GRAVITATION

1. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

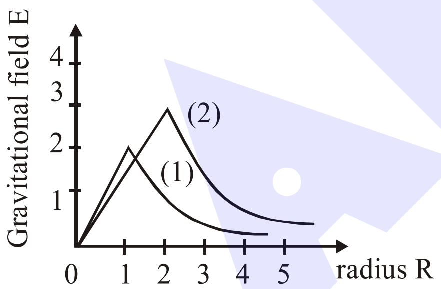
(1) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$

(2) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

(3) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$

(4) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$

2. Consider two solid spheres of radii R_1 = 1m, R_2 = 2m and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is :



(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{1}{6}$

3. An asteroid is moving directly towards the centre of the earth. When at a distance of 10R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s) ? Give your answer to the nearest integer in kilometer/s _____.

4. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body :

(1) starts moving in an elliptical orbit around the planet.

(2) continues to move in a circular orbit

(3) Falls vertically downwards towards the planet

(4) Escapes from the Planet's Gravitational field.

5. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are ν_A and ν_B, respectively, then $\frac{\nu_A}{\nu_B}=\frac{n}{4}$.

The value of n is :

(1) 4 (2) 1 (3) 2 (4) 3

6. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as :

(1) $T \propto R$ (2) $T^2 \propto \frac{1}{R^3}$

(3) $T^2 \propto R$ (4) $T^2 \propto R^3$

7. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected):

(1) $\frac{\sqrt{5}R-R}{2}$ (2) $\frac{\sqrt{5}}{2}R-R$

(3) $\frac{R}{2}$ (4) $\frac{\sqrt{3}R-R}{2}$

8. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that is become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R, value of R is :

(1) $4R_e$ (2) $3R_e$ (3) $2R_e$ (4) $2.5R_e$

9. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at:

(1) $r=\frac{1}{\sqrt{3}}R$ (2) $r=\sqrt{\frac{5}{9}}R$

(3) $r=\sqrt{\frac{3}{4}}R$ (4) r = R

10. On the x-axis and a dsitance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is :

(1) $\frac{A}{(x^2+a^2)^{1/2}}$ (2) $\frac{A}{(x^2+a^2)^{3/2}}$

(3) $A(x^2 + a^2)^{3/2}$ (4) $A(x^2 + a^2)^{1/2}$

11. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is :

(1) 1 (2) 2 (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

12. The value of the acceleration due to gravity is g_1 at a height $h=\frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals :

(1) $\frac{7}{9}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{5}{9}$

13. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is : (h<<R, where R is the radius of the earth)

(1) $\frac{R^2\omega^2}{8g}$ (2) $\frac{R^2\omega^2}{4g}$

(3) $\frac{R^2\omega^2}{g}$ (4) $\frac{R^2\omega^2}{2g}$

14. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is :

(1) 1 : 6 (2) 3 : 4 (3) 1 : 3 (4) 1 : 2

15. Two planets have masses M and 16 M and their radii are a and 2a, respectively. The separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is :

(1) $\sqrt{\frac{GM^2}{ma}}$ (2) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

(3) $4\sqrt{\frac{GM}{a}}$ (4) $2\sqrt{\frac{GM}{a}}$

SOLUTION

1. NTA Ans. (2)

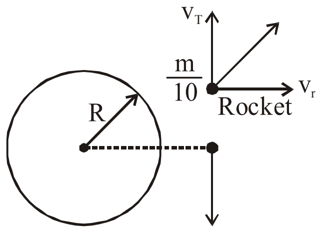
Sol. Applying energy conservation

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mu^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$v = \sqrt{u^2 - \frac{GM}{R}} \quad ....(i)$$

By momentum conservation, we have



$$\frac{m}{10}v_T = \frac{9m}{10}\sqrt{\frac{GM}{2R}} \quad ....(ii)$$

& $\frac{m}{10}v_r = mv$

$$\Rightarrow \frac{m}{10}v_r = m\sqrt{u^2 - \frac{GM}{R}} \quad ....(iii)$$

Kinetic energy of rocket

$$= \frac{1}{2}m\left(v_T^2 + v_r^2\right)$$

$$= \frac{m}{20}\left(81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R}\right)$$

