KINEMATICS

1. A particle is moving along the x-axis with its coordinate with the time 't' given be $x(t) = 10 + 8t - 3t^2$. Another particle is moving the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is __________.

2. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle :

(1) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

(2) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(3) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(4) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

3. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _____

4. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is __________ .

5. A particle starts from the origin at t = 0 with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $\left(6.0\hat{i} + 4.0\hat{j}\right) m/s^2$. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is :-

(1) 50 (2) 32 (3) 60 (4) 40

6. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hr. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(1) 30.5 ms^{-1} (2) 29.5 ms^{-1}

(3) 31.5 ms^{-1} (4) 28.5 ms^{-1}

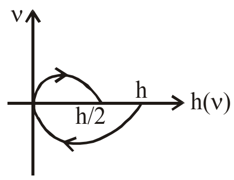
7. Starting from the origin at time t = 0, with initial velocity $5\hat{j}\ ms^{-1}$, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})ms^{-2}$. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0, are respectively :

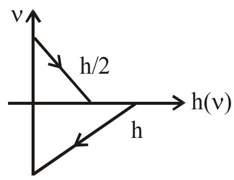
(1) 4s and 52 m (2) 2s and 24 m

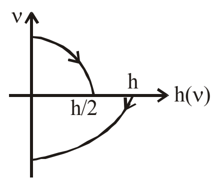
(3) 2s and 18 m (4) 5s and 25 m

8. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :

(graph are drawn schematically and on not to scale)

(1)


(2)


(3)


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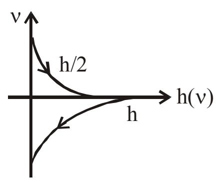
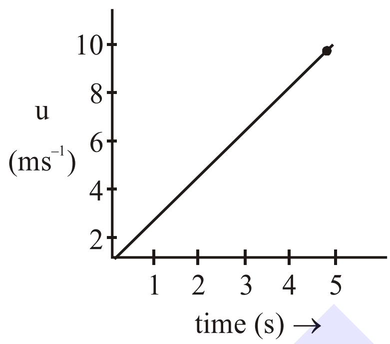


9. A small ball of mass is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is :

(1) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$ (2) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$

(3) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$ (4) $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$

10. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5s will be ______ :



11. A helicopter reises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] :

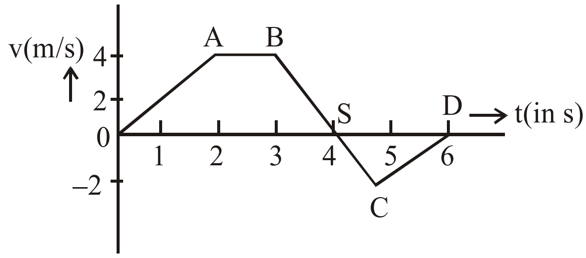
(1) $t=\sqrt{\frac{2h}{3g}}$

(2) $t=1.8\sqrt{\frac{h}{g}}$

(3) $t=3.4\sqrt{\left(\frac{h}{g}\right)}$

(4) $t=\frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

12. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6s is :



(1) 12m (2) $\frac{49}{4}$ m

(3) 11 m (4) $\frac{37}{3}$ m

13. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)$v, this angle changes to 45°. The value of β is close to:

(1) 0.41 (2) 0.50

(3) 0.37 (4) 0.73

SOLUTION

1. NTA Ans. (580.00)

Sol. $x = 10 + 8t - 3t^2$

$v_x = 8 - 6t$

$(v_x)_{t=1} = 2\hat{i}$

$y = 5 - 8t^3$

$v_y = -24t^2$

$(v_y)_{t=1} = -24\hat{j}$

Now

$$\sqrt{v} = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$$

$\therefore$ $v = 580\ m^2/s^2$

2. NTA Ans. (1)

Sol. $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$

On diff. we get

$$\vec{v} = -\omega\sin\omega t\ \hat{i} + \omega\cos\omega t\ \hat{j}$$

$$\vec{a} = -\omega^2\vec{r}$$

$$\vec{v}\,.\,\vec{r} = 0$$

3. NTA Ans. (8 or 2888)

Sol. Time to travel 81 m is t sec.

