)$ (2) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

(3) $\left(2, \pm\frac{3}{2}\right)$ (4) $\left(1, \pm 2\right)$

PARABOLA

1. Let the tangent to the parabola $S : y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to:

(1) $\frac{25}{2}$ (2) $\frac{35}{2}$ (3) $\frac{15}{2}$ (4) 25

2. Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}\,(m + c)$ is equal to_______.

3. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line $y = x$ is :

(1) $(3x - y)^2 + (x - 3y) + 2 = 0$

(2) $2(3x - y)^2 + (x - 3y) + 2 = 0$

(3) $(3x - y)^2 + 2(x - 3y) + 2 = 0$

(4) $2(x - 3y)^2 + (3x - y) + 2 = 0$

4. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to_______.

5. Let a parabola P be such that its vertex and focus lie on the positive x–axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to :

(1) $16\sqrt{2}$ (2) 16 (3) 32 (4) $8\sqrt{2}$

6. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is :

(1) 5 (2) 7 (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

7. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :

(1) $y^3(x-2) = x^2$ (2) $x^3(x-2) = y^2$

(3) $y^2(x-2) = x^3$ (4) $x^2(x-2) = y^3$

8. A tangent and a normal are drawn at the point $P(2, -4)$ on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a, b)$ is a point such that AQBP is a square, then $2a + b$ is equal to :

(1) –16 (2) –18 (3) –12 (4) –20

9. If two tangents drawn from a point P to the parabola $y^2 = 16(x - 3)$ are at right angles, then the locus of point P is :

(1) $x + 3 = 0$ (2) $x + 1 = 0$

(3) $x + 2 = 0$ (4) $x + 4 = 0$

10. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x–axis at a distance R and S (>R) respectively from the origin, is :

(1) $4(S + R)$ (2) $2(S - R)$

(3) $4(S - R)$ (4) $2(S + R)$

11. A tangent line L is drawn at the point $(2, -4)$ on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to_______.

12. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to :

(1) $\frac{75}{8}$ (2) $\frac{125}{16}$ (3) $\frac{25}{2}$ (4) $\frac{15}{2}$

13. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(1) (3, 13) (2) (1, 5) (3) (–2, 8) (4) (2, 8)

14. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :

(1) $x = -\frac{a}{2}$ (2) $x = \frac{a}{2}$

(3) $x = 0$ (4) $x = a$

15. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it?

(1) (–6, 0) (2) (4, 5) (3) (5, 4) (4) (0, 3)

16. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) 0

17. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to ______.

18. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2,1) is :

(1) $x - y = 1$ (2) $2x + y = 5$

(3) $x + 3y = 5$ (4) $x + 2y = 4$

19. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0)$ $a \neq 0$, then 'a' must be greater than :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) –1 (4) 1

ELLIPSE

1. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is :

(1) $\frac{-1+\sqrt{5}}{2}$ (2) $\frac{-1+\sqrt{8}}{2}$

(3) $\frac{-1+\sqrt{3}}{2}$ (4) $\frac{-1+\sqrt{6}}{2}$

2. Let an ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus F(α, 0), α > 0, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

(1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{16}{3}$ (4) 3

3. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point :

(1) $(\sqrt{3}, 0)$ (2) $(\sqrt{2}, 0)$

(3) (1, 1) (4) (–1, 1)

4. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, – 4), one focus at (4, – 4) and one vertex at (5, – 4). If mx – y = 4, m > 0 is a tangent to the ellipse E, then the value of $5m^2$ is equal to______.

5. On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line x + 2y = 0. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2)$. A is :

(1) 6 (2) 12 (3) 14 (4) 24

6. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to _______.

7. The line 12x cosθ + 5y sinθ = 60 is tangent to which of the following curves?

(1) $x^2 + y^2 = 169$

(2) $144x^2 + 25y^2 = 3600$

(3) $25x^2 + 12y^2 = 3600$

(4) $x^2 + y^2 = 60$

8. The locus of mid–points of the line segments joining (–3, –5) and the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is :

(1) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

(2) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

(3) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

(4) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

9. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then tanθ is equal to :

(1) $\frac{5}{2\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{4}{\sqrt{3}}$ (4) 2

10. If the curve $x^2 + 2y^2 = 2$ intersects the line x + y = 1 at two points P and Q, then the angle subtended by the line segment PQ at the origin is :

(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

(3) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (4) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

11. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is ______.

12. If the point of intersections of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to:

(1) 12 (2) 5 (3) 6 (4) 10

13. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$, then the value of b is equal to :

(1) 11 (2) 14 (3) 16 (4) 20

14. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta,\sin\theta\right)$ where $\theta\in\left(0,\frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

HYPERBOLA

1. Let a line L : $2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to :

(1) 12 (2) –12 (3) 24 (4) –24

2. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

(1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

(3) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

(4) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

3. The point $P\left(-2\sqrt{6},\sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to :

(1) $4\sqrt{3}$ (2) 6 (3) $6\sqrt{3}$ (4) $3\sqrt{6}$

4. Let A (secθ, 2tanθ) and B (secϕ, 2tanϕ), where θ + ϕ = π/2, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to ___________.

5. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is TRUE?

(1) $a + b = c + d$ (2) $a - b = c - d$

(3) $a - c = b + d$ (4) $ab=\frac{c+d}{a+b}$

6. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x)=\frac{x-\frac{1}{2}}{x-1}$.

Then the composition function $f(g(x))$ is :

(1) onto but not one-one

(2) both one-one and onto

(3) one-one but not onto

(4) neither one-one nor onto

7. The locus of the point of intersection of the lines $(\sqrt{3})kx+ky-4\sqrt{3}=0$ and $\sqrt{3}x-y-4(\sqrt{3})k=0$ is a conic, whose eccentricity is ___________.

8. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities in one, then the equation of the hyperbola is :

(1) $\frac{x^2}{9}-\frac{y^2}{25}=1$ (2) $\frac{x^2}{9}-\frac{y^2}{16}=1$

(3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9}-\frac{y^2}{4}=1$

9. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

10. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

11. Consider a hyperbola H : $x^2 - 2y^2 = 4$. Let the tangent at a point $P\left(4,\sqrt{6}\right)$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6}-1$

(3) $\frac{7}{\sqrt{6}}-2$ (4) $4\sqrt{6}-1$

PERMUTATION & COMBINATION

1. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is__________.

2. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _______.

3. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _______.

4. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to:

(1) 1 (2) 4 (3) 2 (4) 3

5. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10}.(11)^{11}.(13)^{13}$ is equal to_____ .

6. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ________.

7. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _______.

8. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _________.

9. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set T = $\{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of 3$\}$ is _________.

10. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _________.

11. Let $P_1, P_2, \ldots, P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is :

(1) 12 (2) 419 (3) 443 (4) 455

12. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is ______.

13. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

14. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :
(1) 1625 (2) 575 (3) 560 (4) 1050

15. The total number of positive integral solutions (x, y, z) such that xyz = 24 is :
(1) 36 (2) 24 (3) 45 (4) 30

16. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ______.

17. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then :
(1) y = 273x (2) 2y = 91x
(3) y = 91x (4) 2y = 273x

18. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ______.

19. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is
(1) 42 (2) 82 (3) 77 (4) 35

20. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, y > z. Then the number of odd divisors of n, including 1, is :
(1) 11 (2) 6 (3) 6x (4) 12

21. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :
(1) 795 (2) 1173 (3) 1890 (4) 717

22. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :
(1) 364 (2) 240 (3) 333 (4) 360

23. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :
(1) 5 (2) 2 (3) 4 (4) 6

24. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:
(1) 26664 (2) 122664
(3) 122234 (4) 22264

25. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

26. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ______.

BINOMIAL THEOREM

1. The coefficient of x^{256} in the expansion of $(1 - x)^{101} (x^2 + x + 1)^{100}$ is :
(1) ${}^{100}C_{16}$ (2) ${}^{100}C_{15}$
(3) $-{}^{100}C_{16}$ (4) $-{}^{100}C_{15}$

2. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is _________.

3. For the natural numbers m, n, if $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \ldots + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of (m + n) is equal to :
(1) 88 (2) 64 (3) 100 (4) 80

4. For $k \in N$,

let $\frac{1}{\alpha(\alpha+1)(\alpha+2).......(\alpha+20)} = \sum_{K=0}^{20} \frac{A_k}{\alpha+k}$,

where $\alpha > 0$. Then the value of

$100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to ________.

5. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _______.

6. The number of elements in the set $\{n \in \{1, 2, 3, ..., 100\} \mid (11)^n > (10)^n + (9)^n\}$ is_______.

7. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + ... + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$,

then the value of γ is :

(1) $\frac{a^2+b}{3a^3}$ (2) $\frac{a+b}{3a^2}$

(3) $\frac{b^2}{3a^3}$ (4) $\frac{a+b^2}{3a^3}$

8. The ratio of the coefficient of the middle term in the expansion of $(1 + x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1 + x)^{19}$ is ________.

9. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to_______.

10. The sum of all those terms which are rational numbers in the expansion of $(2^{1/3}+3^{1/4})^{12}$ is:

(1) 89 (2) 27 (3) 35 (4) 43

11. If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to :

(1) –1 (2) 1 (3) –2 (4) 2

12. The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is ________.

(1) 3 (2) 4 (3) 2 (4) 1

13. Let $n \in \mathbf{N}$ and [x] denote the greatest integer less than or equal to x. If the sum of (n + 1) terms ${}^nC_0, 3\cdot{}^nC_1, 5\cdot{}^nC_2, 7\cdot{}^nC_3, ...$ is equal to $2^{100}\cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to _________.

14. If the co-efficient of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then the value of n is equal to _______.

15. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to:

(1) 2 (2) –1 (3) 1 (4) –2

16. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}} + 3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}$ is equal to 180, is :

(1) 0 (2) –1 (3) 2 (4) 1

17. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1 + x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\,{}^{20}C_r$ is equal to :

(1) 420×2^{19} (2) 380×2^{19}

(3) 380×2^{18} (4) 420×2^{18}

18. If $^1P_1 + 2.^2P_2 + 3.^3P_3 + ... + 15 \cdot {}^{15}P_{15} = {}^qP_r - s$, $0 \le s \le 1$, then $^{q+s}C_{r-s}$ is equal to ________.

19. Let $\binom{n}{k}$ denotes ^nC_k and $\left[\begin{matrix} n \\ k \end{matrix}\right] = \begin{cases} \binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}$

If $A_k = \sum_{i=0}^{9} \binom{9}{i} \left[\begin{matrix} 12 \\ 12-k+i \end{matrix}\right] + \sum_{i=0}^{8} \binom{8}{i} \left[\begin{matrix} 13 \\ 13-k+i \end{matrix}\right]$

and $A_4 - A_3 = 190$ p, then p is equal to :

20. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...$, is equal to :

(1) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(2) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

(3) $\frac{1-x}{1+x} + \log_e(1-x)$

(4) $\frac{1+x}{1-x} + \log_e(1-x)$

21. $\sum_{k=0}^{20} \left({}^{20}C_k\right)^2$ is equal to :

(1) $^{40}C_{21}$ (2) $^{40}C_{19}$ (3) $^{40}C_{20}$ (4) $^{41}C_{20}$

22. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder __________.

23. If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$, then k is equal to _________.

24. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K.2^{16}$, then K is equal to _____.

25. If the sum of the coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is _______.

26. If $n \ge 2$ is a positive integer, then the sum of the series $^{n+1}C_2 + 2(^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2)$ is:

(1) $\frac{n(n-1)(2n+1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$

(3) $\frac{n(2n+1)(3n+1)}{6}$ (4) $\frac{n(n+1)^2(n+2)}{12}$

27. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \ge r \ge 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to ________.

28. The value of $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ...$ $-15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is :

(1) $2^{16} - 1$ (2) $2^{13} - 14$

(3) 2^{14} (4) $2^{13} - 13$

29. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _______.

30. Let $m,n \in N$ and $\gcd(2,n) = 1$. If

$30\binom{30}{0} + 29\binom{30}{1} + ... + 2\binom{30}{28} + 1\binom{30}{29} = n.2^m$,

then n + m is equal to

(Here $\binom{n}{k} = {}^nC_k$)

31. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is :

(1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(3) $\frac{2.10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

32. Let n be a positive integer. Let

$A = \sum_{k=0}^{n} (-1)^k {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to _______.

33. If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then$(n - 1)$ is divisible by :

(1) 26 (2) 30 (3) 8 (4) 7

34. Let [x] denote greatest integer less than or equal to x. If for $n \in \mathbb{N}$, $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$ is equal to :

(1) 2 (2) 2^{n-1} (3) 1 (4) n

35. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to :

(1) 1124 (2) 1324 (3) 1024 (4) 924

36. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x+\frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _______.

37. Two dices are rolled. If both dices have six faces numbered 1,2,3,5,7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

(1) $\frac{4}{9}$ (2) $\frac{17}{36}$ (3) $\frac{5}{12}$ (4) $\frac{1}{2}$

38. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where $x \in N$ is equal to :

(1) 2 (2) 4 (3) 3 (4) 1

39. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\ldots\infty}}}}$ is :

(1) $2+\frac{2}{5}\sqrt{30}$ (2) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(3) $4+\frac{4}{\sqrt{5}}\sqrt{30}$ (4) $5+\frac{2}{5}\sqrt{30}$

40. If $(2021)^{3762}$ is divided by 17, then the remainder is _______ .

41. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \ldots + a_{40}x^{40}$. then $a_1 + a_3 + a_5 + \ldots + a_{37}$ is equal to

(1) $2^{20}(2^{20} - 21)$ (2) $2^{19}(2^{20} - 21)$

(3) $2^{19}(2^{20} + 21)$ (4) $2^{20}(2^{20} + 21)$

42. If $\sum_{r=1}^{10} r!(r^3+6r^2+2r+5) = \alpha(11!)$, then the value of α is equal to _______.

43. Let P(x) be a real polynomial of degree 3 which vanishes at $x = -3$. Let P(x) have local minima at $x = 1$, local maxima at $x = -1$ and $\int_{-1}^{1} P(x)dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to _______.

44. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum_{k=0}^{10}(2^2+3k)\,{}^nC_k = \alpha.3^{10} + \beta.2^{10}$, $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to ______.

SET

1. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :

(1) {80, 83, 86, 89} (2) {84, 86, 88, 90}

(3) {79, 81, 83, 85} (4) {84, 87, 90, 93}

2. If $A = \{x \in \mathbf{R} : |x - 2| > 1\}$, $B = \{x \in \mathbf{R} : \sqrt{x^2-3} > 1\}$, $C = \{x \in \mathbf{R} : |x-4| \geq 2\}$ and **Z** is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap \mathbf{Z}$ is ________.

3. Let A = {n ∈ N : n is a 3-digit number}

B = {9k + 2 : k ∈ N}

and C = {9k + l : k ∈ N} for some l ($0 < l < 9$)

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274 × 400, then l is equal to ______.

4. Let R = {(P,Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set :

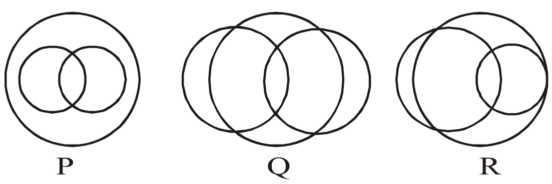
(1) S = $\{(x,y) \mid x^2 + y^2 = 4\}$

(2) S = $\{(x,y) \mid x^2 + y^2 = 1\}$

(3) S = $\{(x,y) \mid x^2 + y^2 = \sqrt{2}\}$

(4) S = $\{(x,y) \mid x^2 + y^2 = 2\}$

5. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?



