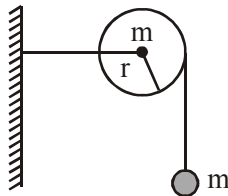# ROTATIONAL MECHANICS

1. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be :

(1) $\dfrac{1}{r}\sqrt{\dfrac{2gh}{3}}$ (2) $r\sqrt{\dfrac{3}{4gh}}$ (3) $\dfrac{1}{r}\sqrt{\dfrac{4gh}{3}}$ (4) $r\sqrt{\dfrac{3}{2gh}}$

2. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\dfrac{l}{4}$ away from the centre of the rod, and perpendicular to it, is :

(1) $\dfrac{1}{8}l$ (2) $\sqrt{\dfrac{7}{48}}l$

(3) $\sqrt{\dfrac{3}{8}}l$ (4) $\dfrac{1}{4}l$

3. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as σ (r) = A + Br. The moment of inertia of the disc about the the axis, perpendicular to the plane and passing through its centre is :

(1) $2\pi a^4 \left(\dfrac{A}{4} + \dfrac{aB}{5} \right)$

(2) $\pi a^4 \left(\dfrac{A}{4} + \dfrac{aB}{5} \right)$

(3) $2\pi a^4 \left(\dfrac{aA}{4} + \dfrac{B}{5} \right)$

(4) $2\pi a^4 \left(\dfrac{A}{4} + \dfrac{B}{5} \right)$

4.
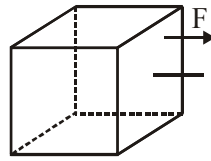
Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is μ = 0.4, the maximum possible value of $100 \times \dfrac{b}{a}$ for a box not to topple before moving is _____ .

5. Consider a uniform rod of mass M = 4m and length ℓ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \dfrac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is :

(1) $\dfrac{3}{7\sqrt{2}} \dfrac{v}{\ell}$ (2) $\dfrac{3\sqrt{2}}{7} \dfrac{v}{\ell}$

(3) $\dfrac{4}{7} \dfrac{v}{\ell}$ (4) $\dfrac{3}{7} \dfrac{v}{\ell}$

6. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is :

(1) 8.75×10^{-4} J

(2) 8.75×10^{-3} J

(3) 6.25×10^{-4} J

(4) 1.13×10^{-3} J

7.
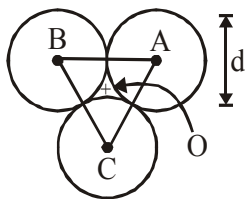
Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is :

(1) $\dfrac{13}{23}$ (2) $\dfrac{15}{13}$

(3) $\dfrac{23}{13}$ (4) $\dfrac{13}{15}$

8. One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\,s^{-1}$, where n is an integer. The value of n is _____ .
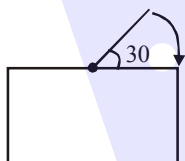

9. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is :

(1) $\left[\dfrac{m_1 + m_2}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$ (2) $\left[\dfrac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

(3) $\left[\dfrac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$ (4) $\left[\dfrac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

10.
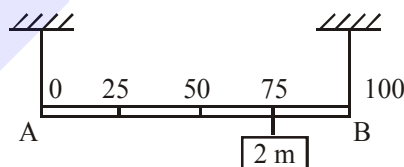
Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is :
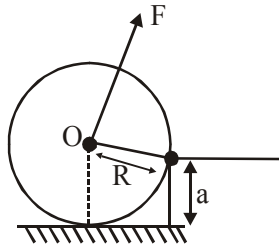
(1) 2 mg   (2) 0.5 mg (3) 0.75 mg (4) 1 mg

11. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :

(1) $Mg\sqrt{1-\dfrac{a^2}{R^2}}$

(2) $Mg\sqrt{\left(\dfrac{R}{R-a}\right)^2-1}$

(3) $Mg\dfrac{a}{R}$

(4) $Mg\sqrt{1-\left(\dfrac{R-a}{R}\right)^2}$

12. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The Kinetic energy of the combined system is :

(1) $\dfrac{10}{3}J$ (2) $\dfrac{2}{3}J$ (3) $\dfrac{5}{3}J$ (4) $\dfrac{20}{3}J$

13. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\dfrac{R^2}{4}+\dfrac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of material, the ratio L/R for it to have minimum possible I is :-

(1) $\sqrt{\dfrac{2}{3}}$ (2) $\dfrac{3}{2}$ (3) $\sqrt{\dfrac{3}{2}}$ (4) $\dfrac{2}{3}$

14. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg, and length *l* = 1 m, the value of θ is approximately : (Take g = 10 m/s^2)
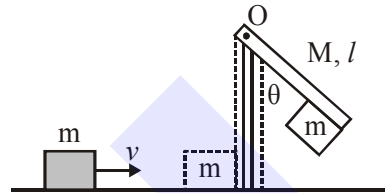
(1) 69°   (2) 63°   (3) 55°   (4) 49°

15. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis as 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre_____.