$$= \frac{m}{20}\left(100u^2 - \frac{119GM}{2R}\right)$$

$$= 5m\left(u^2 - \frac{119GM}{200R}\right).$$

2. NTA Ans. (4)

Sol. Gravitational field on the surface of a solid sphere $I_g = \frac{GM}{R^2}$

By the graph

$$\frac{GM_1}{(1)^2} = 2 \quad \text{and} \quad \frac{GM_2}{(2)^2} = 3$$

On solving

$$\frac{M_1}{M_2} = \frac{1}{6}$$

3. NTA Ans. (16)

Sol. $U_1 + K_1 = U_2 + K_2$

$$-\frac{GM_em}{10R} + \frac{1}{2}mv_0^2 = -\frac{GM_em}{R} + \frac{1}{2}mv^2$$

$$+\frac{9}{10} \times \frac{GM_em}{R} + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$\frac{9}{10} \times \frac{1}{2}M \times v_e^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$v^2 = \frac{9}{10}v_e^2 + v_0^2 \qquad \Rightarrow = \frac{9}{10} \times (11.2)^2 + (12)^2$$

$v^2 = 112.896 + 144$

$v = 16.027$

$v = 16$ km/s

4. NTA Ans. (1)

Sol. Initially, the body of mass m is moving in a circular orbit of radius R. So it must be moving with orbital speed.

$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision, let the combined mass moves with speed v_1

$$mv_0 + \frac{m}{2}\frac{v_0}{2} = \left(\frac{3m}{2}\right)v_1 \qquad \Rightarrow v_1 = \frac{5v_0}{6}$$

Since after collision, the speed is not equal to orbital speed at that point. So motion cannot be circular. Since velocity will remain tangential, so it cannot fall vertically towards the planet.

Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field.

So their motion will be elliptical around the planet.

5. NTA Ans. (1)

Sol. $V_e = \sqrt{\frac{2GM}{R}}$ (Escape velocity)

$$V_A = \sqrt{\frac{2GM}{R}}$$

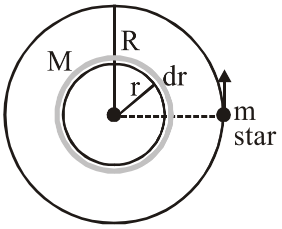
$$V_B = \sqrt{\frac{2G[M/2]}{R/2}} = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_A}{V_B} = 1 = \frac{n}{4} \Rightarrow n = 4$$

$\therefore$ Correct answer (1)

6. Official Ans. by NTA (3)

Sol.



dm = ρdv

$$dm = \left(\frac{k}{r}\right)(4\pi r^2 dr)$$

dm = 4πkrdr

$$M = \int_0^R dm = \int_0^R 4\pi krdr$$

$$M = 4\pi k \left.\frac{r^2}{2}\right|_0^R$$

$M = 2\pi k(R^2 - 0)$

$M = 2\pi kR^2$

for circular motion gravitational force will provide required centripital force or

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{G(2\pi kR^2)m}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{2\pi GkR}$$

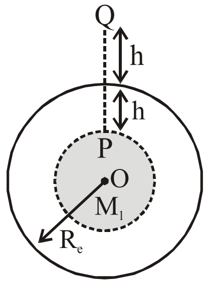
Time period $T = \frac{2\pi R}{v}$

$$T = \frac{2\pi R}{\sqrt{2\pi GkR}} \propto \sqrt{R}$$

or $T^2 \propto R$

7. Official Ans. by NTA (1)

Sol.



- M = mass of earth

M_1 = mass of shaded portion

R = Radius of earth

- $M_1 = \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi(R-h)^3$

$$= \frac{M(R-h)^3}{R}$$

- Weight of body is same at P and Q

i.e. $mg_P = mg_Q$

$g_P = g_Q$

$$\frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$$

$$\frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$$

$(R - h)(R + h)^2 = R^3$

$R^3 - hR^2 - h^2R - h^3 + 2R^2h - 2Rh^2 = R^3$

$R^2 - Rh^2 - h^3 = 0$

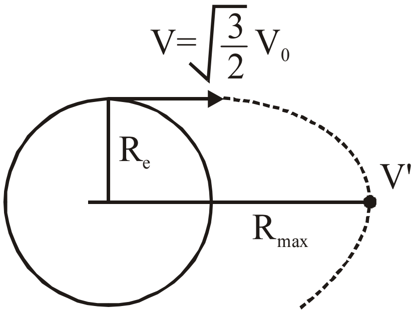
$R^2 - Rh - h^2 = 0$

$$h^2 + Rh - R^2 = 0 \Rightarrow h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$$

$$\text{ie } h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5}-1}{2}\right)R$$