(1) P and Q (2) P and R

(3) None of these (4) Q and R

6. The number of elements in the set

$\{x \in \mathbb{R} : (|x| - 3)\,|x + 4| = 6\}$ is equal to

(1) 3 (2) 2 (3) 4 (4) 1

RELATION

1. Let **N** be the set of natural numbers and a relation R on **N** be defined by

R = $\{(x,y) \in \mathbf{N} \times \mathbf{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$.

Then the relation R is :

(1) symmetric but neither reflexive nor transitive

(2) reflexive but neither symmetric nor transitive

(3) reflexive and symmetric, but not transitive

(4) an equivalence relation

2. Which of the following is **not** correct for relation R on the set of real numbers ?

(1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric.

(2) $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric and transitive.

(3) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric.

(4) $(x, y) \in R \Leftrightarrow |x - y| \le 1$ is reflexive and symmetric.

3. Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by $(a, b) \simeq (c, d)$, if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 5 (2) 6 (3) 8 (4) 7

4. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive,

(2) R is reflexive, symmetric but not transitive

(3) R is an equivalence relation

(4) R is reflexive, transitive but not symmetric

FUNCTION

1. Let [x] denote the greatest integer ≤ x, where $x \in \mathbf{R}$. If the domain of the real valued function $f(x) = \sqrt{\dfrac{|[x]| - 2}{|[x]| - 3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty), a < b < c$, then the value of a + b + c is :

(1) 8 (2) 1 (3) –2 (4) –3

2. Let $f : \mathbf{R} - \left\{\frac{\alpha}{6}\right\} \to \mathbf{R}$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of α for which $(fof)(x) = x$, for all $x \in \mathbf{R} - \left\{\frac{\alpha}{6}\right\}$, is :

(1) No such α exists (2) 5

(3) 8 (4) 6

3. Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in \mathbf{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

(1) $\left[0, \frac{1}{e}\right)$ (2) $[\log_e 2, \log_e 3)$

(3) $[1, e)$ (4) $[0, \log_e 2)$

4. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

5. Let $g : \mathbf{N} \to \mathbf{N}$ be defined as

$g(3n + 1) = 3n + 2$,

$g(3n + 2) = 3n + 3$,

$g(3n + 3) = 3n + 1$, for all $n \geq 0$.

Then which of the following statements is true ?

(1) There exists an onto function $f : \mathbf{N} \to \mathbf{N}$ such that fog = f

(2) There exists a one–one function $f: \mathbf{N} \to \mathbf{N}$ such that fog = f

(3) gogog = g

(4) There exists a function $f : \mathbf{N} \to \mathbf{N}$ such that gof = f

6. If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100} \left[\frac{(-1)^n n}{2}\right]$ is equal to :

(1) 0 (2) 4 (3) –2 (4) 2

7. Consider function $f : A \to B$ and $g : B \to C$ (A, B, C $\subseteq$ **R**) such that $(gof)^{-1}$ exists, then:

(1) f and g both are one-one

(2) f and g both are onto

(3) f is one-one and g is onto

(4) f is onto and g is one-one

8. Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to______.

9. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$f(x + y) + f(x - y) = 2\, f(x)\, f(y)$, $f\left(\frac{1}{2}\right) = -1$.

Then,

the value of $\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k+f(k))}$ is equal to :

(1) $\operatorname{cosec}^2(21) \cos(20) \cos(2)$

(2) $\sec^2(1) \sec(21) \cos(20)$

(3) $\operatorname{cosec}^2(1) \operatorname{cosec}(21) \sin(20)$

(4) $\sec^2(21) \sin(20) \sin(2)$

10. The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

(1) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$ (2) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

(3) $\left(-\frac{1}{2}, \infty\right) - \{0\}$ (4) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

11. Let $f : \mathbf{N} \to \mathbf{N}$ be a function such that $f(m + n) = f(m) + f(n)$ for every $m, n \in \mathbf{N}$. If $f(6) = 18$, then $f(2) \cdot f(3)$ is equal to :

(1) 6 (2) 54 (3) 18 (4) 36

12. The range of the function,

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$

is :

(1) $\left(0, \sqrt{5}\right)$ (2) $[-2, 2]$

(3) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$ (4) $[0, 2]$

13. Let f(x) be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for k = 2, 3, 4, 5. Then the value of $52 - 10\, f(10)$ is equal to :

14. Let f, g : N → N such that

f(n + 1) = f(n) + f(1) ∀ n ∈ N and g be any arbitrary function. Which of the following statements is NOT true ?

(1) If fog is one-one, then g is one-one

(2) If f is onto, then f(n) = n ∀ n∈N

(3) f is one-one

(4) If g is onto, then fog is one-one

15. A function f(x) is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series

$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+......+f\left(\frac{39}{20}\right)$ is equal to :

(1) $\frac{19}{2}$ (2) $\frac{49}{2}$ (3) $\frac{29}{2}$ (4) $\frac{39}{2}$

16. Let A = {1, 2, 3, ..., 10} and f : A → A be defined as $f(k)=\begin{cases} k+1 & \text{if k is odd} \\ k & \text{if k is even}\end{cases}$

Then the number of possible functions g : A → A such that go$f$ = f is

(1) 10^5 (2) ${}^{10}C_5$ (3) 5^5 (4) 5!

17. Let $f(x) = \sin^{-1}x$ and $g(x)=\frac{x^2-x-2}{2x^2-x-6}$. If $g(2)=\lim_{x\to 2} g(x)$, then the domain of the function $f$og is :

(1) $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

(2) $(-\infty,-2]\cup[-1,\infty)$

(3) $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$

(4) $(-\infty,-1]\cup[2,\infty)$

18. Let f be any function defined on R and let it satisfy the condition :

$|f(x) - f(y)| \le |(x-y)^2|, \forall (x,y) \in R$

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0, \forall x \in R$

(3) $f(x) = 0, \forall x \in R$

(4) $f(x) > 0, \forall x \in R$

19. If a + α = 1, b + β = 2 and

$af(x)+\alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, x ≠ 0, then the value of expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is _______.

20. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval [0, 2π] is :

(1) 3 (2) 4 (3) 2 (4) 5

21. The inverse of $y = 5^{\log x}$ is :

(1) $x = 5^{\log y}$ (2) $x = y^{\log 5}$

(3) $x = y^{\frac{1}{\log 5}}$ (4) $x = 5^{\frac{1}{\log y}}$

22. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :

$f + g, f - g, f/g, g/f, g - f$ where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$

(1) $0 \le x \le 1$ (2) $0 \le x < 1$

(3) $0 < x < 1$ (4) $0 < x \le 1$

23. Let f : R – {3} → R – {1} be defined by $f(x)=\frac{x-2}{x-3}$. Let g : R → R be given as g(x) = 2x – 3. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(1) 7 (2) 2 (3) 5 (4) 3

INVERSE TRIGONOMETRY FUNCTION

1. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is :

(1) 1 (2) 2 (3) 4 (4) 0

2. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to :

(1) $\frac{-181}{69}$ (2) $\frac{220}{21}$ (3) $\frac{-291}{76}$ (4) $\frac{151}{63}$

3. If the domain of the function $f(x)=\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha+\beta$ is equal to :

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{1}{2}$ (4) 1

4. If $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2}=p$, then the value of tan p is :

(1) $\frac{101}{102}$ (2) $\frac{50}{51}$ (3) 100 (4) $\frac{51}{50}$

5. If $(\sin^{-1}x)^2-(\cos^{-1}x)^2=a;\ 0<x<1,\ a\neq 0$, then the value of $2x^2-1$ is :

(1) $\cos\left(\frac{4a}{\pi}\right)$ (2) $\sin\left(\frac{2a}{\pi}\right)$

(3) $\cos\left(\frac{2a}{\pi}\right)$ (4) $\sin\left(\frac{4a}{\pi}\right)$

6. Let M and m respectively be the maximum and minimum values of the function $f(x)=\tan^{-1}(\sin x+\cos x)$ in $\left[0,\frac{\pi}{2}\right]$, Then the value of tan(M – m) is equal to:

(1) $2+\sqrt{3}$ (2) $2-\sqrt{3}$

(3) $3+2\sqrt{2}$ (4) $3-2\sqrt{2}$

7. The domain of the function

$$f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$$ is :

(1) $\left[0,\frac{1}{4}\right]$ (2) $[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$

(3) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$ (4) $\left[0,\frac{1}{2}\right]$

8. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$ is equal to :

(The inverse trigonometric functions take the principal values)

(1) $3\pi-11$ (2) $4\pi-9$

(3) $4\pi-11$ (4) $3\pi+1$

9. $\text{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to :

(1) $\frac{56}{33}$ (2) $\frac{65}{56}$ (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

10. If $0<a, b<1$, and $\tan^{-1}a+\tan^{-1}b=\frac{\pi}{4}$, then the value of

$$(a+b)-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)-\left(\frac{a^4+b^4}{4}\right)+\ldots$$ is:

(1) $\log_e 2$ (2) e^2-1

(3) e (4) $\log_e\left(\frac{e}{2}\right)$

11. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c};0<x<1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1-y^2$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

12. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to:

(1) 2 (2) 1 (3) 3 (4) 0

13. Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty}S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$ (2) $\frac{\pi}{2}$

(3) $\cot^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}(3)$

14. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for x ∈ [–1, 1], and [x] denotes the greatest integer less than or equal to x, is :

(1) 2 (2) 0

(3) 4 (4) Infinite

15. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + ...$ upto 100 terms, then α is :

(1) 1.01 (2) 1.00 (3) 1.02 (4) 1.03

16. The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is :

(1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$ (3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

17. The real valued function $f(x)=\frac{\operatorname{cosec}^{-1}x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :

(1) all reals except integers

(2) all non-integers except the interval [–1, 1]

(3) all integers except 0, –1, 1

(4) all reals except the Interval [–1,1]

LIMIT

1. If the value of $\lim_{x\to0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to ________.

2. If $\lim_{x\to0}\frac{\alpha xe^x-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2x}=10$, $\alpha,\beta,\gamma\in\mathbf{R}$, then the value of α + β + γ is ________.

3. The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

(1) $5+\log_e\left(\frac{3}{2}\right)$ (2) $2-\log_e\left(\frac{2}{3}\right)$

(3) $3+2\log_e\left(\frac{2}{3}\right)$ (4) $1+2\log_e\left(\frac{3}{2}\right)$

4. The value of $\lim_{x\to0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to :

(1) 0 (2) 4 (3) – 4 (4) –1

5. $\lim_{x\to2}\left(\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to :

(1) $\frac{9}{44}$ (2) $\frac{5}{24}$ (3) $\frac{1}{5}$ (4) $\frac{7}{36}$

6. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to:

(1) $b^2 + 4c$ (2) $2(b^2 + 4c)$

(3) $2(b^2 – 4c)$ (4) $b^2 – 4c$

7. If 0 < x < 1 and $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+...$, then the value of e^{1+y} at $x=\frac{1}{2}$ is:

(1) $\frac{1}{2}e^2$ (2) 2e (3) $\frac{1}{2}\sqrt{e}$ (4) $2e^2$

8. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is:

(1) $\left(1,\frac{1}{2}\right)$ (2) $\left(1,-\frac{1}{2}\right)$

(3) $\left(-1,\frac{1}{2}\right)$ (4) $\left(-1,-\frac{1}{2}\right)$

9. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to :

(1) π^2 (2) $2\pi^2$ (3) $4\pi^2$ (4) 4π

10. If $\alpha=\lim\limits_{x\to\pi/4}\dfrac{\tan^3 x-\tan x}{\cos\left(x+\dfrac{\pi}{4}\right)}$ and $\beta=\lim\limits_{x\to 0}(\cos x)^{\cot x}$ are the roots of the equation, $ax^2+bx-4=0$, then the ordered pair (a, b) is :

(1) (1, –3) (2) (–1, 3)

(3) (–1, –3) (4) (1, 3)

11. Let $f:\mathbf{R}\to\mathbf{R}$ be a continuous function. Then $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)\,dx}{x^2-\frac{\pi^2}{16}}$ is equal to :

(1) $f(2)$ (2) $2f(2)$

(3) $2f(\sqrt{2})$ (4) $4f(2)$

12. Let $f(x)=x^6+2x^4+x^3+2x+3$, $x\in\mathbf{R}$. Then the natural number n for which $\lim\limits_{x\to 1}\dfrac{x^n f(1)-f(x)}{x-1}=44$ is________.

13. $\lim\limits_{n\to\infty}\tan\left\{\sum\limits_{r=1}^{n}\tan^{-1}\left(\dfrac{1}{1+r+r^2}\right)\right\}$ is equal to ____.

14. $\lim\limits_{n\to\infty}\left(1+\dfrac{1+\frac{1}{2}+\dots\dots+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\dfrac{1}{2}$ (2) 0 (3) $\dfrac{1}{e}$ (4) 1

15. If $\lim\limits_{x\to 0}\dfrac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of a – 2b is _______.

16. The value of

$$\lim_{h\to 0}2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$$ is

(1) $\dfrac{4}{3}$ (2) $\dfrac{2}{\sqrt{3}}$ (3) $\dfrac{3}{4}$ (4) $\dfrac{2}{3}$

17. Let $\alpha\in R$ be such that the function

$$f(x)=\begin{cases}\dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x\neq 0\\ \alpha, & x=0\end{cases}$$

is continuous at x = 0, where {x} = x – [x], [x] is the greatest integer less than or equal to x. Then :

(1) $\alpha=\dfrac{\pi}{\sqrt{2}}$ (2) $\alpha=0$

(3) no such α exists (4) $\alpha=\dfrac{\pi}{4}$

18. Let $f:(0,2)\to\mathbb{R}$ be defined as

$$f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right).$$

Then, $\lim\limits_{n\to\infty}\dfrac{2}{n}\left(f\left(\dfrac{1}{n}\right)+f\left(\dfrac{2}{n}\right)+\dots+f(1)\right)$ is equal to _______ .

19. If $\lim\limits_{x\to 0}\dfrac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then a + b + c is equal to _______.

20. The value of $\lim\limits_{n\to\infty}\dfrac{[r]+[2r]+\dots+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to :

(1) $\dfrac{r}{2}$ (2) r (3) 2r (4) 0

21. The value of the limit $\lim\limits_{\theta\to 0}\dfrac{\tan\left(\pi\cos^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$ is equal to :

(1) $-\dfrac{1}{2}$ (2) $-\dfrac{1}{4}$ (3) 0 (4) $\dfrac{1}{4}$

22. The value of

$\lim\limits_{x\to 0^+}\dfrac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3}$, where [x] denotes the greatest integer $\le x$ is :

(1) π (2) 0 (3) $\dfrac{\pi}{4}$ (4) $\dfrac{\pi}{2}$

23. If $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$ (3) 6 (4) 2

CONTINUITY

1. Let a function $f: \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\sin x-e^x & \text{if } x\le 0\\ a+[-x] & \text{if } 0<x<1\\ 2x-b & \text{if } x\ge 1\end{cases}$$

Where [x] is the greatest integer less than or equal to x. If f is continuous on **R**, then (a + b) is equal to:

(1) 4 (2) 3 (3) 2 (4) 5

2. Let f : **R** → **R** be defined as

$$f(x)=\begin{cases}\dfrac{\lambda|x^2-5x+6|}{\mu(5x-x^2-6)}, & x<2\\ e^{\frac{\tan(x-2)}{x-[x]}} & ,x>2\\ \mu & ,x=2\end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then $\lambda+\mu$ is equal to :

(1) e(–e + 1) (2) e(e – 2)

(3) 1 (4) 2e – 1

3. Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}} & , -\frac{\pi}{4}<x<0\\ b & , x=0\\ e^{\cot 4x/\cot 2x} & , 0<x<\frac{\pi}{4}\end{cases}$$

If f is continuous at x = 0, then the value of $6a+b^2$ is equal to :

(1) 1 – e (2) e –1 (3) 1 + e (4) e

4. Let a, b ∈ **R**, b ≠ 0, Define a function

$$f(x)=\begin{cases}a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0\\ \dfrac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0.\end{cases}$$

If f is continuous at x = 0, then 10 – ab is equal to _________.