16.
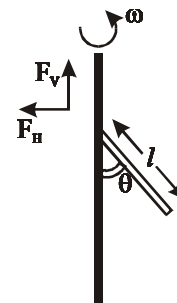
A uniform rod of length '*l*' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper)

$\dfrac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that:

(1) $\cos\theta = \dfrac{g}{2\ell\omega^2}$ (2) $\cos\theta = \dfrac{3g}{2\ell\omega^2}$

(3) $\cos\theta = \dfrac{2g}{3\ell\omega^2}$ (4) $\cos\theta = \dfrac{g}{\ell\omega^2}$

17. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of intertia of the system about the line EX perpendicular to EG in the plane of EFG is

$\dfrac{N}{20} ma^2$ where N is an integer. The value of N

is _____.
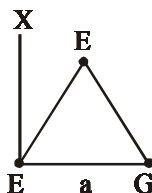
18. ABC is a plane lamina of the shape of an equilateral triagnle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same

axis is $\dfrac{NI_0}{16}$ where N is an integer. Value of N

is _____.
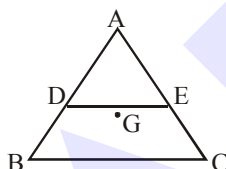
19. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1.

If another stationary disc having radius $\dfrac{R}{2}$ and

same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is _____.

20. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is :

(1) $\sqrt{2}$ (2) 2   (3) 4   (4) $2\sqrt{2}$
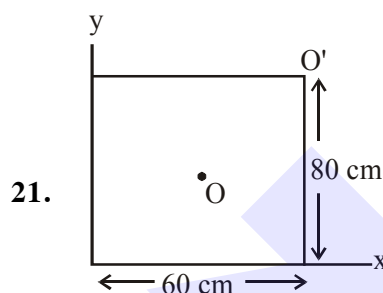
21.

For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is :

(1) 1/2   (2) 2/3

(3) 1/8   (4) 1/4

22. A wheel is rotaing freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of momet of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is :

(1) 0   (2) $\dfrac{1}{4}$ (3) $\dfrac{3}{4}$ (4) $\dfrac{5}{6}$

23. A force $\vec{F} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) N$ acts at a point

$\left(4\hat{i} + 3\hat{j} - \hat{k}\right) m$. Then the magnitude of torque

about the point $\left(\hat{i} + 2\hat{j} + \hat{k}\right) m$ will be

$\sqrt{x}$ N-m. The value of x is _____.

24. A thin rod of mass 0.9 kg and length 1m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be _____.
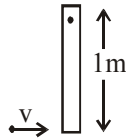

25. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is:

(1) $\dfrac{MR^2}{2}$

(2) $\dfrac{MH^2}{3}$

(3) $\dfrac{MR^2}{3}$

(4) $\dfrac{M(R^2 + H^2)}{4}$
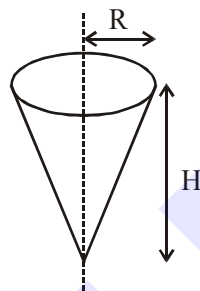
26. Four point masses, each of mass m, are fixed at the corners of a square of side ℓ. The square is rotating with angular frequency ω, about an axis passing through oneof the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is:
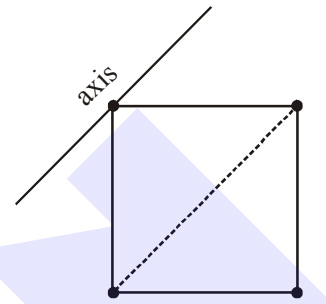
(1) $2m\ell^2\omega$ (2) $3m\ell^2\omega$ (3) $m\ell^2\omega$ (4) $4m\ell^2\omega$

27. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1 + \dfrac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is:

