GEOMETRICAL OPTICS

1. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to :

(1) 22 mm (2) 12 mm
(3) 33 mm (4) 2 mm

2. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1, then the ratio f_1/f is closest to the integer :

(1) 1 (2) 5 (3) 9 (4) 17

3. The magnifying power of a telescope with tube 60 cm is 5. What is the focal length of its eye piece ?

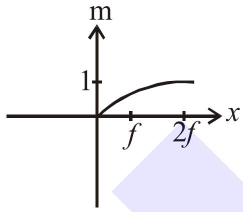
(1) 30 cm (2) 40 cm (3) 20 cm (4) 10 cm

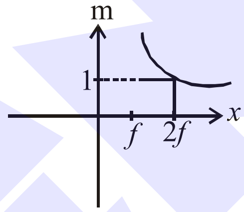
4. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be :

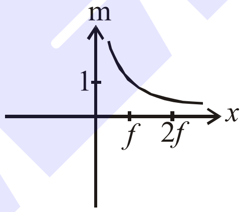
(1) 60° (2) 15° (3) 45° (4) 30°

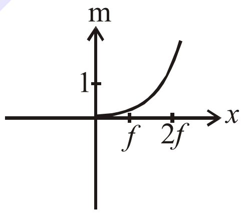
5. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is ----.

6. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by :
(Graphs are drawn schematically and are not to scale)

(1) 

(2) 

(3) 

(4) 

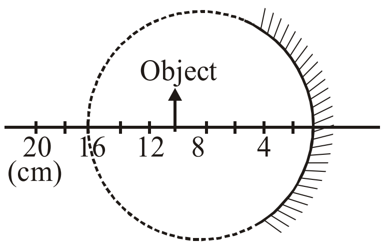
7. A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be :

(1) $\frac{h}{\sqrt{2}}$ (2) $\frac{3}{4}h\sqrt{2}$ (3) $\frac{h}{2\left(\sqrt{2}+1\right)}$ (4) $\frac{h}{3\sqrt{2}}$

8. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly) :
[Use the fact that surface area of a spherical cap of height h and radius of curvature r is $2\pi rh$]:

(1) 17% (2) 21%
(3) 34% (4) 50%

9.

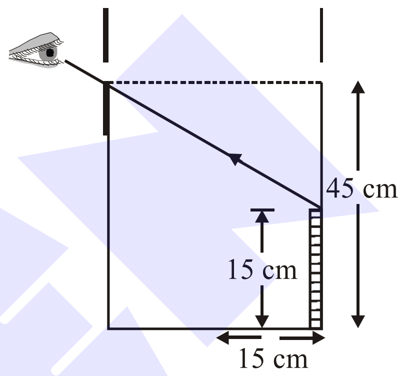


A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object ? (Figure drawn as schematic and not to scale)

(1) Inverted, real and magnified
(2) Erect, virtual and magnified
(3) Erect, virtual and unmagnified
(4) Inverted, real and unmagnified

10. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is______.

11. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid N/100, where N is an integer, the value of N is___________.

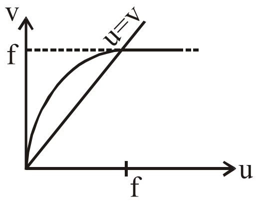


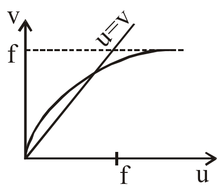
12. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is _______.

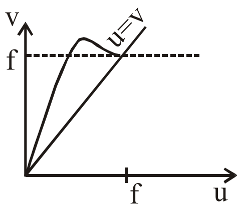
13. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ________.

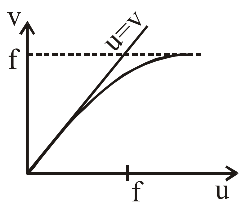
14. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$D where N is an integer, the value of N is ______.