5. If the function $f(x)=\begin{cases}\dfrac{1}{x}\log_e\left(\dfrac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , x<0\\ k & , x=0\\ \dfrac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , x>0\end{cases}$

is continuous at x = 0, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

(1) –5 (2) 5 (3) – 4 (4) 4

6. Let [t] denote the greatest integer ≤ t. The number of points where the function

$$f(x)=[x]|x^2-1|+\sin\left(\frac{\pi}{[x]+3}\right)-[x+1], x\in(-2,2)$$

is not continuous is _______.

7. If f : R → R is a function defined by $f(x)=[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is :

(1) discontinuous at all integral values of x except at x = 1

(2) continuous only at x = 1

(3) continuous for every real x

(4) discontinuous only at x = 1

8. Let f : R → R be defined as

$$f(x)=\begin{cases}2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1\\ |ax^2+x+b|, & \text{if } -1\le x\le 1\\ \sin(\pi x), & \text{if } x>1\end{cases}$$

If $f(x)$ is continuous on R, then a + b equals:

(1) –3 (2) –1 (3) 3 (4) 1

9. Let f : R → R and g : R → R be defined as

$$f(x)=\begin{cases} x+a, & x<0 \\ |x-1|, & x\ge 0 \end{cases} \text{ and}$$

$$g(x)=\begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x\ge 0 \end{cases}$$

where a, b are non-negative real numbers. If (gof)(x) is continuous for all x ∈ R, then a + b is equal to ____________.

10. If the function $f(x)=\dfrac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0)=\dfrac{1}{k}$, then k is _________ .

11. Let f : R → R be a function defined as

$$f(x)=\begin{cases} \dfrac{\sin(a+1)x+\sin 2x}{2x} & , \text{if } x<0 \\ b & , \text{if } x=0 \\ \dfrac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}} & , \text{if } x>0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to :

(1) $-\dfrac{5}{2}$ (2) –2 (3) –3 (4) $-\dfrac{3}{2}$

DIFFERENTIABILITY

1. Let a function g : [0, 4] → **R** be defined as

$$g(x)=\begin{cases} \max\limits_{0\le t\le x}\{t^3-6t^2+9t-3\}, & 0\le x\le 3 \\ 4-x & , 3<x\le 4 \end{cases},$$

then the number of points in the interval (0, 4) where g(x) is NOT differentiable, is _______.

2. Let f : **R** → **R** be defined as

$$f(x)=\begin{cases} \dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , x\ne 0 \\ \alpha & , x=0 \end{cases}.$$

If f is continuous at x = 0, then α is equal to :

(1) 1 (2) 3 (3) 0 (4) 2

3. Let f : **R** → **R** be a function defined as

$$f(x)=\begin{cases} 3\left(1-\dfrac{|x|}{2}\right) & \text{if } |x|\le 2 \\ 0 & \text{if } |x|>2 \end{cases}$$

Let g : **R** → **R** be given by g(x) = f(x + 2) – f(x – 2). If n and m denote the number of points in **R** where g is not continuous and not differentiable, respectively, then n + m is equal to _______.

4. Let f : **R** → **R** be a function such that f(2) = 4 and $f'(2)=1$. Then, the value of $\lim\limits_{x\to 2}\dfrac{x^2f(2)-4f(x)}{x-2}$ is equal to :

(1) 4 (2) 8 (3) 16 (4) 12

5. Let f : [0, 3] → **R** be defined by

f(x) = min {x – [x], 1 + [x] – x}

where [x] is the greatest integer less than or equal to x. Let P denote the set containing all x ∈ [0, 3] where f is discontinuous, and Q denote the set containing all x ∈ (0, 3) where f is not differentiable. Then the sum of number of elements in P and Q is equal to _________.

माना f : [0, 3] → **R**

6. Let f : [0, ∞) → [0, 3] be a function defined by

$$f(x)=\begin{cases} \max\{\sin t: 0\le t\le x\}, & 0\le x\le \pi \\ 2+\cos x, & x>\pi \end{cases}$$

Then which of the following is true ?

(1) f is continuous everywhere but not differentiable exactly at one point in (0, ∞)

(2) f is differentiable everywhere in (0, ∞)

(3) f is not continuous exactly at two points in (0, ∞)

(4) f is continuous everywhere but not differentiable exactly at two points in (0, ∞)

7. Let [t] denote the greatest integer less than or equal to t. Let f(x) = x – [x], g(x) = 1 – x + [x], and h(x) = min{f(x), g(x)}, x ∈ [–2, 2]. Then h is :

(1) continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

(2) not continuous at exactly three points in [–2, 2]

(3) continuous in [–2, 2] but not differentiable at exactly three points in (–2, 2)

(4) not continuous at exactly four points in [–2, 2]

8. The function $f(x)=|x^2-2x-3|\cdot e^{|9x^2-12x+4|}$ is not differentiable at exactly :

(1) four points (2) three points

(3) two points (4) one point

9. The number of points, at which the function f(x) = |2x + 1| – 3|x + 2| + |x² + x – 2|, x ∈ R is not differentiable, is _______.

10. A function f is defined on [–3, 3] as

$$f(x)=\begin{cases}\min\{|x|,2-x^2\}, & -2\le x\le 2\\ [|x|] & ,2<|x|\le 3\end{cases}$$

where [x] denotes the greatest integer ≤ x. The number of points, where f is not differentiable in (–3, 3) is ______.

11. Let the functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined as :

$$f(x)=\begin{cases}x+2, & x<0\\ x^2, & x\ge 0\end{cases}\text{ and } g(x)=\begin{cases}x^3, & x<1\\ 3x-2, & x\ge 1\end{cases}$$

Then, the number of points in $\mathbb{R}$ where $(f\text{o}g)(x)$ is NOT differentiable is equal to :

(1) 3 (2) 1 (3) 0 (4) 2

12. If $f(x)=\begin{cases}\dfrac{1}{|x|} & ; |x|\ge 1\\ ax^2+b & ; |x|<1\end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

(1) $\frac{1}{2},\frac{1}{2}$ (2) $\frac{1}{2},-\frac{3}{2}$

(3) $\frac{5}{2},-\frac{3}{2}$ (4) $-\frac{1}{2},\frac{3}{2}$

13. Let f : R → R satisfy the equation $f(x+y)=f(x).f(y)$ for all x, y ∈ R and f(x) ≠ 0 for any x ∈ R. If the function f is differentiable at x = 0 and $f'(0)=3$, then $\lim_{h\to 0}\frac{1}{h}(f(h)-1)$ is equal to ______.

METHOD OF DIFFERENTIATION

1. Consider the function $f(x)=\dfrac{P(x)}{\sin(x-2)},\ x\ne 2$
$=7\qquad ,\ x=2$

Where P(x) is a polynomial such that P″ (x) is always a constant and P(3) = 9. If f(x) is continuous at x = 2, then P(5) is equal to ______.

2. Let f(x) = $\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$, 0 < x < 1. Then :

(1) $(1-x)^2 f'(x)-2(f(x))^2=0$

(2) $(1+x)^2 f'(x)+2(f(x))^2=0$

(3) $(1-x)^2 f'(x)+2(f(x))^2=0$

(4) $(1+x)^2 f'(x)-2(f(x))^2=0$

3. If y = y(x) is an implicit function of x such that $\log_e(x+y)=4xy$, then $\frac{d^2y}{dx^2}$ at x = 0 is equal to _______.

4. If $y^{1/4}+y^{-1/4}=2x$, and $(x^2-1)\frac{d^2y}{dx^2}+\alpha x\frac{dy}{dx}+\beta y=0$, then |α – β| is equal to __________.

5. If $y(x)=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x\in\left(\frac{\pi}{2},\pi\right)$, then $\frac{dy}{dx}$ at $x=\frac{5\pi}{6}$ is :

(1) $-\frac{1}{2}$ (2) -1 (3) $\frac{1}{2}$ (4) 0

6. $\lim_{x\to 0}\frac{\int_0^{x^2}\left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 0 (4) $\frac{1}{15}$

7. Let $f(x)$ be a differentiable function at x = a with $f'(a) = 2$ and $f(a) = 4$. Then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals :

(1) 2a + 4 (2) 4 – 2a (3) 2a – 4 (4) a + 4

8. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point

(1) (2,2) (2) (0,0)

(3) (2,9) (4) $\left(3,\frac{21}{2}\right)$

9. Let f be a twice differentiable function defined on R such that f(0) = 1, f '(0) = 2 and $f'(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x)\end{vmatrix}=0$, for all $x \in R$, then the value of f(1) lies in the interval:

(1) (9, 12) (2) (6, 9) (3) (0, 3) (4) (3, 6)

10. Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that f(x + 1) = xf(x). If $g : S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of |g''(5) – g''(1)| is equal to :

(1) $\frac{205}{144}$ (2) $\frac{197}{144}$ (3) $\frac{187}{144}$ (4) 1

11. If $f(x)=\sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is _______ .

AOD (TANGENT & NORMAL)

1. An angle of intersection of the curves, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab$, a > b, is :

(1) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (2) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

(3) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$ (4) $\tan^{-1}\left(2\sqrt{ab}\right)$

2. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is y = x, then the possible values of a, b, c are :

(1) $a=\frac{1}{2}$, $b=\frac{1}{2}$, c = 1

(2) a = 1, b = 0, c= 1

(3) a = 1, b = 1, c = 0

(4) a = –1, b = 1, c = 1

3. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) 0 (3) $-t^3$ (4) $2t^3$

4. If the curves $x = y^4$ and xy = k cut at right angles, then $(4k)^6$ is equal to ______.

5. If the normal to the curve $y(x)=\int_0^x(2t^2-15t+10)dt$ at a point (a,b) is parallel to the line x + 3y = –5, a > 1, then the value of |a + 6b| is equal to ________ .

AOD (MONOTONICITY)

1. Let $f:\mathbf{R}\to\mathbf{R}$ be defined as

$$f(x)=\begin{cases}-\frac{4}{3}x^3+2x^2+3x & , \quad x>0\\ 3xe^x & , \quad x\le 0\end{cases}$$. Then f is increasing function in the interval

(1) $\left(-\frac{1}{2},2\right)$ (2) (0,2)

(3) $\left(-1,\frac{3}{2}\right)$ (4) (–3, –1)

2. Let $f(x) = 3\sin^4x + 10\sin^3x + 6\sin^2x - 3$, $x\in\left[-\frac{\pi}{6},\frac{\pi}{2}\right]$. Then, f is :

(1) increasing in $\left(-\frac{\pi}{6},\frac{\pi}{2}\right)$

(2) decreasing in $\left(0,\frac{\pi}{2}\right)$

(3) increasing in $\left(-\frac{\pi}{6},0\right)$

(4) decreasing in $\left(-\frac{\pi}{6},0\right)$

3. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is :

(1) 2 (2) 4 (3) 1 (4) 0

4. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R-S|$ is _________.

5. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If f(0) = 0, f(1) = 1 and f(2) = 2, then

(1) $f''(x) = 0$ for all $x \in (0, 2)$

(2) $f''(x) = 0$ for some $x \in (0, 2)$

(3) $f'(x) = 0$ for some $x \in [0, 2]$

(4) $f''(x) > 0$ for all $x \in (0, 2)$

6. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

(S1) there exists $x_1, x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) there exists $x_3, x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$. Then

(1) both (S1) and (S2) are true

(2) (S1) is false and (S2) is true

(3) both (S1) and (S2) are false

(4) (S1) is true and (S2) is false

7. Let $f:\mathbf{R}\to\mathbf{R}$ be defined as,

$$f(x)=\begin{cases}-55x, & \text{if } x<-5\\ 2x^3-3x^2-120x, & \text{if } -5\le x\le 4\\ 2x^3-3x^2-36x-336, & \text{if } x>4,\end{cases}$$

Let A = {$x \in \mathbf{R}$: f is increasing}. Then A is equal to :

(1) $(-\infty, -5) \cup (4, \infty)$

(2) $(-5, \infty)$

(3) $(-\infty, -5) \cup (-4, \infty)$

(4) $(-5, -4) \cup (4, \infty)$

8. The function

$$f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x:$$

(1) increases in $\left[\frac{1}{2},\infty\right)$

(2) increases in $\left(-\infty,\frac{1}{2}\right]$

(3) decreases in $\left[\frac{1}{2},\infty\right)$

(4) decreases in $\left(-\infty,\frac{1}{2}\right]$

9. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4,\ x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (5, 8) (2) (–5, 8)

(3) (5, –8) (4) (–5, –8)

10. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval (a, a + 1). Then, |a| is equal to ______.

11. Let f be a real valued function, defined on R – {–1, 1} and given by

$$f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function f(x) is increasing?

(1) $(-\infty,-1)\cup\left(\left[\frac{1}{2},\infty\right)-\{1\}\right)$

(2) $(-\infty, \infty) - \{-1, 1\}$

(3) $\left(-1,\frac{1}{2}\right]$

(4) $\left(-\infty,\frac{1}{2}\right]-\{-1\}$

12. Consider the function f : R → R defined by

$$f(x)=\begin{cases}\left(2-\sin\left(\frac{1}{x}\right)\right)|x|, & x\neq 0\\ 0, & x=0\end{cases}$$. Then f is :

(1) monotonic on (–∞, 0) ∪ (0, ∞)

(2) not monotonic on (–∞, 0) and (0, ∞)

(3) monotonic on (0, ∞) only

(4) monotonic on (–∞, 0) only

AOD (MAXIMA & MINIMA)

1. Let A = $[a_{ij}]$ be a 3 × 3 matrix, where

$$a_{ij}=\begin{cases}1, & \text{if } i=j\\ -x, & \text{if } |i-j|=1\\ 2x+1, & \text{otherwise.}\end{cases}$$

Let a function f : **R** → **R** be defined as f(x) = det(A). Then the sum of maximum and minimum values of f on **R** is equal to:

(1) $-\frac{20}{27}$ (2) $\frac{88}{27}$ (3) $\frac{20}{27}$ (4) $-\frac{88}{27}$

2. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15,\ x \in \mathbf{R}$ is increasing in $\left(-\infty,\frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4},\infty\right)$. Then the function $g(x) = ax^2 - 6x + 15,\ x \in \mathbf{R}$ has a:

(1) local maximum at $x = -\frac{3}{4}$

(2) local minimum at $x = -\frac{3}{4}$

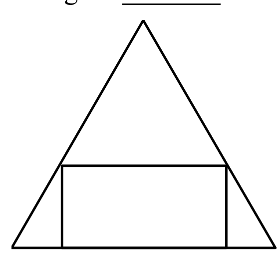
(3) local maximum at $x = \frac{3}{4}$

(4) local minimum at $x = \frac{3}{4}$

3. The sum of all the local minimum values of the twice differentiable function f : **R** → **R** defined by $f(x)=x^3-3x^2-\frac{3f''(2)}{2}x+f''(1)$ is :

(1) –22 (2) 5 (3) –27 (4) 0

4. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is_______.



5. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi}+1\right)k$ is equal to _________.