(1) $\dfrac{5}{12}ML^2$

(2) $\dfrac{3}{7}ML^2$

(3) $\dfrac{2}{5}ML^2$

(4) $\dfrac{7}{18}ML^2$

SOLUTION

1.   **NTA Ans. (3)**

Sol. $mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2} \times \dfrac{1}{2}mr^2 \times \dfrac{v^2}{r^2} = \dfrac{3}{4}mv^2$

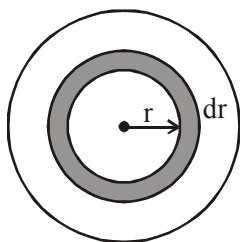$u = \sqrt{\dfrac{4}{3}gh}$

$\omega = \dfrac{v}{r}$

2.   **NTA Ans. (2)**

Sol. $m\dfrac{l^2}{12} + m\dfrac{l^2}{16} = mk^2$

$\dfrac{7l^2}{48} = k^2$

3.   **NTA Ans. (1)**

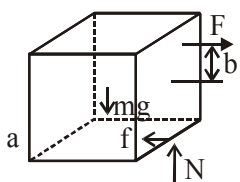**Sol.**

$dI = dmr^2$

$dI = \sigma 2\pi r\, dr\, r^2$

$dI = 2\pi(A + Br)\, r^3\, dr$

$\displaystyle\int dI = 2\pi \int_0^a (Ar^3 + Br^4)\,dr$

$I = 2\pi a^4 \left(\dfrac{A}{4} + \dfrac{B9}{5} \right)$

4.   **NTA Ans. (75)**

Sol.

$F = \mu mg$ ...(1)

$F\left(b + \dfrac{a}{2} \right) = mg\ \dfrac{a}{2}$ ...(2)

$\mu mg\ \left(b + \dfrac{a}{2} \right) = mg \times \dfrac{a}{2}$

$\left(b + \dfrac{a}{2} \right)\mu = \dfrac{a}{2}$
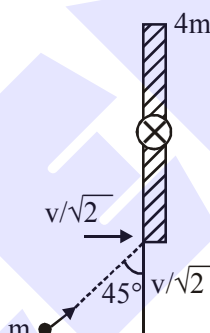
$0.4\ =\mu\ =\ \dfrac{a}{2b+a}$

$0.8b + 0.4a = a$

$0.8\ b = 0.6\ a$

$\dfrac{b}{a}\ =\ \dfrac{3}{4}$

5.   **NTA Ans. (2)**

Sol.

Let angular velocity of the system after collision be ω.

By conservation of angular momentum about the hinge :

$m\left(\dfrac{v}{\sqrt{2}} \right)\left(\dfrac{\ell}{2} \right) = \left[\dfrac{4m\ell^2}{12} + \dfrac{m\ell^2}{4} \right]\omega$

On solving

$\omega = \dfrac{3\sqrt{2}}{7}\left(\dfrac{v}{\ell} \right)$

6.   **NTA Ans. (1)**

Sol. m = 0.5 kg, v = 5 cm/s

KE in rolling = $\dfrac{1}{2}\,mv^2 + \dfrac{1}{2}I\omega^2$

$= \dfrac{1}{2}mv^2\left(1 + \dfrac{K^2}{R^2} \right)$

$= 8.75 \times 10^{-4}$ J

7.   **NTA Ans. (1)**

Sol. From parallel axis theorem

$$I_0 = 3 \times \left[\frac{2}{5} M \left(\frac{d}{2} \right)^2 + M \left(\frac{d}{\sqrt{3}} \right)^2 \right] = \frac{13}{10} Md^2$$
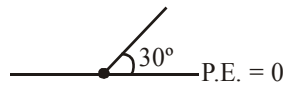
$$I_A = I_0 + 3M \left(\frac{d}{\sqrt{3}} \right)^2$$

$$= \frac{13}{10} Md^2 + Md^2$$

$$= \frac{23}{10} Md^2 \qquad \Rightarrow \frac{I_0}{I_A} = \frac{13}{23}$$

8. **NTA Ans. (15.00)**

Sol.