Time to travel 100 m is $t + \frac{1}{2}$ sec.

$$81 = \frac{1}{2}\times a\times t^2 \qquad \Rightarrow t = 9\sqrt{\frac{2}{a}}$$

$$100 = \frac{1}{2}\times a\times\left(t+\frac{1}{2}\right)^2 \qquad \Rightarrow t + \frac{1}{2} = 10\sqrt{\frac{2}{a}}$$

$$9\sqrt{\frac{2}{a}} + \frac{1}{2} = 10\sqrt{\frac{2}{a}}$$

$$\frac{1}{2} = \sqrt{\frac{2}{a}}$$

$$\boxed{a = 8\ m/s^2}$$

4. NTA Ans. (3.00)

Sol. $x = \sqrt{at^2 + 2bt + c}$

Differentiating w.r.t. time

$$\frac{dx}{dt} = v = \frac{1}{2\sqrt{at^2+2bt+c}}\times(2at+2b)$$

$$\Rightarrow v = \frac{at+b}{x}$$

$\Rightarrow vx = at + b$

Differentiating w.r.t. x

$$\Rightarrow \frac{dv}{dx}\times x + v = a\times\frac{dt}{dx}$$

Multiply both side by v

$$\Rightarrow \left(v\frac{dv}{dx}\right)x + v^2 = a$$

$\Rightarrow a'x = a - v^2$ [Here a' is acceleration]

$$\Rightarrow a'x = a - \left(\frac{at+b}{x}\right)^2$$

$$\Rightarrow a'x = \frac{ax^2 - (at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{a(at^2+2bt+c) - (at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{ac - b^2}{x^2}$$

$$\Rightarrow a' = \frac{ac - b^2}{x^3}$$

$$\therefore a' \propto \frac{1}{x^3} \qquad \therefore n = 3$$

5. NTA Ans. (3)

Sol. $x = u_x t + \frac{1}{2} a_x t^2$

$y = u_y t + \frac{1}{2} a_y t^2$

$32 = 0 \times t + \frac{1}{2}(4)(t)^2$

$t^2 = 16$

$t = 4$ sec

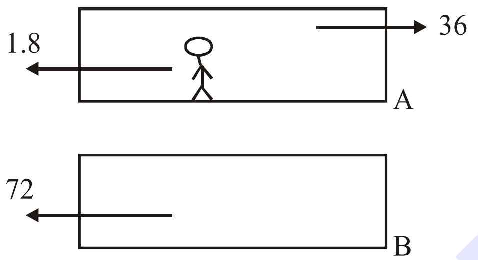
$x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2$

$= 12 + 48 = 60$ m

$\therefore$ Correct answer (3)

6. Official Ans. by NTA (2)

Sol.



Velocity of man with respect to ground

$\vec{V}_{m/g} = \vec{V}_{m/A} + \vec{V}_A = -1.8 + 36$

Velocity of man w.r.t. B

$\vec{V}_{m/B} = \vec{V}_m - \vec{V}_B$

$= -1.8 + 36 - (-72)$

$= 106.2$ km/hr

$= 29.5$ m/s

7. Official Ans. by NTA (3)

Sol. Given $\vec{u} = 5\hat{j}\,m/s$, $\vec{a} = 10\hat{i} + 4\hat{j}$, final coordinate $(20, y_0)$ in time t

$S_x = 4_x t + \frac{1}{2} a_x t^2$

$20 - 0 = 0 + \frac{1}{2} \times 10 \times t^2$

$t = 2$sec

$S_y = u_y \times t + \frac{1}{2} a_y t^2$

$y_0 = 5 \times 2 + \frac{1}{2} 4 \times 2^2 = 18$m

2 sec and 18 m

8. Official Ans. by NTA (3)

Sol. Velocity at ground (means zero height) is non-zero therefore one is incorrect and velocity versus height is non-linear therefore two is also incorrect.