6. The local maximum value of the function $f(x)=\left(\frac{2}{x}\right)^{x^2}, x>0$, is

(1) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$ (2) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(3) $(e)^{\frac{2}{e}}$ (4) 1

7. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is:

(1) $\frac{5}{2+\sqrt{3}}$ (2) $\frac{10}{2+3\sqrt{3}}$

(3) $\frac{5}{3+\sqrt{3}}$ (4) $\frac{10}{3+2\sqrt{3}}$

8. A box open from top is made from a rectangular sheet of dimension a × b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to :

(1) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

(2) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

(3) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

(4) $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$

9. Let f(x) be a cubic polynomial with f(1) = –10, f(–1) = 6, and has a local minima at x = 1, and f'(x) has a local minima at x = –1. Then f(3) is equal to _____.

10. A man starts walking from the point P(–3,4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50\left((PR)^2+(RQ)^2\right)$ is equal to _____.

11. The minimum value of α for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=\alpha$ has at least one solution in $\left(0,\frac{\pi}{2}\right)$ is _______.

12. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = –1 and x = 1. If $\lim_{x\to0}\frac{f(x)}{x^3}=1$, then 5·f(2) is equal to _______.

13. The minimum value of $f(x)=a^{a^x}+a^{1-a^x}$, where a, x ∈ R and a > 0, is equal to :

(1) 2a (2) $2\sqrt{a}$

(3) $a+\frac{1}{a}$ (4) a + 1

14. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :

(1) An isosceles triangle with base equal to 2r.

(2) An equilateral triangle of height $\frac{2r}{3}$.

(3) An equilateral triangle having each of its side of length $\sqrt{3}r$.

(4) A right angle triangle having two of its sides of length 2r and r.

15. The range of a ∈ ℝ for which the function

$$f(x)=(4a-3)(x+\log_e 5)+2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),$$

$x\neq 2n\pi, n\in\mathbb{N}$, has critical points, is :

(1) (–3, 1) (2) $\left[-\frac{4}{3},2\right]$ (3) [1, ∞) (4) (–∞,–1]

INDEFINITE INTEGRATION

1. If $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$, $x > 0$ where C is the constant of integration, then the value of $9(\sqrt{3}a+b)$ is equal to _______.

2. If $\int \frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx = \frac{1}{14}\left(ux + v\log_e(4e^x+7e^{-x})\right) + C$, where C is a constant of integration, then u + v is equal to __________.

3. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}} + C$ (2) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}} + C$

(3) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$ (4) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}} + C$

4. If $\int \frac{\sin x}{\sin^3 x + \cos^3 x}dx =$

$\alpha\log_e|1+\tan x| + \beta\log_e|1-\tan x+\tan^2 x| + \gamma\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$,

when C is constant of integration, then the value of $18(\alpha+\beta+\gamma^2)$ is _______.

5. If $\int \frac{\cos x - \sin x}{\sqrt{8-\sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (–1, 3) (2) (3, 1)

(3) (1, 3) (4) (1, –3)

6. The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$$

is :

(where c is a constant of integration)

(1) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}} + c$

7. The integral $\int \frac{e^{3\log_e 2x}+5e^{2\log_e 2x}}{e^{4\log_e x}+5e^{3\log_e x}-7e^{2\log_e x}}dx$, $x > 0$, is equal to : (where c is a constant of integration)

(1) $\log_e|x^2+5x-7| + c$

(2) $4\log_e|x^2+5x-7| + c$

(3) $\frac{1}{4}\log_e|x^2+5x-7| + c$

(4) $\log_e\sqrt{x^2+5x-7} + c$

8. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$= \alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)$$

$$+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right) + \delta\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

where C is an arbitrary constant, then the value of $10(\alpha+\beta\gamma+\delta)$ is equal to __________.

9. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to (where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5} + c$

(2) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5} + c$

(3) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5} + c$

(4) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5} + c$

10. If $f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \geq 0), f(0) = 0$ and $f(1) = \frac{1}{K}$, then the value of K is

DEFINITE INTEGRATION

1. Let a be a positive real number such that $\int_0^a e^{x-[x]}\,dx = 10e - 9$ where [x] is the greatest integer less than or equal to x. Then a is equal to :

(1) $10 - \log_e(1 + e)$ (2) $10 + \log_e 2$

(3) $10 + \log_e 3$ (4) $10 + \log_e(1+ e)$

2. The value of the integral $\int_{-1}^{1} \log_e(\sqrt{1-x}+\sqrt{1+x})\,dx$ is equal to :

(1) $\frac{1}{2}\log_e 2+\frac{\pi}{4}-\frac{3}{2}$ (2) $2\log_e 2+\frac{\pi}{4}-1$

(3) $\log_e 2+\frac{\pi}{2}-1$ (4) $2\log_e 2+\frac{\pi}{2}-\frac{1}{2}$

3. Let $g(t)=\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$, where $f(x) = \log_e\left(x+\sqrt{x^2+1}\right)$, $x \in \mathbf{R}$. Then which one of the following is correct ?

(1) $g(1) = g(0)$ (2) $\sqrt{2}g(1)=g(0)$

(3) $g(1)=\sqrt{2}g(0)$ (4) $g(1)+g(0)=0$

4. If $\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{\alpha\pi^3}{1+4\pi^2},\alpha\in\mathbf{R}$ where [x] is the greatest integer less than or equal to x, then the value of α is :

(1) $200\,(1 - e^{-1})$ (2) $100\,(1 - e)$

(3) $50\,(e - 1)$ (4) $150\,(e^{-1}-1)$

5. The value of the definite integral $\int_{\pi/24}^{5\pi/24}\frac{dx}{1+\sqrt[3]{\tan 2x}}$ is :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{18}$

6. Let $f : [0, \infty) \to [0, \infty)$ be defined as

$f(x) = \int_0^x [y]\,dy$

where [x] is the greatest integer less than or equal to x. Which of the following is true?

(1) f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points.

(2) f is both continuous and differentiable except at the integer points in $[0, \infty)$.

(3) f is continuous everywhere except at the integer points in $[0, \infty)$.

(4) f is differentiable at every point in $[0, \infty)$.

7. If $f(x)=\begin{cases}\int_0^x(5+|1-t|)\,dt, & x>2\\ 5x+1, & x\le 2\end{cases}$, then

(1) f(x) is not continuous at x = 2

(2) f(x) is everywhere differentiable

(3) f(x) is continuous but not differentiable at x = 2

(4) f(x) is not differentiable at x = 1

8. The value of the integral $\int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$ is:

(1) 2 (2) 0 (3) –1 (4) 1

9. The value of the definite integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$ is equal to :

(1) $-\frac{\pi}{2}$ (2) $\frac{\pi}{2\sqrt{2}}$ (3) $-\frac{\pi}{4}$ (4) $\frac{\pi}{\sqrt{2}}$

10. Let the domain of the function

$f(x)=\log_4\left(\log_5\left(\log_3\left(18x-x^2-77\right)\right)\right)$ be (a, b).

Then the value of the integral

$\int_a^b\frac{\sin^3 x}{(\sin^3 x+\sin^3(a+b-x))}dx$ is equal to ______.

11. Let f : (a,b) $\rightarrow$ **R** be twice differentiable function such that $f(x) = \int_a^x g(t)\,dt$ for a differentiable function g(x). If $f(x) = 0$ has exactly five distinct roots in (a, b), then $g(x)g'(x) = 0$ has at least :

(1) twelve roots in (a, b) (2) five roots in (a, b)

(3) seven roots in (a, b) (4) three roots in (a, b)

12. If $\int_0^{\pi}(\sin^3 x)e^{-\sin^2 x}\,dx = \alpha - \frac{\beta}{e}\int_0^1 \sqrt{t}\,e^t dt$, then $\alpha + \beta$ is equal to _____ .

13. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{1/2} dx$ is :

(1) $\log_e 4$ (2) $\log_e 16$

(3) $2\log_e 16$ (4) $4\log_e(3+2\sqrt{2})$

14. The value of $\lim\limits_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is :

(1) $\frac{1}{2}\tan^{-1}(2)$ (2) $\frac{1}{2}\tan^{-1}(4)$

(3) $\tan^{-1}(4)$ (4) $\frac{1}{4}\tan^{-1}(4)$

15. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in$ **R**, $5\alpha + 6\beta = 0$, and [x] denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to :

(1) 100 (2) 25 (3) 16 (4) 36

16. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ is

(1) $\frac{\pi}{2}$ (2) $\frac{5\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

17. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \ldots\left(1+\frac{n^2}{n^2}\right)^n$, then $\lim\limits_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ is equal to :

(1) $\frac{e^2}{16}$ (2) $\frac{4}{e}$ (3) $\frac{16}{e^2}$ (4) $\frac{4}{e^2}$

18. $\int_6^{16}\frac{\log_e x^2}{\log_e x^2 + \log_e(x^2-44x+484)}dx$ is equal to :

(1) 6 (2) 8 (3) 5 (4) 10

19. The value of the integral $\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$ is :

(1) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$ (2) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

(3) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$ (4) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$

20. Let [t] denote the greatest integer $\le$ t. Then the value of $8\cdot\int_{-\frac{1}{2}}^{1}([2x]+|x|)\,dx$ is _________.

21. If $x\,\phi(x) = \int_5^x (3t^2 - 2\phi'(t))\,dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is _________.

22. If [x] is the greatest integer $\le$ x, then $\pi^2\int_0^2\left(\sin\frac{\pi x}{2}\right)(x-[x])^{[x]}dx$ is equal to :

(1) $2(\pi - 1)$ (2) $4(\pi - 1)$

(3) $4(\pi + 1)$ (4) $2(\pi + 1)$

23. The function f(x), that satisfies the condition $f(x) = x + \int_0^{\pi/2}\sin x\cdot\cos y\, f(y)\,dy$, is :

(1) $x + \frac{2}{3}(\pi-2)\sin x$ (2) $x + (\pi + 2)\sin x$

(3) $x + \frac{\pi}{2}\sin x$ (4) $x + (\pi - 2)\sin x$

24. The value of the integral, $\int_1^3 [x^2-2x-2]\,dx$, where [x] denotes the greatest integer less than or equal to x, is :

(1) $-\sqrt{2}-\sqrt{3}+1$ (2) $-\sqrt{2}-\sqrt{3}-1$

(3) –5 (4) –4

25. Let f(x) be a differentiable function defined on [0, 2] such that f '(x) = f '(2 – x) for all x ∈ (0, 2), f(0) = 1 and f(2) = e^2. Then the value of $\int_0^2 f(x)\,dx$ is :

(1) $1 - e^2$ (2) $1 + e^2$

(3) $2(1 - e^2)$ (4) $2(1 + e^2)$

26. If $\int_{-a}^{a} (|x|+|x-2|)dx = 22, (a>2)$ and [x] denotes the greatest integer ≤ x, then $\int_{a}^{-a} (x+[x])\,dx$ is equal to _______.

27. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer ≤ t, is :

(1) $\frac{e-1}{3e}$ (2) $\frac{e+1}{3}$ (3) $\frac{e+1}{3e}$ (4) $\frac{1}{3e}$

28. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+........+\frac{n}{(2n-1)^2}\right]$ is equal to :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

29. The value of $\int_{-2}^{2} |3x^2-3x-6|\,dx$ is ______.

30. For x > 0, if $f(x)=\int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e)+f\left(\frac{1}{e}\right)$ is equal to :

(1) 1 (2) –1 (3) $\frac{1}{2}$ (4) 0

31. Let $f(x)=\int_0^x e^t f(t)dt+e^x$ be a differentiable function for all x ∈ R. Then $f(x)$ equals :

(1) $2e^{(e^x-1)}-1$ (2) $e^{e^x}-1$

(3) $2e^{e^x}-1$ (4) $e^{(e^x-1)}$

32. If $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, for m, n ≥ 1 and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha\, I_{m,n}$, $\alpha \in R$, then α equals ____.

33. The value of the integral $\int_0^{\pi} |\sin 2x|\,dx$ is

34. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}dx$ is

(1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

35. If $I_n=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x\,dx$, then :

(1) $\frac{1}{I_2+I_4},\frac{1}{I_3+I_5},\frac{1}{I_4+I_6}$ are in G.P.

(2) $I_2+I_4, I_3+I_5, I_4+I_6$ are in A.P.

(3) $I_2+I_4, (I_3+I_5)^2, I_4+I_6$ are in G.P.

(4) $\frac{1}{I_2+I_4},\frac{1}{I_3+I_5},\frac{1}{I_4+I_6}$ are in A.P.

36. The value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer ≤ x, is

(1) 100(e – 1) (2) 100(1 – e)

(3) 100e (4) 100 (1 + e)

37. Consider the integral

$$I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx,$$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(1) 9(e – 1) (2) 45(e + 1)

(3) 45(e – 1) (4) 9(e + 1)

38. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by (x – 2). Then the value of 9(b + c) is equal to:

(1) 9 (2) 15 (3) 7 (4) 11

39. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) + f(x + 1) = 2$, for all $x \in \mathbb{R}$. If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^3 f(x)dx$, then the value of $I_1 + 2I_2$ is equal to ________.

40. Let $f : R \to R$ be defined as $f(x) = e^{-x} \sin x$. If $F : [0, 1] \to R$ is a differentiable function such that $F(x) = \int_0^x f(t)\,dt$, then the value of $\int_0^1 (F'(x) + f(x))e^x dx$ lies in the interval

(1) $\left[\frac{327}{360}, \frac{329}{360}\right]$ (2) $\left[\frac{330}{360}, \frac{331}{360}\right]$

(3) $\left[\frac{331}{360}, \frac{334}{360}\right]$ (4) $\left[\frac{335}{360}, \frac{336}{360}\right]$

41. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where α, β, γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of $\alpha + \beta + \gamma$ is equal to :

(1) 0 (2) 20 (3) 25 (4) 10

42. Let $I_n = \int_1^e x^{19} (\log|x|)^n dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then α – β equal to _______.

43. Which of the following statements is incorrect for the function g(α) for α ∈ R such that

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^\alpha x}{\cos^\alpha x + \sin^\alpha x} dx$$

(1) g(α) is a strictly increasing function

(2) g(α) has an inflection point at $\alpha = -\frac{1}{2}$

(3) g(α) is a strictly decreasing function

(4) g(α) is an even function

44. Let $f(x)$ and g(x) be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^4 f(x)^2 dx$ is

45. Let $g(x) = \int_0^x f(t)\,dt$, where f is continuous function in [0, 3] such that $\frac{1}{3} \le f(t) \le 1$ for all $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which g(3) lies is :

(1) $\left[-1, -\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2}, -1\right]$

(3) $\left[\frac{1}{3}, 2\right]$ (4) [1, 3]

DIFFERENTIAL EQUATION

1. Let y = y(x) be the solution of the differential equation

$$x \tan\left(\frac{y}{x}\right) dy = \left(y \tan\left(\frac{y}{x}\right) - x\right) dx,$$

$-1 \le x \le 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves $x = 0, x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is:

(1) $\frac{1}{8}(\pi - 1)$ (2) $\frac{1}{12}(\pi - 3)$

(3) $\frac{1}{4}(\pi - 2)$ (4) $\frac{1}{6}(\pi - 1)$

2. Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}dx+\left(\frac{y}{x}\right)dy=0, y(1)=-1$.