30° P.E. = 0

From mechanical energy conservation,

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow mg \frac{\ell}{2} \sin 30° + 0 = 0 + \frac{1}{2} I\omega^2$$
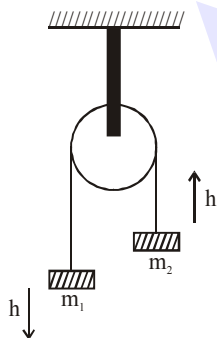
$$\Rightarrow mg \times \frac{1}{2} \times \frac{1}{2} + 0 = 0 + \frac{1}{2} \times \frac{m(1)^2}{3} \omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{2} \Rightarrow \omega = \sqrt{15}$$

$\therefore$ n = 15

9. **NTA Ans. (2)**

Sol.

by using work energy theorem
Wg = ΔKE

$$(m_1 - m_2) gh = \frac{1}{2} (m_1 + m_2) V^2 + \frac{1}{2} I\omega^2$$

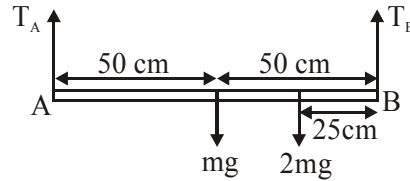$$(m_1 - m_2) gh = \frac{1}{2} (m_1 + m_2)(\omega R)^2 + \frac{1}{2} I\omega^2$$

$$(m_1 - m_2) gh = \frac{\omega^2}{2} \left[(m_1 + m_2) R^2 + I \right]$$

$$\omega = \sqrt{ \frac{2(m_1 - m_2) gh}{(m_1 + m_2) R^2 + I} }$$

$\therefore$ Correct answer (2)

10. **Official Ans. by NTA (4)**

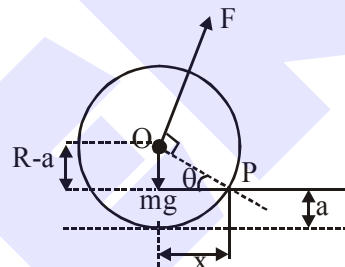**Sol.**

$\tau_B = 0$ (torque about point B is zero)
$(T_A) \times 100 - (mg) \times 50 - (2mg) \times 25 = 0$
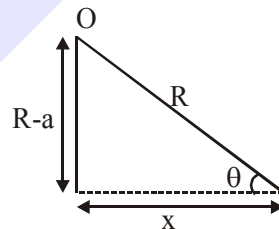$100 \, T_A = 100 \, mg$
$T_A = 1 \, mg$

11. **Official Ans. by NTA (4)**

Sol.

$(\tau)_P = 0$
F.R. – mgx = 0

$$x = \sqrt{R^2 - (R-a)^2}$$

$$F = mg \frac{x}{R}$$

$$F = mg \sqrt{1 - \left(\frac{R-a}{R} \right)^2}$$

= minimum value of force to pull

12. **Official Ans. by NTA (4)**

Sol. ♦ Both discs are rotating in same sense
♦ Angular momentum conserved for the system
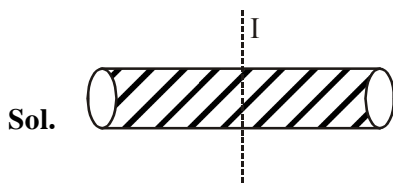i.e. $L_1 + L_2 = L_{final}$
$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$
$0.1 \times 10 + 0.2 \times 5 = (0.1+0.2) \times \omega_f$

$$\omega_f = \frac{20}{3}$$

♦ Kinetic energy of combined disc system

$$\Rightarrow \frac{1}{2}(I_1 + I_2)\omega_f^2$$

$$= \frac{1}{2}(0.1 + 0.2).\left(\frac{20}{3}\right)^2$$

$$= \frac{0.3}{2} \times \frac{400}{9} = \frac{120}{18} = \frac{20}{3} J$$

13.   Official Ans. by NTA (3)

Sol.

$$I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) \qquad .....(1)$$

as mass is constant $\Rightarrow$ m = ρV = constant

V = constant

$\pi^2 R l$ = constant $\Rightarrow R^2 L$ = constant

$$2RL + R^2\frac{dL}{dR} = 0 \qquad .....(2)$$

From equation (1)

$$\frac{dI}{dR} = M\left(\frac{2R}{4} + \frac{2L}{12} \times \frac{dL}{dr}\right) = 0$$

$$\frac{R}{2} + \frac{L}{6}\frac{dL}{dR} = 0$$

Substituting value of $\frac{dL}{dR}$ from eqution (2)

$$\frac{R}{2} + \frac{L}{6}\left(\frac{-2L}{R}\right) = 0$$

$$\frac{R}{2} = \frac{L^2}{3R} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