$v^2 = 2gh$

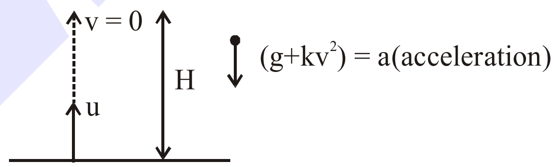
$v\frac{dv}{dh} = 2g = \text{const.}$

$\frac{dv}{dh} = \frac{\text{constant}}{v}$

Here we can see slope is very high when velocity is low therefore at Maximum height the slope should be very large which is in option 3 and as velocity increases slope must decrease there for option 3 is correct.

9. Official Ans. by NTA (2)

Sol.



$\vec{F} = mkv^2 - mg$

$\vec{a} = \frac{\vec{F}}{m} = -\left[kv^2 + g\right]$

$\Rightarrow v.\frac{dv}{dh} = -[kv^2 + g]$

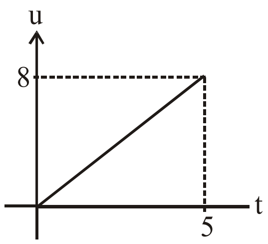
$\Rightarrow \int_u^0 \frac{v.dv}{kv^2 + g} = -\int_0^H dh$

$\frac{1}{2K} \ln\left[kv^2 + g\right]_u^0 = -H$

$\Rightarrow \frac{1}{2K} \ln\left[\frac{ku^2 + g}{g}\right] = H$

10. Official Ans. by NTA (20)

Sol.

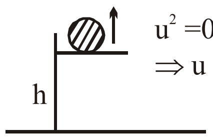


Distance = $\int v\,dt$

Area under graph = $\frac{1}{2}\times5\times8 = 20$

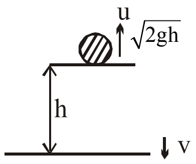
11. Official Ans. by NTA (3)

Sol.



$u^2 = 0 + 2gh$

$\Rightarrow u = \sqrt{2gh}$



$v^2 = u^2 + 2as$

$v^2 = 2gh + 2gh$

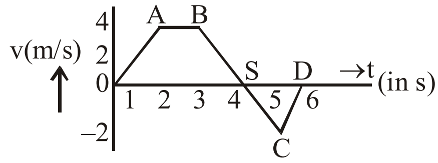
$v = \sqrt{4gh}$

$$\Rightarrow \sqrt{4gh} = \sqrt{2gh} + gt$$

$$\Rightarrow t = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}} \Rightarrow 3.4\sqrt{\frac{h}{g}}$$

12. Official Ans. by NTA (4)

Sol.



$OS = 4 + \frac{1}{3} = \frac{13}{3}$

$SD = 2 - \frac{1}{3} = \frac{5}{3}$

Area of OABS is A_1

Area of SCD is A_2

Distance = $|A_1| + |A_2|$

$A_1 = \frac{1}{2}\left[\frac{13}{3}+1\right]4 = \frac{32}{3}$

$A_2 = \frac{1}{2}\times\frac{5}{3}\times2 = \frac{5}{3}$

Distance = $|A_1| + |A_2|$

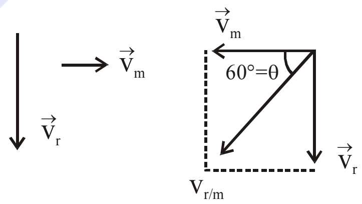
$= \frac{32}{3} + \frac{5}{3}$

$= \frac{37}{3}$

13. Official Ans. by NTA (4)

Sol. Rain is falling vertically downwards.

$\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$

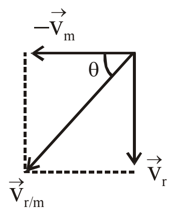


$\tan 60° = \frac{v_r}{v_m} = \sqrt{3}$

$v_r = v_m\sqrt{3} = v\sqrt{3}$

Now, $v_m = (1 + B)v$

and $\theta = 45°$



$\tan 45 = \frac{v_r}{v_m} = 1$

$v_r = v_m$

$v\sqrt{3} = (1+\beta)v$

$\sqrt{3} = 1 + \beta$

$\Rightarrow \beta = \sqrt{3} - 1 = 0.73$