Then the value of $(y(3))^2$ is equal to:

(1) $1-4e^3$ (2) $1-4e^6$

(3) $1+4e^3$ (4) $1+4e^6$

3. If [x] denotes the greatest integer less than or equal to x, then the value of the integral

$\int_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$ is equal to :

(1) $-\pi$ (2) π (3) 0 (4) 1

4. If $f:\mathbf{R}\to\mathbf{R}$ is given by $f(x) = x + 1$, then the value of

$\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+.....+f\left(\frac{5(n-1)}{n}\right)\right]$,

is :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{1}{2}$ (4) $\frac{7}{2}$

5. Let a curve y = y(x) be given by the solution of the differential equation $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx=\sqrt{e^{2x}-1}\,dy$

If it intersects y-axis at y = –1, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to ________.

6. Let y = y(x) be the solution of the differential equation $\operatorname{cosec}^2x\,dy + 2dx = (1 + y\cos2x)\operatorname{cosec}^2x\,dx$, with $y\left(\frac{\pi}{4}\right)=0$. Then, the value of $(y(0) + 1)^2$ is equal to :

(1) $e^{1/2}$ (2) $e^{-1/2}$ (3) e^{-1} (4) e

7. Let y = y(x) be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx = (x + 2)\,dy$, y(1) = 1. If the domain of y = y(x) is an open interval (α, β), then $|\alpha + \beta|$ is equal to _______.

8. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=1+xe^{y-x}, -\sqrt{2}<x<\sqrt{2}, y(0)=0$ then, the minimum value of y(x), $x\in\left(-\sqrt{2},\sqrt{2}\right)$ is equal to :

(1) $\left(2-\sqrt{3}\right)-\log_e 2$

(2) $\left(2+\sqrt{3}\right)+\log_e 2$

(3) $\left(1+\sqrt{3}\right)-\log_e\left(\sqrt{3}-1\right)$

(4) $\left(1-\sqrt{3}\right)-\log_e\left(\sqrt{3}-1\right)$

9. Let y = y(x) be solution of the following differential equation

$e^y\frac{dy}{dx}-2e^y\sin x+\sin x\cos^2x=0,\ y\left(\frac{\pi}{2}\right)=0$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha + \beta)$ is equal to ________.

10. Let y = y (x) be the solution of the differential equation $x\,dy = (y + x^3\cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(1) $\frac{\pi^2}{4}+\frac{\pi}{2}$ (2) $\frac{\pi^2}{2}+\frac{\pi}{4}$

(3) $\frac{\pi^2}{2}-\frac{\pi}{4}$ (4) $\frac{\pi^2}{4}-\frac{\pi}{2}$

11. Let a curve y = f(x) pass through the point $(2, (\log_e2)^2)$ and have slope $\frac{2y}{x\log_e x}$ for all positive real value of x. Then the value of f(e) is equal to _______.

12. Let y = y(x) be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right)=3x+4y$, with y(0) = 0.

If $y\left(-\frac{2}{3}\log_e 2\right)=\alpha\log_e 2$, then the value of α is equal to:

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) 2 (4) $-\frac{1}{2}$

13. Let F : [3, 5] → **R** be a twice differentiable function on (3, 5) such that

$$F(x)=e^{-x}\int_3^x (3t^2+2t+4F'(t))\,dt.$$

If $F'(4)=\dfrac{\alpha e^{\beta}-224}{(e^{\beta}-4)^2}$, then $\alpha+\beta$ is equal to _________.

14. If y = y(x), $y\in\left[0,\dfrac{\pi}{2}\right)$ is the solution of the differential equation

$\sec y\dfrac{dy}{dx}-\sin(x+y)-\sin(x-y)=0$, with y(0) = 0,

then $5y'\left(\dfrac{\pi}{2}\right)$ is equal to _________.

15. Let y = y(x) be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, x > 2. If y(3) = 3, then y(4) is equal to :

(1) 4 (2) 12 (3) 8 (4) 16

16. Let y= y(x) be the solution of the differential equation $dy = e^{\alpha x+y}\,dx$; $\alpha \in \mathbf{N}$. If $y(\log_e 2)= \log_e 2$ and $y(0) = \log_e\left(\dfrac{1}{2}\right)$, then the value of α is equal to____.

17. Let y = y(x) be a solution curve of the differential equation $(y + 1)\tan^2x\,dx + \tan x\,dy + y\,dx = 0$, $x\in\left(0,\dfrac{\pi}{2}\right)$. If $\lim\limits_{x\to 0+} xy(x)=1$, then the value of $y\left(\dfrac{\pi}{4}\right)$ is :

(1) $-\dfrac{\pi}{4}$ (2) $\dfrac{\pi}{4}-1$ (3) $\dfrac{\pi}{4}+1$ (4) $\dfrac{\pi}{4}$

18. Let y(x) be the solution of the differential equation $2x^2dy+(e^y-2x)dx=0$, x > 0. If y(e) = 1, then y(1) is equal to :

(1) 0 (2) 2

(3) $\log_e 2$ (4) $\log_e(2e)$

19. Let y = y(x) be the solution of the differential equation $\dfrac{dy}{dx}$ = 2 (y + 2 sin x – 5) x – 2 cosx such that y(0) = 7. Then y(π) is equal to :

(1) $2e^{\pi^2}+5$ (2) $e^{\pi^2}+5$

(3) $3e^{\pi^2}+5$ (4) $7e^{\pi^2}+5$

20. Let us consider a curve, y = f(x) passing through the point (–2, 2) and the slope of the tangent to the curve at any point (x, f(x)) is given by $f(x)+xf'(x)=x^2$. Then :

(1) $x^2 + 2xf(x) - 12 = 0$ (2) $x^3 + xf(x) + 12 = 0$

(3) $x^3 - 3xf(x) - 4 = 0$ (4) $x^2 + 2xf(x) + 4 = 0$

21. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) form the line 3x + 4y = 5, is given by :

(1) $10\dfrac{d^2y}{dx^2}=11$ (2) $11\dfrac{d^2x}{dy^2}=10$

(3) $10\dfrac{d^2x}{dy^2}=11$ (4) $11\dfrac{d^2y}{dx^2}=10$

22. If the solution curve of the differential equation $(2x - 10y^3)\,dy + y\,dx = 0$, passes through the points (0, 1) and (2, β), then β is a root of the equation :

(1) $y^5 - 2y - 2 = 0$ (2) $2y^5 - 2y - 1 = 0$

(3) $2y^5 - y^2 - 2 = 0$ (4) $y^5 - y^2 - 1 = 0$

23. Let f be a non–negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x\sqrt{1-(f'(t))^2}\,dt=\int_0^x f(t)\,dt$, $0\le x\le 1$ and f(0) = 0, then $\lim\limits_{x\to 0}\dfrac{1}{x^2}\int_0^x f(t)\,dt$:

(1) equals 0 (2) equals 1

(3) does not exist (4) equals $\dfrac{1}{2}$

24. If $\frac{dy}{dx}=\frac{2^{x+y}-2^x}{2^y}$, y(0) = 1, then y(1) is equal to :

(1) $\log_2(2+e)$ (2) $\log_2(1+e)$

(3) $\log_2(2e)$ (4) $\log_2(1+e^2)$

25. If $\frac{dy}{dx}=\frac{2^x y+2^y\cdot 2^x}{2^x+2^{x+y}\log_e 2}$, y(0) = 0, then for y = 1, the value of x lies in the interval :

(1) (1, 2) (2) $\left(\frac{1}{2},1\right]$

(3) (2, 3) (4) $\left(0,\frac{1}{2}\right]$

26. If $y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x>0, \phi>0$, and $y(1)=-1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to :

(1) $4\,\phi(2)$ (2) $4\,\phi(1)$

(3) $2\,\phi(1)$ (4) $\phi(1)$

27. If y = y (x) is the solution curve of the differential equation $x^2dy+\left(y-\frac{1}{x}\right)dx=0$; x > 0 and y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{3}{2}-\frac{1}{\sqrt{e}}$ (2) $3+\frac{1}{\sqrt{e}}$

(3) 3 + e (4) 3 – e

28. If a curve y = f(x) passes through the point (1, 2) and satisfies $x\frac{dy}{dx}+y=bx^4$, then for what value of b, $\int_1^2 f(x)dx=\frac{62}{5}$?

(1) 5 (2) 10 (3) $\frac{62}{5}$ (4) $\frac{31}{5}$

29. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt}$ = 0.5P – 450. If P(0) = 850, then the time at which population becomes zero is :

(1) $\log_e 18$ (2) $\log_e 9$

(3) $\frac{1}{2}\log_e 18$ (4) $2\log_e 18$

30. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point:

(1) (5, 4) (2) (4, 5) (3) (4, 4) (4) (5, 5)

31. If the curve, y = y(x) represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, 2x – 3y = 1 and 3x + 2y = 8, then |y(1)| is equal to ______.

32. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line x + 2y = 4 at x = –2, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $\frac{18}{35}$ (2) $-\frac{4}{3}$ (3) $-\frac{18}{19}$ (4) $-\frac{18}{11}$

33. The rate of growth of bacteria in a culture is proportional to the number of bacteris present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4 (2) 8 (3) 2 (4) 16

34. If y = y(x) is the solution of the equaiton $e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x, y(0)=0$;

then $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to

35. The difference between degree and order of a differential equation that represents the family of curves given by $y^2=a\left(x+\frac{\sqrt{a}}{2}\right), a>0$ is

36. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+(\tan x)y=\sin x,\ 0\le x\le\frac{\pi}{3}$, with y(0) = 0, then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\frac{1}{4}\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\log_e 2$ (4) $\frac{1}{2}\log_e 2$

37. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2$, x > 0. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1,1), then the area enclosed by the curves C_1 and C_2 is equal to :

(1) π – 1 (2) $\frac{\pi}{2}-1$ (3) π + 1 (4) $\frac{\pi}{4}+1$

38. If y = y(x) is the solution of the differential equation, $\frac{dy}{dx}+2y\tan x=\sin x, y\left(\frac{\pi}{3}\right)=0$, then the maximum value of the function y(x) over $\mathbb{R}$ is equal to :

(1) 8 (2) $\frac{1}{2}$ (3) $-\frac{15}{4}$ (4) $\frac{1}{8}$

39. Let y = y(x) be the solution of the differential equation cosx (3sinx + cosx + 3)dy = (1 + y sinx (3sinx + cosx + 3))dx, $0\le x\le\frac{\pi}{2}$, y(0) = 0. Then , $y\left(\frac{\pi}{3}\right)$ is equal to:

(1) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (2) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

(3) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$ (4) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

40. If the curve y = y(x) is the solution of the differential equation
$2(x^2+x^{5/4})dy-y(x+x^{1/4})dx=2x^{9/4}dx$, x > 0
which passes through the point $\left(1, 1-\frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to :

(1) $4\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$ (2) $\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$

(3) $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$ (4) $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

41. Which of the following is true for y(x) that satisfies the differential equation
$\frac{dy}{dx}=xy-1+x-y$; $y(0)=0$:

(1) $y(1)=e^{-\frac{1}{2}}-1$ (2) $y(1)=e^{\frac{1}{2}}-e^{-\frac{1}{2}}$

(3) y(1) = 1 (4) $y(1)=e^{\frac{1}{2}}-1$

42. If $[\cdot]$ represents the greatest integer function, then the value of
$\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[[x^2]-\cos x\right]dx\right|$ is ________ .

43. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2-2x\left(\frac{dy}{dx}\right)-y=0$

(2) $y\left(\frac{dy}{dx}\right)^2-2x\left(\frac{dy}{dx}\right)+y=0$

(3) $y\left(\frac{dy}{dx}\right)^2+2x\left(\frac{dy}{dx}\right)-y=0$

(4) $y\left(\frac{dy}{dx}\right)+2x\left(\frac{dy}{dx}\right)-y=0$

44. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=(y+1)\left((y+1)e^{x^2/2}-x\right)$, 0 < x < 2.1, with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{-e^{3/2}}{(e^2+1)^2}$ (2) $-\frac{2e^2}{(1+e^2)^2}$

(3) $\frac{e^{5/2}}{(1+e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2+1)^2}$

45. Let y = y(x) be the solution of the differential equation $xdy-ydx=\sqrt{(x^2-y^2)}\,dx$, $x \geq 1$, with y(1) = 0. If the area bounded by the line x = 1, $x = e^{\pi}$, y = 0 and y = y(x) is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to ____

AREA UNDER THE CURVE

1. Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha+\beta+\gamma|$ is equal to __________.

2. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is _______.

3. The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

(1) $\frac{8}{3}$ (2) $\frac{17}{3}$ (3) $\frac{13}{3}$ (4) $\frac{7}{3}$

4. If the area of the bounded region

$$R=\left\{(x,y): \max\{0,\log_e x\} \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2\right\}$$

is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha+\beta-2\gamma)^2$ is equal to :

(1) 8 (2) 2 (3) 4 (4) 1

5. The area of the region bounded by y – x = 2 and $x^2 = y$ is equal to :-

(1) $\frac{16}{3}$ (2) $\frac{2}{3}$ (3) $\frac{9}{2}$ (4) $\frac{4}{3}$

6. The area of the region

$S = \{(x, y) : 3x^2 \leq 4y \leq 6x + 24\}$ is _________.

7. Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by y = f(x), the x-axis and the lines x = a and x = b, then 4A is equal to ________.

8. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is :

(1) 9 (2) 10 (3) 4 (4) 6

9. If the line y = mx bisects the area enclosed by the lines x = 0, y = 0, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12 m is equal to _______.

10. The area, enclosed by the curves y = sin x + cos x and $y = |\cos x - \sin x|$ and the lines x = 0, $x = \frac{\pi}{2}$, is :

(1) $2\sqrt{2}(\sqrt{2}-1)$ (2) $2(\sqrt{2}+1)$

(3) $4(\sqrt{2}-1)$ (4) $2\sqrt{2}(\sqrt{2}+1)$

11. The area of the region :

$R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$ is :

(1) $11\sqrt{3}$ square units (2) $12\sqrt{3}$ square units

(3) $9\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

12. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :

(1) $24\pi+3\sqrt{3}$ (2) $12\pi-3\sqrt{3}$

(3) $24\pi-3\sqrt{3}$ (4) $12\pi+3\sqrt{3}$

13. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to ______.

14. Let A_1 be the area of the region bounded by the curves y = sinx, y = cosx and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sinx, y = cosx, x-axis and $x = \frac{\pi}{2}$ in the first quadrant. Then,

(1) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

(2) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(3) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

(4) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

15. The area bounded by the lines $y = ||x - 1| - 2|$ is

16. Let the curve y = y(x) be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to _______.

17. Let f : [–3, 1] → R be given as

$$f(x) = \begin{cases} \min\{(x+6), x^2\}, & -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\}, & 0 \le x \le 1. \end{cases}$$

If the area bounded by y = f(x) and x-axis is A, then the value of 6A is equal to ______.

18. The area bounded by the curve $4y^2 = x^2(4 - x)(x - 2)$ is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{16}$

MATRICES

1. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, a ∈ **R** be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

(1) 36 (2) 24 (3) 45 (4) 18

2. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3 × 3. If B = [b_{ij}], then b_{13} is equal to _________.

3. Let y = y(x) satisfies the equation $\frac{dy}{dx} - |A| = 0$, for all x > 0, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$.

If y(π) = π +2, then the value of $y\left(\frac{\pi}{2}\right)$ is :

(1) $\frac{\pi}{2} + \frac{4}{\pi}$ (2) $\frac{\pi}{2} - \frac{1}{\pi}$

(3) $\frac{3\pi}{2} - \frac{1}{\pi}$ (4) $\frac{\pi}{2} - \frac{4}{\pi}$

4. Let $A = \{a_{ij}\}$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then $\det(3\text{Adj}(2A^{-1}))$ is equal to ______.