14.   Official Ans. by NTA (2)

Sol.   Angular momentum conservation

$$mvl = \frac{Ml^2}{3}\omega + ml^2\omega$$

$$\Rightarrow \omega = \frac{1 \times 6 \times 1}{\frac{2}{3} + 1} = \frac{18}{5}$$

Now using energy consevation

$$\frac{1}{2}\left(M\frac{l^2}{3}\right)\omega^2 + \frac{1}{2}(ml^2)\omega^2$$

$$= (m + M)r_{cm}(1 - \cos\theta)$$

$$= (m + M)\left(\frac{ml + \frac{Ml}{2}}{m + M}\right)g(1 - \cos\theta)$$

$$\frac{5}{6} \times \left(\frac{18}{5}\right)^2 = 20(1 - \cos\theta)$$

$$\Rightarrow 1 - \cos\theta = \frac{18}{5} \times \frac{3}{20}$$

$$\cos\theta = 1 - \frac{27}{50}$$

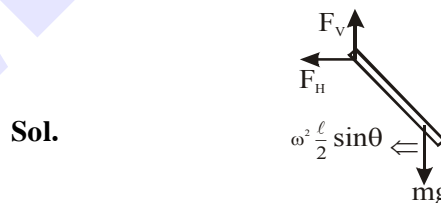$$\cos\theta = \frac{23}{50} \Rightarrow \theta \simeq 63°$$

15.   Official Ans. by NTA (9)

Sol. $L_i = L_f$

$$\left(80R^2 + \frac{200R^2}{2}\right)\omega = \left(0 + \frac{200R^2}{2}\right)\omega_1$$

$180\omega_0 = 100\omega_1$
$\omega_1 = 1.8\omega_0 = 1.8 \times 5$
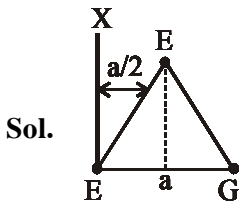   = 9 rpm

16.   Official Ans. by NTA (2)

Sol.

$F_V = mg$

$$F_H = m\omega^2\frac{\ell}{2}\sin\theta$$

$$mg\frac{\ell}{2}\sin\theta - m\omega^2\frac{\ell}{2}\sin\theta\frac{\ell}{2}\cos\theta = \frac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$$

$$\cos\theta = \frac{3}{2}\frac{g}{\omega^2\ell} \qquad .....(ii)$$
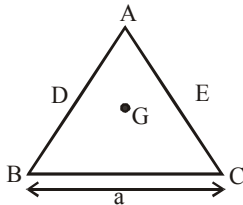
17. Official Ans. by NTA (25)

Sol.

$$I = 0 + m\left(\frac{a}{2}\right)^2 + ma^2$$

$$= \frac{5}{4}ma^2$$

18. Official Ans. by NTA (11)

Sol. Let side of triangle is a and mass is m
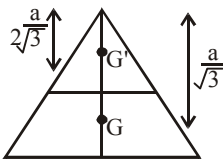

MOI of plate ABC about centroid

$$I_0 = \frac{m}{3}\left(\left(\frac{a}{2\sqrt{3}}\right)^2 \times 3\right) = \frac{ma^2}{12}$$

triangle ADE is also an equilateral triangle of side a/2.

Let moment of inertia of triangular plate ADE about it's centroid (G') is I_1 and mass is m_1

$$m_1 = \frac{m}{\frac{\sqrt{3}a^2}{4}} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

$$I_1 = \frac{m_1}{12}\left(\frac{a}{2}\right)^2 = \frac{m}{4\times12}\frac{a^2}{4} = \frac{ma^2}{192}$$

distance GG' = $\frac{a}{\sqrt{3}} - \frac{a}{2\sqrt{3}} = \frac{a}{2\sqrt{3}}$

so MOI of part ADE about centroid G is

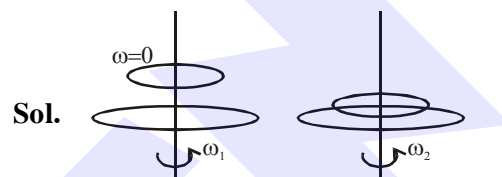$$I_2 = I_1 + m_1\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{ma^2}{192} + \frac{m}{4}\cdot\frac{a^2}{12}$$

$$= \frac{5ma^2}{192}$$

now MOI of remaining part

$$= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{12\times16} = \frac{11I_0}{16}$$

$$\Rightarrow \quad N = 11$$

19. Official Ans. by NTA (20)

Sol.