8. Official Ans. by NTA (2)

Sol.



$$V_0 = \sqrt{\frac{GM}{R_e}}$$

$$\frac{-GMm}{R_e} + \frac{1}{2}mv^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mv'^2 \quad ....(i)$$

$$VR_e = V'R_{max} \quad ....(ii)$$

Solving (i) & (ii)

$$\boxed{R_{max} = 3R_e}$$

9. Official Ans. by NTA (2)

Sol. $E\,4\pi r^2 = \int \rho_0 4\pi r^2 dr$

$$\Rightarrow Er^2 = 4\pi G\int_0^r \rho_0\left(1-\frac{r^2}{R^2}\right)r^2 dr$$

$$\Rightarrow E = 4\pi G\rho_0\left(\frac{r^3}{3}-\frac{r^5}{5R^2}\right)$$

$$\frac{dE}{dr}=0 \quad \therefore \quad r=\sqrt{\frac{5}{9}}\,R$$

10. Official Ans. by NTA (1)

Sol. Given $E_G = \frac{Ax}{(x^2+a^2)^{3/2}},\ V_\infty = 0$

$$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G . \vec{d}_x$$

$$V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}}dx$$

put $x^2 + a^2 = z$

$2x\,dx = dz$

$$V_x - 0 = -\int_\infty^x \frac{A\,dz}{2(z)^{3/2}} = \left[\frac{A}{z^{1/2}}\right]_\infty^x = \left[\frac{A}{(x^2+a^2)^{1/2}}\right]_\infty^x$$

$$V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$$

11. Official Ans. by NTA (3)

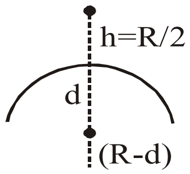
Sol. $V_{orbit} = \sqrt{\frac{GM}{R}}$

$$V_{escape} = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_{orbit}}{V_{escape}} = \frac{1}{\sqrt{2}}$$

12. Official Ans. by NTA (4)

Sol.



$$g_1 = \frac{GM}{\left(R+\frac{R}{2}\right)^2} \quad ... (1)$$

$$g_2 = \frac{GM(R-d)}{R^3} \quad ... (2)$$

$$g_1 = g_2$$

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3}$$

$$\Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$$

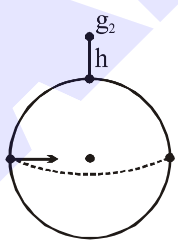
$4R = 9R - 9d$

$$5R = 9d \Rightarrow \frac{d}{R} = \frac{5}{9}$$

13. Official Ans. by NTA (4)

Sol. $g_e = g - R\omega^2$

$$g_2 = g\left(1-\frac{2h}{R}\right) \qquad g_1 = ge$$



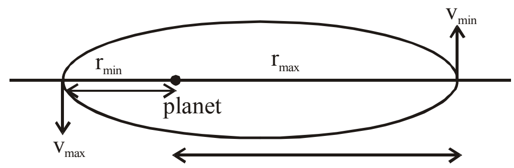
$$g_2 = g - \frac{2gh}{R}$$

$$\text{Now } R\omega^2 = \frac{2gh}{R}$$

$$h = \frac{R^2\omega^2}{2g}$$

14. Official Ans. by NTA (1)

Sol.



By angular momentum conservation

$r_{min}v_{max} = r_{max}v_{min}$ ... (i)

$$\text{Given } v_{min} = \frac{v_{max}}{6}$$

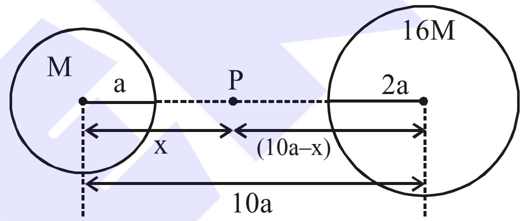
from equation (i)

$$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$$

Ans. (1)

15. Official Ans. by NTA (2)

Sol.



$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$

$$\frac{1}{x} = \frac{4}{(10a-x)} \qquad \Rightarrow 4x = 10a - x$$

$x = 2a$ ....(i)

COME

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE$$

$$= -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$KE = GMm\left[\frac{1}{8a}+\frac{16}{2a}-\frac{1}{2a}-\frac{16}{8a}\right]$$

$$KE = GMm\left[\frac{1+64-4-16}{8a}\right]$$

$$\frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right]$$

$$V = \sqrt{\frac{90GM}{8a}}$$

$$V = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$