5. Let A = [a_{ij}] be a real matrix of order 3 × 3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for i = 1, 2, 3. Then, the sum of all the entries of the matrix A^3 is equal to :

(1) 2 (2) 1 (3) 3 (4) 9

6. Let $A=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$. Then the number of 3 × 3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is _______.

7. Let $M=\left\{A=\begin{pmatrix}a&b\\c&d\end{pmatrix}: a,b,c,d\in\{\pm3,\pm2,\pm1,0\}\right\}$.

Define $f: M\to \mathbf{Z}$, as $f(A)=\det(A)$, for all $A\in M$, where **Z** is set of all integers. Then the number of $A\in M$ such that $f(A)=15$ is equal to ________.

8. If $P=\begin{bmatrix}1&0\\ \frac{1}{2}&1\end{bmatrix}$, then P^{50} is:

(1) $\begin{bmatrix}1&0\\25&1\end{bmatrix}$ (2) $\begin{bmatrix}1&50\\0&1\end{bmatrix}$

(3) $\begin{bmatrix}1&25\\0&1\end{bmatrix}$ (4) $\begin{bmatrix}1&0\\50&1\end{bmatrix}$

9. Let $A=\begin{bmatrix}1&2\\-1&4\end{bmatrix}$. If $A^{-1}=\alpha I+\beta A$, $\alpha,\beta\in\mathbf{R}$, I is a 2 × 2 identity matrix, then $4(\alpha-\beta)$ is equal to :

(1) 5 (2) $\frac{8}{3}$ (3) 2 (4) 4

10. Let A and B be two 3 × 3 real matrices such that (A^2-B^2) is invertible matrix. If $A^5=B^5$ and $A^3B^2=A^2B^3$, then the value of the determinant of the matrix A^3+B^3 is equal to :

(1) 2 (2) 4 (3) 1 (4) 0

11. If $A=\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}$ and $M=A+A^2+A^3+\dots+A^{20}$, then the sum of all the elements of the matrix M is equal to______.

12. If $A=\begin{pmatrix}\frac{1}{\sqrt5}&\frac{2}{\sqrt5}\\ \frac{-2}{\sqrt5}&\frac{1}{\sqrt5}\end{pmatrix}$, $B=\begin{pmatrix}1&0\\i&1\end{pmatrix}$, $i=\sqrt{-1}$, and $Q=A^TBA$, then the inverse of the matrix $A\,Q^{2021}A^T$ is equal to :

(1) $\begin{pmatrix}\frac{1}{\sqrt5}&-2021\\2021&\frac{1}{\sqrt5}\end{pmatrix}$ (2) $\begin{pmatrix}1&0\\-2021i&1\end{pmatrix}$

(3) $\begin{pmatrix}1&0\\2021i&1\end{pmatrix}$ (4) $\begin{pmatrix}1&-2021i\\0&1\end{pmatrix}$

13. Let $A=\begin{pmatrix}1&0&0\\0&1&1\\1&0&0\end{pmatrix}$. Then $A^{2025}-A^{2020}$ is equal to :

(1) A^6-A (2) A^5

(3) A^5-A (4) A^6

14. Let A be a 3 × 3 real matrix.

If $\det(2\mathrm{Adj}(2\ \mathrm{Adj}(\mathrm{Adj}(2A))))=2^{41}$, then the value of $\det(A^2)$ equal _____.

15. If the matrix $A=\begin{pmatrix}0&2\\K&-1\end{pmatrix}$ satisfies $A(A^3+3I)=2I$, then the value of K is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) –1 (4) 1

16. The number of elements in the set $\left\{A=\begin{pmatrix}a&b\\0&d\end{pmatrix}: a,b,d\in\{-1,0,1\}\text{ and }(I-A)^3=I-A^3\right\}$, where I is 2 × 2 identity matrix, is :

17. Let $J_{n,m}=\int_0^{\frac{1}{2}}\frac{x^n}{x^m-1}dx$, $\forall\ n>m$ and $n,m\in N$.

Consider a matrix $A=[a_{ij}]_{3\times3}$ where $a_{ij}=\begin{cases}J_{6+i,3}-J_{i+3,3}, & i\le j\\ 0, & i>j\end{cases}$. Then $\left|\mathrm{adj}A^{-1}\right|$ is :

(1) $(15)^2\times2^{42}$ (2) $(15)^2\times2^{34}$

(3) $(105)^2\times2^{38}$ (4) $(105)^2\times2^{36}$

18. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × 1 null matrix, has :

(1) no solution

(2) exactly two solutions

(3) infinitely many solutions

(4) a unique solution

19. Let M be any 3 × 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is _______.

20. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$.

Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _______.

21. Let A be a 3 × 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to :

(1) 16 (2) 80 (3) 128 (4) 64

22. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is :

(1) 4 (2) 2 (3) 3 (4) 1

23. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is_____.

24. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and

$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to ________ .

25. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then β – α is equal to ______.

26. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

(1) 4 (2) 1 (3) 6 (4) 12

27. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2+1)b_2^2 \neq -2b_1b_2$, then the value of k is _______.

28. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$. Then, the system of linear equations $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(1) A unique solution

(2) Infinitely many solutions

(3) No solution

(4) Exactly two solutions

29. The total number of 3 × 3 matrices A having enteries from the set (0, 1, 2, 3) such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

30. If x, y, z are in arithmetic progression with common difference d, x ≠ 3d, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is

(1) 72 (2) 12 (3) 36 (4) 6

31. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B=\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ________.

32. If $A=\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2-\frac{1}{2}I\right)=0$, then a possible value of α is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

33. If $A=\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4)+\det\left(A^{10}-(\text{Adj}(2A))^{10}\right)$ is equal to _________ .

34. Let $A+2B=\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A-B=\begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to

(1) 1 (2) 2 (3) 0 (4) 3

35. Let I be an identity matrix of order 2 × 2 and $P=\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of n ∈ N for which $P^n = 5I - 8P$ is equal to _______.

VECTORS

1. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|, |\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ is :

(1) $\frac{2}{3}$ (2) 4 (3) 3 (4) $\frac{3}{2}$

2. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to _________.

3. In a triangle ABC, if $|\overrightarrow{BC}|=3$, $|\overrightarrow{CA}|=5$ and $|\overrightarrow{BA}|=7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to

(1) $\frac{19}{2}$ (2) $\frac{13}{2}$ (3) $\frac{11}{2}$ (4) $\frac{15}{2}$

4. For p > 0, a vector $\vec{v}_2=2\hat{i}+(p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1=\sqrt{3}p\hat{i}+\hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta=\frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to _________.

5. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$, and $|\vec{a}|=\sqrt{10}$. Then a possible value of $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{b}\ \vec{d}]+[\vec{a}\ \vec{c}\ \vec{d}]$ is equal to :

(1) – 42 (2) – 40 (3) – 29 (4) – 38

6. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following is **not** true ?

(1) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))=\vec{0}$

(2) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2

(3) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=8$

(4) $|3\vec{a}+\vec{b}-2\vec{c}|^2=51$

7. Let the vectors

$(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k}$,

$(1+b)\hat{i}+2b\hat{j}-b\hat{k}$ and $(2+b)\hat{i}+2b\hat{j}+(1-b)\hat{k}$ a, b, c, $\in \mathbf{R}$

be co-planar. Then which of the following is true?

(1) 2b = a + c (2) 3c = a + b

(3) a = b + 2c (4) 2a = b + c

8. Let $\vec{p}=2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}+2\hat{j}+\hat{k}$ be two vectors. If a vector $\vec{r}=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$, and $|\vec{r}|=\sqrt{3}$, then $|\alpha|+|\beta|+|\gamma|$ is equal to _______.

9. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co-planar, then c is equal to:

(1) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ (2) $\frac{a+b}{2}$

(3) $\frac{1}{a}+\frac{1}{b}$ (4) $\sqrt{ab}$

10. If $|\vec{a}|=2, |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$, then $|\vec{a}\cdot\vec{b}|$ is equal to :

(1) 6 (2) 4 (3) 3 (4) 5

11. If $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is ________.

12. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product $(\vec{a}+\vec{b})\times((\vec{a}\times((\vec{a}-\vec{b})\times\vec{b}))\times\vec{b})$ is equal to :

(1) $5(34\hat{i}-5\hat{j}+3\hat{k})$ (2) $7(34\hat{i}-5\hat{j}+3\hat{k})$

(3) $7(30\hat{i}-5\hat{j}+7\hat{k})$ (4) $5(30\hat{i}-5\hat{j}+7\hat{k})$

13. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}$ and $\vec{c}=\hat{j}-\hat{k}$ be three vectors such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}\cdot\vec{b}=1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a}\times\vec{c}$ is l, then the value of $3l^2$ is equal to _______.

14. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}=\vec{b}\times(\vec{b}\times\vec{c})$. If magnitudes of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is θ $\left(0<\theta<\frac{\pi}{2}\right)$, then the value of $1+\tan\theta$ is equal to :

(1) $\sqrt{3}+1$ (2) 2

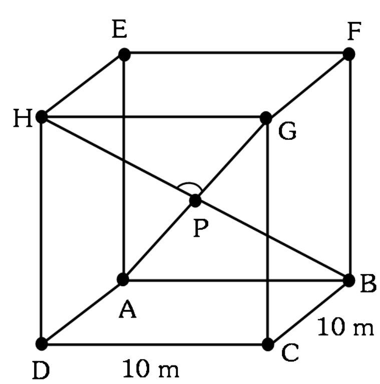
(3) 1 (4) $\frac{\sqrt{3}+1}{\sqrt{3}}$

15. Let $\vec{a}=\hat{i}-\alpha\hat{j}+\beta\hat{k}$, $\vec{b}=3\hat{i}+\beta\hat{j}-\alpha\hat{k}$ and $\vec{c}=-\alpha\hat{i}-2\hat{j}+\hat{k}$, where α and β are integers. If $\vec{a}\cdot\vec{b}=-1$ and $\vec{b}\cdot\vec{c}=10$, then $(\vec{a}\times\vec{b})\cdot\vec{c}$ is equal to____ .

16. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$, then $\vec{a}.(\vec{b}\times\vec{c})$ is equal to :

(1) –2 (2) –6 (3) 6 (4) 2

17. A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :



(1) 5 (2) $2\sqrt{10}$ (3) $5\sqrt{3}$ (4) $5\sqrt{2}$

18. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to ______.

19. Let $\vec{a}=\hat{i}+5\hat{j}+\alpha\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _______.

20. The equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to the x-axis is :

(1) $\vec{r}.(\hat{j}-3\hat{k})+6=0$ (2) $\vec{r}.(\hat{i}+3\hat{k})+6=0$

(3) $\vec{r}.(\hat{i}-3\hat{k})+6=0$ (4) $\vec{r}.(\hat{j}-3\hat{k})-6=0$

21. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}|=|3\vec{a}+\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 4 (2) 6 (3) 5 (4) 8

22. Let $\vec{a},\vec{b},\vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude.
If a vector $\vec{r}$ satisfies. $\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$, then $\vec{r}$ is equal to :

(1) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$ (2) $\frac{1}{3}(2\vec{a}+\vec{b}-\vec{c})$

(3) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$ (4) $\frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$

23. Let $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i}+2\hat{j}-\hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to _____.

24. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is :

(1) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$

(2) $\vec{r}.(3\hat{i}+7\hat{j}+3\hat{k})=7$

(3) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=7$

(4) $\vec{r}.(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

25. Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c}=7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+\hat{k}$, then the value of $2|\vec{a}+\vec{b}+\vec{c}|^2$ is ______.

26. Let $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}\cdot\vec{b}=0$, then $\vec{r}\cdot\vec{a}$ is equal to _________.

27. Let $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to ______:

28. If vectors $\vec{a}_1=x\hat{i}-\hat{j}+\hat{k}$ and $\vec{a}_2=\hat{i}+y\hat{j}+z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i}+y\hat{j}+z\hat{k}$ is

(1) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (2) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$

(3) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (4) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$

29. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4\vec{b}$

(3) $\vec{a}\times\vec{b}$ (4) $|\vec{a}|^4\vec{b}$

30. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}.\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=-1$, $\alpha\in R$, then the value of $\alpha+|\vec{r}|^2$ is equal to :

(1) 9 (2) 15 (3) 13 (4) 11

31. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

If $\vec{c}.\left(\hat{i}+\hat{j}+3\hat{k}\right)=8$ then the value of $\vec{c}.\left(\vec{a}\times\vec{b}\right)$ is equal to _________.

32. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

33. Let O be the origin. Let $\overrightarrow{OP}=x\hat{i}+y\hat{j}-\hat{k}$ and $\overrightarrow{OQ}=-\hat{i}+2\hat{j}+3x\hat{k}$, x, y $\in$ R, x > 0, be such that $|\overrightarrow{PQ}|=\sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR}=3\hat{i}+z\hat{j}-7\hat{k}$, z $\in$ R, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2+y^2+z^2$ is equal to

(1) 7 (2) 9 (3) 2 (4) 1

34. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i}+2\hat{j}-\hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to ________.

35. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=7\hat{i}+\hat{j}-6\hat{k}$.

If $\vec{r}\times\vec{a}=\vec{r}\times\vec{b}$, $\vec{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right)=-3$, then $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ is equal to :

(1) 12 (2) 8 (3) 13 (4) 10

36. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

$\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

$\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

such that $\vec{a}\cdot\vec{b}=1$ and $\vec{b}\cdot\vec{c}=-3$, then $\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to _______.

37. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to:

(1) 1 (2) $-\frac{5}{4}$ (3) $\frac{4}{5}$ (4) –1

38. In a triangle ABC, if $|\overrightarrow{BC}|=8$, $|\overrightarrow{CA}|=7$, $|\overrightarrow{AB}|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

39. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a}\times\vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

40. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)-b=0$ be (–3, 5, 2). Then the value of |a + b| is equal to _______.

3D

1. Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). Let a vector $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\vec{a}\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)=2$, then $(\alpha-\beta+\gamma)^2$ equals _______.

2. If the shortest distance between the lines $\vec{r_1}=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$, $\lambda \in \mathbf{R}$, $\alpha > 0$ and $\vec{r_2}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$, $\mu \in \mathbf{R}$ is 9, then α is equal to ________.

3. The lines x = ay – 1 = z – 2 and x = 3y – 2 = bz – 2, (ab ≠ 0) are coplanar, if :

(1) b = 1, a ∈ R – {0}

(2) a = 1, b ∈ R – {0}

(3) a = 2, b = 2

(4) a = 2, b = 3

4. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, –1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ?

(1) (–1, 1, 2) (2) (1, 1, 1)

(3) (1, 1, 2) (4) (1, 2, 2)

5. Let L be the line of intersection of planes $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=2$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)=2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point (1,2,0), then the value of $35(\alpha+\beta+\gamma)$ is equal to :

(1) 101 (2) 119 (3) 143 (4) 134

6. If the shortest distance between the straight lines 3(x – 1) = 6(y – 2) = 2(z – 1) and 4(x – 2) = 2(y – λ) = (z – 3), λ ∈ **R** is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

(1) 3 (2) 2 (3) 5 (4) –1

7. Let the foot of perpendicular from a point P(1, 2, –1) to the straight line $L:\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane x + y + 2z = 0 which meets L at point Q. If α is the acute angle between the lines PN and PQ, then cosα is equal to _________.

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2\sqrt{3}}$

8. If the lines $\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$ are co-planar, then the value of k is _______.

9. Let the plane passing through the point (–1, 0, –2) and perpendicular to each of the planes 2x + y – z = 2 and x – y – z = 3 be ax + by + cz + 8 = 0. Then the value of a + b + c is equal to :

(1) 3 (2) 8 (3) 5 (4) 4

10. For real numbers α and β, consider the following system of linear equations :

x + y – z = 2, x + 2y + αz = 1, 2x – y + z = β.