Let moment of inertia of bigger disc is $I = \frac{MR^2}{2}$

$$\Rightarrow \text{MOI of small disc } I_2 = \frac{M\left(\frac{R}{2}\right)^2}{2} = \frac{I}{4}$$

by angular momentum conservation

$$I\omega_1 + \frac{I}{4}(0) = I\omega_2 + \frac{I}{4}\omega_2 \Rightarrow \omega_2 = \frac{4\omega_1}{5}$$

initial kinetic energy $K_1 = \frac{1}{2}I\omega_1^2$

final kinetic energy K_2

$$= \frac{1}{2}\left(I + \frac{I}{4}\right)\left(\frac{4\omega_1}{5}\right)^2 = \frac{1}{2}I\omega_1^2\left(\frac{4}{5}\right)$$

$$P\% = \frac{K_1 - K_2}{K_1} \times 100\% = \frac{1 - 4/5}{1} \times 100 = 20\%$$
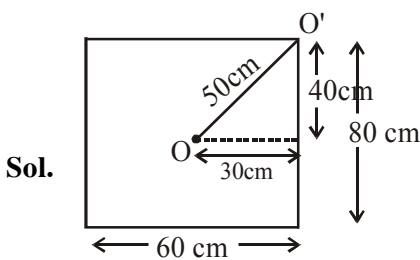
20. **Official Ans. by NTA (2)**

Sol. $I_1 = \dfrac{MR^2}{2} = \dfrac{\rho(\pi R^2)t.R^2}{2}$

$I \propto R^4$

$\dfrac{I_1}{I_2} = \dfrac{R_1^4}{R_2^4} = \dfrac{1}{16}$

$\therefore \dfrac{R_1}{R_2} = \dfrac{1}{2}$

21. **Official Ans. by NTA (4)**
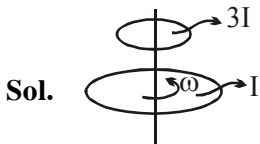
Sol.

Rectangular sheet

$I_O = \dfrac{M}{12}[L^2 + B^2] = \dfrac{M}{12}[80^2 + 60^2]$

$I_{O'} = I_0 + Md^2 \ \{\text{parallel axis theorem}\}$
$= \dfrac{M}{12}\Big[80^2 + 60^2\Big] + M[50]^2$

$\dfrac{I_O}{I_{O'}} = \dfrac{M/12[80^2 + 60^2]}{\dfrac{M}{12}[80^2 + 60^2] + M[50]^2} = \dfrac{1}{4}$

22. **Official Ans. by NTA (3)**

Sol.

By anglar momentum conservation

$\omega I + 3I \times 0 = 4I\omega' \Rightarrow \omega' = \dfrac{\omega}{4}$

$(KE)_i = \dfrac{1}{2}I\omega^2$

$(KE)_f = \dfrac{1}{2} \times (4I) \times \left(\dfrac{\omega}{4}\right)^2 = \dfrac{I\omega^2}{8}$

$\Delta KE = \dfrac{3}{8}I\omega^2$

fractional loss $= \dfrac{\Delta KE}{KE_i} = \dfrac{\dfrac{3}{8}I\omega^2}{\dfrac{1}{2}I\omega^2} = \dfrac{3}{4}$

23. **Official Ans. by NTA (195)**

Sol. $\vec{\tau} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$

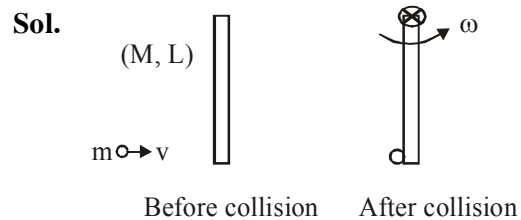$= [(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})] \times \vec{F}$

$= (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$

$= 7\hat{i} - 11\hat{j} + 5\hat{k}$

$|\vec{\tau}| = \sqrt{195}$

24. **Official Ans. by NTA (20.00)**

Sol.