If the system has infinite solutions, then α + β is equal to ________

11. Let a plane P pass through the point (3, 7, –7) and contain the line, $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to _________.

12. For real numbers α and β ≠ 0, if the point of intersection of the straight lines $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$, lies on the plane x + 2y – z = 8, then α – β is equal to :

(1) 5 (2) 9 (3) 3 (4) 7

13. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points. Q(3, –4, –5) and R(2, –3, 1) and the plane 2x + y + z = 7, is equal to______.

14. A plane P contains the line

x + 2y + 3z + 1 = 0 = x – y – z – 6, and is perpendicular to the plane –2x + y + z + 8 = 0. Then which of the following points lies on P ?

(1) (–1, 1, 2) (2) (0, 1, 1)

(3) (1, 0, 1) (4) (2, –1, 1)

15. Let the line L be the projection of the line $\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$ in the plane x – 2y – z = 3. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to _______.

16. Let P be the plane passing through the point (1,2,3) and the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+4\hat{k})=16$ and $\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=6$.

Then which of the following points does **NOT** lie on P ?

(1) (3, 3, 2) (2) (6, –6, 2)

(3) (4, 2, 2) (4) (–8, 8, 6)

17. Let Q be the foot of the perpendicular from the point P(7,–2,13) on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$.

Then $(PQ)^2$, is equal to _______.

18. The distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to a line, whose direction ratios are 2, 3, –6 is :

(1) 3 (2) 5 (3) 2 (4) 1

19. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes x – y – z – 1 = 0 and 2x + y – 3z + 4 = 0, is :

(1) 3x – y – 5z + 2 = 0 (2) 3x – 4z + 3 = 0

(3) –x + 2y + 2z – 3 = 0 (4) 4x – y – 5z + 2 = 0

20. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is :

(1) $\frac{\pi}{2}$ (2) $\pi-\cos^{-1}\left(\frac{4}{9}\right)$

(3) $\cos^{-1}\left(\frac{8}{9}\right)$ (4) $\frac{\pi}{3}$

21. Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane 2x – y + z + 3 = 0 and let R (3, 5, γ) be a point of this plane. Then the square of the length of the line segment SR is _________.

22. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes 3x – 2y + 4z – 7 = 0 and x + 5y – 2z + 9 = 0, be αx + βy + γz + 3 = 0, then α + β + γ is equal to :

(1) –23 (2) – 15 (3) 23 (4) 15

23. The square of the distance of the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and the plane 2x – y + z = 6 from the point (–1, –1, 2) is _________.

24. The distance of the point (–1, 2, –2) from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{5}{2}$ (3) $\frac{\sqrt{42}}{2}$ (4) $\frac{\sqrt{34}}{2}$

25. Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. Then $(\alpha + \beta)$ is equal to ____.

26. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P ?

(1) $\left(3,1,-\frac{1}{2}\right)$ (2) $\left(-2,0,-\frac{1}{2}\right)$

(3) (0, 2, –4) (4) (4, 0, – 2)

27. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point (2, – 1, 6) is :

(1) $\sqrt{26}$ (2) $2\sqrt{5}$ (3) $2\sqrt{6}$ (4) $4\sqrt{2}$

28. Let a, b $\in$ R. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, –a –9), then $|a + b|$ is equal to :

(1) 88 (2) 86 (3) 84 (4) 90

29. Let λ be an interger. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _______.

30. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is

(1) $3x - 10y - 2z + 11 = 0$

(2) $6x - 5y - 2z - 2 = 0$

(3) $11x + y + 17z + 38 = 0$

(4) $6x - 5y + 2z + 10 = 0$

31. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane $x + y + z = 17$ is :

(1) $2\sqrt{19}$ (2) $19\sqrt{2}$

(3) 38 (4) $\sqrt{38}$

32. The equation of the line through the point (0,1,2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is :

(1) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$ (2) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$

(3) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$ (4) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

33. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{8}$ (3) $\frac{5}{8}$ (4) $\frac{1}{2}$

34. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length :

(1) $\sqrt{\frac{2}{7}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{5}}$

35. A line 'l' passing through origin is perpendicular to the lines

$l_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

If the co-ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c), then 18(a + b + c) is equal to ______.

36. Let L be a line obtained from the intersection of two planes x + 2y + z = 6 and y + 2z = 4. If point P(α, β, γ) is the foot of perpendicular from (3, 2, 1) on L, then the value of 21(α + β + γ) equals :

(1) 142 (2) 68 (3) 136 (4) 102

37. If the mirror image of the point (1, 3, 5) with respect to the plane 4x – 5y + 2z = 8 is (α,β, γ), then 5(α + β + γ) equals :

(1) 47 (2) 43 (3) 39 (4) 41

38. Consider the three planes

$P_1 : 3x + 15y + 21z = 9$,

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true ?

(1) P_1 and P_2 are parallel

(2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel

(4) P_1,P_2 and P_3 all are parallel

39. Let (λ, 2, 1) be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular to the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to

40. If (1,5,35), (7,5,5), (1,λ,7) and (2λ,1,2) are coplanar, then the sum of all possible values of λ is

(1) $\frac{39}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $-\frac{44}{5}$

41. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

(1) 0 (2) 3 (3) 39 (4) –45

42. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{1}{\sqrt{3}}$

43. If the distance of the point (1, –2, 3) from the plane x + 2y – 3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to ______.

44. If for a > 0, the feet of perpendiculars from the points A(a, –2a, 3) and B(0, 4, 5) on the plane $lx + my + nz = 0$ are points C(0, –a, –1) and D respectively, then the length of line segment CD is equal to :

(1) $\sqrt{31}$ (2) $\sqrt{41}$ (3) $\sqrt{55}$ (4) $\sqrt{66}$

45. Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, –2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{482}$ (2) $\sqrt{171}$ (3) $\sqrt{5}$ (4) $\sqrt{227}$

46. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio k : 1 then the value of k is equal to :

(1) 1.5 (2) 3 (3) 2 (4) 4

47. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ and containing the line $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to :

(1) 20 (2) 19 (3) 18 (4) 21

48. Let P be an arbitrary point having sum of the squares of the distance from the planes $x + y + z = 0$, $lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $l - n$ is equal to _______.

49. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is :

(1) $x + 3z = 10$ (2) $x + 3z = 0$

(3) $3x + z = 6$ (4) $3x - z = 0$

50. If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point (–2, 1, 3) is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is _____ .

51. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points (4,–3,1) and (2, 3, –5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

52. The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point (1, 2, 3) is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is

53. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point (1, –1, α) lies on the plane P, then the value of $|5\alpha|$ is equal to ______.

COMPLEX NUMBER

1. If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is : (Here arg(z) denotes the principal argument of complex number z)

(1) $\frac{\pi}{4}$ (2) $-\frac{3\pi}{4}$ (3) $-\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

2. If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^{\theta} \sin x\, dx$ is equal to :

(1) 1 (2) 2 (3) –1 (4) 0

3. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is equal to

(1) 1 (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) 2

4. Let

$$S = \left\{ n \in \mathbf{N} \middle| \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a,b,c,d \in \mathbf{R} \right\},$$

where $i = \sqrt{-1}$. Then the number of 2–digit numbers in the set S is ______.

5. The equation of a circle is $\text{Re}(z^2) + 2(\text{Im}(z))^2 + 2\text{Re}(z) = 0$, where $z = x + iy$. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to ____.

6. Let C be the set of all complex numbers. Let

$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$,

$S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ and

$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$.

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to

(1) 1 (2) 0

(3) 2 (4) Infinite

7. Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z-2| \le 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \ge 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to :

(1) $\frac{3+2\sqrt{2}}{4}$ (2) $\frac{5+2\sqrt{2}}{2}$

(3) $\frac{3+2\sqrt{2}}{2}$ (4) $\frac{5+2\sqrt{2}}{4}$

8. If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to______.

9. The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with:

(1) centre at (0, –1) and radius $\sqrt{2}$

(2) centre at (0, 1) and radius $\sqrt{2}$

(3) centre at (0, 0) and radius $\sqrt{2}$

(4) centre at (0, 1) and radius 2

10. Let $z = \frac{1-i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of $21 + \left(z+\frac{1}{z}\right)^3 + \left(z^2+\frac{1}{z^2}\right)^3 + \left(z^3+\frac{1}{z^3}\right)^3 + ... + \left(z^{21}+\frac{1}{z^{21}}\right)^3$ is_______.

11. If $(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation :

(1) $x^2 - (\sqrt{3}-1)x - \sqrt{3} = 0$

(2) $x^2 + (\sqrt{3}+1)x + \sqrt{3} = 0$

(3) $x^2 + (\sqrt{3}-1)x - \sqrt{3} = 0$

(4) $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

12. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$ is a positive integer, is______.

13. If $S = \left\{z \in \mathbb{C} : \frac{z-i}{z+2i} \in \mathbb{R}\right\}$, then :

(1) S contains exactly two elements

(2) S contains only one element

(3) S is a circle in the complex plane

(4) S is a straight line in the complex plane

14. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to _________.

15. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to _________.

16. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is :

(1) $2\sqrt{2} - 1$ (2) $3\sqrt{2}$

(3) $6\sqrt{2}$ (4) $2\sqrt{2}$

17. If for the complex numbers z satisfying $|z - 2 - 2i| \le 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to ________.

18. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and $n = [|k|]$ be the greatest integral part of $|k|$.

Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to________.

19. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ____

20. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is:

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $3\sqrt{2}$ (4) $\frac{3}{2\sqrt{2}}$

21. Let z be those complex numbers which satisfy $|z + 5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10, i = \sqrt{-1}$.

If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _______.

22. The sum of 162[th] power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

23. The least value of |z| where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \ge \log_{\sqrt{2}}\left|5\sqrt{7} + 9i\right|,$$

$i = \sqrt{-1}$, is equal to :

(1) 3 (2) $\sqrt{5}$ (3) 2 (4) 8

24. Let a complex number z, $|z| \ne 1$,

satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \le 2$. Then, the largest value of |z| is equal to _____ .

(1) 8 (2) 7 (3) 6 (4) 5

25. Let

$$P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix} \text{ and } A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$$

Where $\omega = \frac{-1 + i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to ______ .

26. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which w^n is real, is equal to _____.

27. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z - 1| \le \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \text{Re}((1 - i)z) \ge 1\}$

$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

(1) is a singleton

(2) has exactly two elements

(3) has infinitely many elements

(4) has exactly three elements

28. The area of the triangle with vertices A(z), B(iz) and C (z + iz) is :

(1) 1 (2) $\frac{1}{2}|z|^2$

(3) $\frac{1}{2}$ (4) $\frac{1}{2}|z + iz|^2$

29. If the equation $a|z|^2 + \overline{\bar{\alpha}z + \alpha\bar{z}} + d = 0$ represents a circle where a,d are real constants then which of the following condition is correct?

(1) $|\alpha|^2 - ad \ne 0$

(2) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(3) $|\alpha|^2 - ad \ge 0$ and $a \in R$

(4) $\alpha = 0$, $a,d \in R^+$

30. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1,z_2 form an equilateral triangle with origin. Then, the value of |a| is

31. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

32. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then P(l) is equal to______.

PROBABILITY

1. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :

(1) $\frac{1}{66}$ (2) $\frac{1}{11}$ (3) $\frac{1}{9}$ (4) $\frac{2}{11}$

2. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$, is :

(1) $\frac{7}{36}$ (2) $\frac{2}{9}$ (3) $\frac{1}{6}$ (4) $\frac{1}{4}$

3. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is :

(1) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

(2) greater than $\frac{1}{2}$

(3) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(4) exactly equal to $\frac{1}{2}$

4. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :

(1) $\frac{45}{162}$ (2) $\frac{23}{81}$ (3) $\frac{22}{81}$ (4) $\frac{43}{162}$

5. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability than B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

(1) $\{x \in \mathbf{R} : |x - 3| < 1\}$
(2) $\{x \in \mathbf{R} : |x - 2| \le 1\}$
(3) $\{x \in \mathbf{R} : |x - 1| < 1\}$
(4) $\{x \in \mathbf{R} : |x - 5| \le 1\}$

6. Let x be a random variable such that the probability function of a distribution is given by $P(X = 0) = \frac{1}{2}$, $P(X = j) = \frac{1}{3^j}$ (j = 1, 2, 3, ..., ∞). Then the mean of the distribution and P(X is positive and even) respectively are :

(1) $\frac{3}{8}$ and $\frac{1}{8}$ (2) $\frac{3}{4}$ and $\frac{1}{8}$

(3) $\frac{3}{4}$ and $\frac{1}{9}$ (4) $\frac{3}{4}$ and $\frac{1}{16}$

7. A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is _______.

8. The probability that a randomly selected 2-digit number belongs to the set $\{n \in N : (2^n - 2)$ is a multiple of 3$\}$ is equal to

(1) $\frac{1}{6}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

9. A student appeared in an examination consisting of 8 true–false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is :

(1) 5 (2) 6 (3) 3 (4) 4

10. Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$. The largest value of p, for which P (exactly one of A, B occurs) $= \frac{5}{9}$, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$ (3) $\frac{4}{9}$ (4) $\frac{5}{12}$

11. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is :

(1) $\frac{125}{216}$ (2) $\frac{11}{36}$ (3) $\frac{5}{6}$ (4) $\frac{25}{36}$

12. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :

(1) $p = \frac{1}{6}$ and $q = \frac{1}{36}$ (2) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(3) $p = \frac{5}{6}$ and $q = \frac{1}{36}$ (4) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

13. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6} - x$ and its opposite face occurs with probability $\frac{1}{6} + x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is:

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{1}{9}$ (4) $\frac{1}{12}$

14. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

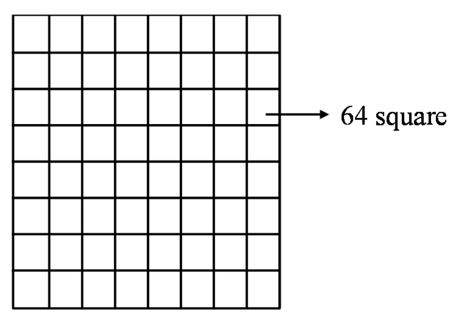
(1) $\frac{1}{8}$ (2) $\frac{5}{8}$ (3) $\frac{5}{16}$ (4) 1

15. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to _________.

16. Let S = {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3) = 2g(1)$ is :

(1) $\frac{1}{10}$ (2) $\frac{1}{15}$ (3) $\frac{1}{5}$ (4) $\frac{1}{30}$

17. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is :



(1) $\frac{2}{7}$ (2) $\frac{1}{18}$ (3) $\frac{1}{7}$ (4) $\frac{1}{9}$

18. Let X be a random variable with distribution.

x	–2	–1	3	4	6
P(X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then 100 σ^2 is equal to :

19. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

(1) $\frac{65}{2^7}$ (2) $\frac{65}{2^8}$ (3) $\frac{135}{2^9}$ (4) $\frac{35}{2^7}$

20. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{1}{32}$ (2) $\frac{5}{16}$ (3) $\frac{3}{16}$ (4) $\frac{1}{2}$

21. Let B_i (i = 1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0,1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to______.

22. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

(1) $\frac{1}{27}$ (2) $\frac{3}{4}$ (3) $\frac{1}{8}$ (4) $\frac{3}{8}$

23. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:

(1) $\frac{1}{72}$ (2) $\frac{5}{216}$ (3) $\frac{1}{36}$ (4) $\frac{1}{54}$

24. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is:

(1) $\frac{7}{45}$ (2) $\frac{14}{45}$ (3) $\frac{28}{45}$ (4) $\frac{8}{45}$

25. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

(1) $\frac{2}{9}$ (2) $\frac{122}{297}$ (3) $\frac{97}{297}$ (4) $\frac{1}{5}$

26. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$ (2) $\frac{1}{7}$ (3) $\frac{3}{7}$ (4) $\frac{4}{7}$

27. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^8}$ (4) $\frac{15}{2^{14}}$

28. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

(1) $\frac{9}{56}$ (2) $\frac{4}{9}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

29. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$ (2) $\frac{52}{867}$ (3) $\frac{39}{50}$ (4) $\frac{22}{425}$

30. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

31. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)\ p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to _____.

32. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{32}{625}$ (2) $\frac{80}{243}$ (3) $\frac{40}{243}$ (4) $\frac{128}{625}$

STATISTICS

1. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:

(1) 10, 11 (2) 3, 18 (3) 8, 13 (4) 1, 20

2. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a - b|$ is equal to :

(1) 9 (2) 11 (3) 7 (4) 1

3. Consider the following frequency distribution :

Class :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to _______.

4. Consider the following frequency distribution :

class :	10–20	20–30	30–40	40–50	50–60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to _________.

5. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is :

(1) 8 (2) 6 (3) 4 (4) 5

6. If the mean and variance of the following data : 6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a - b)^2$ is equal to :

(1) 24 (2) 12 (3) 32 (4) 16

7. Let the mean and variance of the frequency distribution

x :	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f :	4	4	α	β

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be :

(1) 4 (2) 5 (3) $\frac{17}{3}$ (4) $\frac{16}{3}$

8. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :

(1) (11, 26) (2) (10.5, 25)
(3) (11, 25) (4) (10.5, 26)

9. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x – y is _______.

10. Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to ________.

11. The probability distribution of random variable X is given by :

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p = P(1 < X < 4 \mid X < 3)$. If $5p = \lambda K$, then λ equal to ________.

12. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to _________.

13. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :

(1) $\frac{92}{5}$ (2) $\frac{134}{5}$ (3) $\frac{536}{25}$ (4) $\frac{112}{5}$

14. If the variance of 10 natural numbers 1, 1, 1,..., 1, k is less than 10, then the maximum possible value of k is ______.

15. Let $X_1, X_2, ..., X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i - \alpha) = 36$ and $\sum_{i=1}^{18}(X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ______.

16. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _________.

17. Consider three observations a, b and c such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d, then which of the following is true ?

(1) $b^2 = 3(a^2 + c^2) + 9d^2$
(2) $b^2 = a^2 + c^2 + 3d^2$
(3) $b^2 = 3(a^2 + c^2 + d^2)$
(4) $b^2 = 3(a^2 + c^2) - 9d^2$

18. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ______.

19. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is_____.

20. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 425 (2) 650 (3) 250 (4) 925

MATHEMATICAL REASONING

1. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $p \Rightarrow q$

(3) $\sim q \Rightarrow p$ (4) $p \Rightarrow \sim q$

2. Consider the following three statements :

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 = 17.

Then, which of the following statements is correct ?

(1) (A) is false, but (B) and (C) are true

(2) (A) and (C) are true while (B) is false

(3) (A) is true while (B) and (C) are false

(4) (A) and (B) are false while (C) is true

3. Which of the following Boolean expressions is **not** a tautology ?

(1) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(2) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

(3) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

(4) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

4. The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) ~q (2) q (3) p (4) ~p

5. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following:

(1) The match will not be played and weather is not good and ground is wet.

(2) If the match will not be played, then either weather is not good or ground is wet.

(3) The match will be played and weather is not good or ground is wet.

(4) The match will not be played or weather is good and ground is not wet.

6. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to:

(1) $P \vee Q$

(2) $P \wedge \sim Q$

(3) $\sim (P \Rightarrow Q)$

(4) $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

7. Which of the following is the negation of the statement "for all M > 0, there exists $x \in S$ such that $x \geq M$" ?

(1) there exists M > 0, such that x < M for all $x \in S$

(2) there exists $M > 0$, there exists $x \in S$ such that $x \geq M$

(3) there exists $M > 0$, there exists $x \in S$ such that $x < M$

(4) there exists $M > 0$, such that $x \geq M$ for all $x \in S$

8. If the truth value of the Boolean expression $((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)) \rightarrow (p \wedge q)$ is false, then the truth values of the statements p, q, r respectively can be :

(1) T F T (2) F F T

(3) T F F (4) F T F

9. Consider the two statements :

$(S1) : (p \rightarrow q) \vee (\sim q \rightarrow p)$ is a tautology.

$(S2) : (p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

Then :

(1) only (S1) is true.

(2) both (S1) and (S2) are false.

(3) both (S1) and (S2) are true.

(4) only (S2) is true.

10. The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is :

(1) a tautology (2) equivalent to $p \rightarrow \sim r$

(3) a fallacy (4) equivalent to $q \rightarrow \sim r$

11. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to :

(1) $(p \wedge q) \Rightarrow (r \wedge q)$ (2) $(q \wedge r) \Rightarrow (p \wedge q)$

(3) $(p \wedge q) \Rightarrow (r \vee q)$ (4) $(p \wedge r) \Rightarrow (p \wedge q)$

12. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

(1) $* = \vee$, $\square = \vee$ (2) $* = \wedge$, $\square = \wedge$

(3) $* = \wedge$, $\square = \vee$ (4) $* = \vee$, $\square = \wedge$

13. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is :

(1) $p \wedge \sim q \wedge \sim r$ (2) $\sim p \wedge q \wedge \sim r$

(3) $\sim p \wedge q \wedge r$ (4) $p \wedge q \wedge r$

14. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

(1) $\sim (q \rightarrow p)$ (2) $\sim p \rightarrow \sim q$

(3) $\sim (p \rightarrow \sim q)$ (4) $\sim (p \rightarrow q)$

15. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.

(2) (a) and (b) both are tautologies.

(3) (a) is a tautology but not (b).

(4) (b) is a tautology but not (a).

16. The statement among the following that is a tautology is :

(1) $A \vee (A \wedge B)$

(2) $A \wedge (A \vee B)$

(3) $B \rightarrow [A \wedge (A \rightarrow B)]$

(4) $[A \wedge (A \rightarrow B)] \rightarrow B$

17. The contrapositive of the statement "If you will work, you will earn money" is :

(1) You will earn money, if you will not work

(2) If you will earn money, you will work

(3) If you will not earn money, you will not work

(4) To earn money, you need to work

18. Let $F_1(A,B,C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

(1) F_1 and F_2 both are tautologies

(2) F_1 is a tautology but F_2 is not a tautology

(3) F_1 is not tautology but F_2 is a tautology

(4) Both F_1 and F_2 are not tautologies

19. The negative of the statement $\sim p \wedge (p \vee q)$ is

(1) $\sim p \vee q$ (2) $p \vee \sim q$

(3) $\sim p \wedge q$ (4) $p \wedge \sim q$

20. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to :

(1) $A \rightarrow (A \wedge B)$ (2) $A \rightarrow (A \rightarrow B)$

(3) $A \rightarrow (A \leftrightarrow B)$ (4) $A \rightarrow (A \vee B)$

21. Which of the following Boolean expression is a tautology ?

(1) $(p \wedge q) \vee (p \vee q)$ (2) $(p \wedge q) \vee (p \rightarrow q)$

(3) $(p \wedge q) \wedge (p \rightarrow q)$ (4) $(p \wedge q) \rightarrow (p \rightarrow q)$

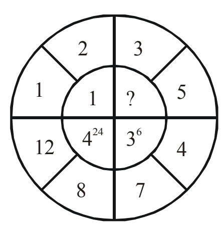
22. If the Boolean expression $(p \wedge q) \circledast (p \otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \vee$ (3) $\vee, \rightarrow$ (4) $\wedge, \rightarrow$

23. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $\sim q \Rightarrow p$ (3) $p \Rightarrow \sim q$ (4) $p \Rightarrow q$

24. The missing value in the following figure is



25. If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

ANSWER KEY

LOGARITHM

Q.No.	1	2
Ans.	1	2

COMPOUND ANGLE

Q.No.	1	2	3	4	5	6	7	8	9
Ans.	2	3	1	3	2	1	1	4	4

QUADRATIC EQUATION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	3	1	1	1	13	3	2	66	18	4	4	1	4	1
Q.No.	15	16	17	18	19	20	21	22	23					
Ans.	2	4	1	2	1	324	1	5	1					

SEQUENCE & PROGRESSION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	4	7	1	1251	1	3	3	832	NTA (4) ALLEN Bonus	2	2021	4	2	2
Q.No.	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Ans.	398	3	5143	305	1	2	4	3	4	9	2	10	3	1
Q.No.	29	30	31	32	33	34	35	36						
Ans.	14	16	3	2	2	2	4	210						

TRIGONOMETRIC EQUATION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	3	4	3	1	56	2	4	2	11	1	2	2	1

SOLUTION OF TRIANGLE

Q.No.	1	2	3
Ans.	3	2	15

HEIGHT & DISTANCE

Q.No.	1	2	3	4	5	6	7	8
Ans.	2	1	3	2	3	4	2	2

DETERMINANT

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	1	2	4	1	2	6	2	5	2	1	4	3	2	3
Q.No.	15	16	17	18	19	20	21	22	23	24	25			
Ans.	4	21	4	2	4	2	3	4	4	2	1			

STRAIGHT LINE

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	9	3	3	2	2	3	3	4	1	3	6	4	56	4
Q.No.	15	16	17	18	19	20	21	22	23					
Ans.	4	3	2	144	2	904	2	1	1					

CIRCLE

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	3	4	3	4	3	2	16	1	61	2	40	165	18	BONUS
Q.No.	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Ans.	3	2	1	9	2	3	1	2	3	1	2	1	3	2
Q.No.	29													
Ans.	3													

PARABOLA

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	1	34	2	9	2	4	3	1	2	3	2	2	4	3
Q.No.	**15**	**16**	**17**	**18**	**19**									
Ans.	3	2	9	1	4									

ELLIPSE

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	1	3	1	3	1	2	2	3	2	4	3	1	2	3

HYPERBOLA

Q.No.	1	2	3	4	5	6	7	8	9	10	11
Ans.	4	1	3	36	2	3	2	2	4	80	3

PERMUTATION & COMBINATION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	777	96	238	3	924	52	7744	100	80	576	3	77	31650	1
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**		
Ans.	4	32	2	45	3	4	4	3	3	1	300	1000		

BINOMIAL THEOREM

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	2	21	4	9	8	96	3	1	210	4	4	1	98	55
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
Ans.	3	4	4	136	49	1	3	15	55	315	924	2	BONUS	2
Q.No.	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**
Ans.	1	45	2	6	1	3	4	4	2	1	1	4	2	160
Q.No.	**43**	**44**												
Ans.	8	NTA (19) ALLEN BONUS												

SET

Q.No.	1	2	3	4	5	6
Ans.	3	256	5	4	3	2

RELATION

Q.No.	1	2	3	4
Ans.	2	2	4	3

FUNCTION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	3	2	4	720	1	2	3	490	3	4	2	4	26	4
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**					
Ans.	4	1	3	4	2	1	NTA (3) ALLEN 1 or 2 or 3	3	3					

INVERSE TRIGONOMETRY FUNCTION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	4	2	1	2	2	4	3	3	2	1	3	3	3	2
Q.No.	**15**	**16**	**17**											
Ans.	1	1	2											

LIMIT

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	3	3	4	3	1	3	1	2	3	4	2	7	1	4
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**					
Ans.	5	1	3	1	4	1	1	4	4					

CONTINUITY

Q.No.	1	2	3	4	5	6	7	8	9	10	11
Ans.	2	1	3	14	1	2	3	2	1	6	4

DIFFERENTIABILITY

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	1	1	4	4	5	2	1	3	2	5	2	4	3

METHOD OF DIFFERENTIATION

Q.No.	1	2	3	4	5	6	7	8	9	10	11
Ans.	39	3	40	17	1	1	2	1	2	1	481

AOD (TANGENT & NORMAL)

Q.No.	1	2	3	4	5
Ans.	3	3	1	4	406

AOD (MONOTONICITY)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	3	4	3	2	2	1	4	1	1	2	1	2

AOD (MAXIMA & MINIMA)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	4	1	3	3	36	3	4	3	22	1250	9	144	2	3
Q.No.	15													
Ans.	2													

INDEFINITE INTEGRATION

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	15	7	3	3	3	4	2	6	1	4

DEFINITE INTEGRATION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	2	NTA (2) ALLEN (3)	2	1	3	1	3	2	2	1	3	5	2	2
Q.No.	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Ans.	2	3	1	3	1	5	4	2	4	2	2	3	3	1
Q.No.	29	30	31	32	33	34	35	36	37	38	39	40	41	42
Ans.	19	3	1	1	2	1	4	1	3	3	16	2	1	1
Q.No.	43	44	45											
Ans.	BONUS	512	3											

DIFFERENTIAL EQUATION

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	1	2	1	4	2	3	4	4	4	1	1	1	16	2
Q.No.	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Ans.	2	2	4	3	1	3	4	4	4	2	1	2	4	2
Q.No.	29	30	31	32	33	34	35	36	37	38	39	40	41	42
Ans.	4	4	1	3	1	1	2	2	2	4	2	3	1	1
Q.No.	43	44	45											
Ans.	3	1	4											

AREA UNDER THE CURVE

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	1	2	4	2	3	27	114	1	26	1	2	3	64	1
Q.No.	15	16	17	18										
Ans.	BONUS	2	41	3										

MATRICES

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Ans.	1	910	1	108	3	3125	16	1	4	4	2020	2	1	4
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
Ans.	1	8	3	3	540	17	4	4	7	13	4	1	1	3
Q.No.	**29**	**30**	**31**	**32**	**33**	**34**	**35**							
Ans.	766	1	2020	3	16	2	6							

VECTORS

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Ans.	4	4	3	6	1	4	1	3	4	1	60	2	2	2
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
Ans.	9	1	4	5	90	1	3	3	1494	3	75	12	2	4
Q.No.	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**		
Ans.	4	2	28	1	2	486	1	2	4	2	2	1		

3D

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Ans.	81	6	1	4	2	1	3	1	4	5	3	4	7	2
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
Ans.	26	3	96	4	4	1	72	1	61	4	7	2	3	1
Q.No.	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**
Ans.	1	3	4	4	3	3	44	4	1	2	8	3	2	2
Q.No.	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**			
Ans.	2	4	2	3	2	0	4	4	28	4	38			

COMPLEX NUMBER

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Ans.	NTA (3) ALLEN (2)	1	2	11	1	1	4	1	2	13	1	6	4	6
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
Ans.	98	4	5	310	10	4	48	3	1	2	36	4	3	2
Q.No.	**29**	**30**	**31**	**32**										
Ans.	2	6	2	0										

PROBABILITY

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Ans.	2	2	2	4	1	2	4	3	1	4	4	2	2	3
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
Ans.	28	1	2	781	3	4	6	3	2	3	3	3	1	2
Q.No.	**29**	**30**	**31**	**32**										
Ans.	3	4	6	1										

STATISTICS

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Ans.	1	4	4	164	3	4	3	4	12	13	30	25	3	11
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**								
Ans.	4	5	4	68	35	1								

MATHEMATICAL REASONING

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Ans.	2	2	4	4	3	4	1	3	3	1	1	3	1	4
Q.No.	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**			
Ans.	2	4	3	3	2	4	4	1	1	4 or 16 or 64	2			