Before collision   After collision

$\vec{L}_i = \vec{L}_f$

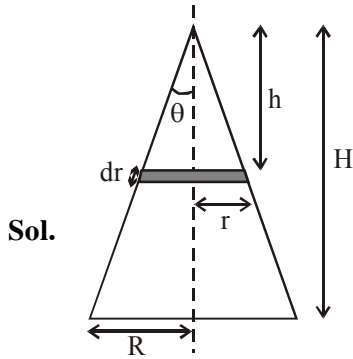$mvL = I\omega$

$mvL = \left(\dfrac{ML^2}{3} + mL^2\right)\omega$

$0.1 \times 80 \times 1 = \left(\dfrac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$

$8 = \left(\dfrac{3}{10} + \dfrac{1}{10}\right)\omega$

$8 = \dfrac{4}{10}\omega$

$\omega = 20 \text{ rad} \dfrac{\text{rad}}{\text{sec}}$

25. Official Ans. by NTA (1)

Sol.

Area $= \pi R\ell = \pi R\left(\sqrt{H^2 + R^2}\right)$

Area of element $dA = 2\pi r\, d\ell = 2\pi r \dfrac{dh}{\cos\theta}$

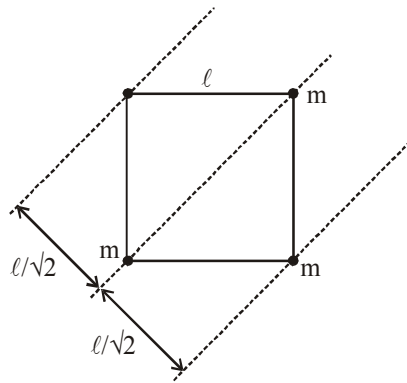mass of element $dm = \dfrac{M}{\pi R\sqrt{H^2 + R^2}} \times \dfrac{2\pi r\, dh}{\cos\theta}$

$dm = \dfrac{2Mh\tan\theta\, dh}{R\sqrt{H^2 + R^2}\cos\theta}$ (here $r = h\tan\theta$)

$I = \int (dm)r^2 = \int \dfrac{h^2\tan^2\theta}{\cos\theta}\left(\dfrac{2m}{R}\dfrac{h\tan\theta}{\sqrt{R^2 + H^2}}\right)dh$

$= \dfrac{2M}{\cos\theta R}\dfrac{\tan^3\theta}{\sqrt{R^2 + H^2}}\int_0^H h^3\, dh = \dfrac{MR^2 H^4}{2RH^3\sqrt{R^2 + H^2}\cos\theta}$

$= \dfrac{MR^2 H\sqrt{R^2 + H^2}}{2\sqrt{R^2 + H^2} \times H} \quad\Rightarrow\quad = \dfrac{MR^2}{2}$
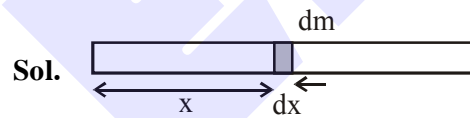
26. Official Ans. by NTA (2)

Sol.

$I = m(0)^2 + m\left(\dfrac{\ell}{\sqrt{2}}\right)^2 \times 2 + m\left(\sqrt{2}\ell\right)^2$

$= \dfrac{2m\ell^2}{2} + 2m\ell^2 = 3m\ell^2$

Angular momentum $L = I\omega$
$= 3m\ell^2\omega$

27. Official Ans. by NTA (4)

Sol.

$I = \int r^2 dm = \int x^2\lambda\, dx \quad\Rightarrow\quad I = \int_0^L x^2\lambda_0\left(1 + \dfrac{x}{L}\right)dx$

$I = \lambda_0\int_0^L \left(x^2 + \dfrac{x^3}{L}\right)dx$

$I = \lambda\left[\dfrac{L^3}{3} + \dfrac{L^3}{4}\right]$

$I = \dfrac{7L^3\lambda_0}{12} \qquad\qquad ....(i)$

$M = \int_0^L \lambda\, dx = \int_0^L \lambda_0\left(1 + \dfrac{x}{L}\right)dx$

$M = \lambda_0\left(L + \dfrac{L}{2}\right) = \lambda_0\dfrac{3L}{2}$

$\dfrac{2}{3}M = (\lambda_0 L) \qquad\qquad ....(ii)$

From (i) & (ii) $\quad I = \dfrac{7}{12}\left(\dfrac{2}{3}M\right)L^2 = \dfrac{7ML^2}{18}$

Ans. (4)