15. For a concave lens of focal length f, the relation between object and image distance u and v, respectively, from its pole can best be represented by (u = v is the reference line):

(1) 

(2) 

(3) 

(4) 

16. A compound microscope consists of an objective lens of focal length 1cm and an eye piece of focal length 5 cm with a separation of 10 cm.
The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is ____.

17. A prism of angle A = 1° has a refractive index μ = 1.5. A good estimate for the minimum angle of deviation (in degrees) is close to N/10. Value of N is ______.

18. A point like object is placed at a distance of 1m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is :
(1) 1 m from the mirror, virtual
(2) 1 m from the mirror, real
(3) 2.6 m from the mirror, real
(4) 2.6 m from the mirror, virtual

19. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5 P is:

(1) $\frac{R}{2}$ (2) 2R (3) $\frac{3R}{2}$ (4) $\frac{R}{3}$

SOLUTION

1. NTA Ans. (1)

Sol. $m = \frac{LD}{f_e \times f_0} = \frac{150 \times 250}{f_e \times 25} = 375$

$f_e = 20$ mm.

2. NTA Ans. (3)

Sol. Using $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{....(1)}$$

and $\frac{1}{f_1} = \left(\frac{1.5}{1.42} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ....(2)

equation (1)/(2),

we get $\frac{f_1}{f} = \frac{0.5}{0.056}$

$= 8.93 \approx 9$

3. NTA Ans. (4)

Sol. $L = f_0 + f_e = 60$ cm

$M = \frac{f_0}{f_e} = 5$

$\Rightarrow f_0 = 5f_e$

$\therefore$ $6f_e = 60$ cm

$f_e = 10$ cm

4. NTA Ans. (4)

Sol. $\sin\theta_C = \frac{1}{\mu} = \frac{1}{\sqrt{3 \times 4/3}}$

$\theta_C = 30°$

5. NTA Ans. (60.00)

Sol. Using Lens-Maker's formula :

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = (1.5 - 1)\left(\frac{1}{30} - 0\right)$$

f = 60 cm

6. NTA Ans. (2)

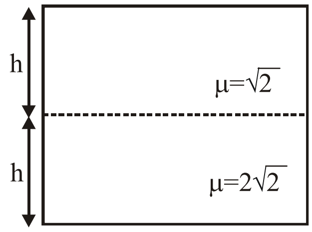
Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

At focus $m = \infty$ $\quad$ $x = f$

At centre $m = -1$ $\quad$ $x = 2f$

7. NTA Ans. (2)

Sol.



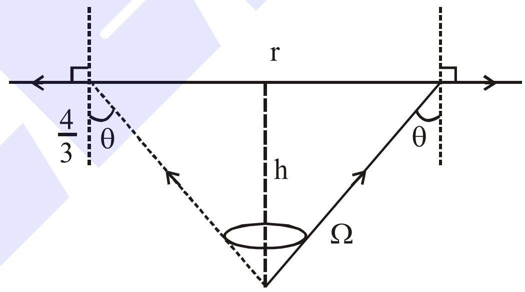
For near normal incidence,

$$h_{app} = \frac{h_{actual}}{\left(\frac{\mu_{in}}{\mu_{ref.}}\right)}$$

$$\therefore \quad h_{apparent} = \frac{\frac{h}{\left(\frac{2\sqrt{2}}{\sqrt{2}}\right)} + h}{\frac{\sqrt{2}}{1}} = \frac{3h}{2\sqrt{2}} = \frac{3}{4}h\sqrt{2}$$

8. NTA Ans. (1)

Sol.



$$\frac{4}{3}\sin\theta = 1\sin 90°$$

$$\sin\theta = \frac{3}{4}$$

Area of sphere in which light spread $= 4\pi R^2$

$\Omega = 2\pi\,(1 - \cos\,\theta)$

$$\Omega = 2\pi\left(1 - \frac{\sqrt{7}}{4}\right)$$

$P \rightarrow 4\pi$ steradians

$$P' \rightarrow \frac{P}{4\pi}(1 - \cos\theta)$$

$$\text{Ratio} = \frac{P'}{P} = \frac{2\pi(1 - \cos\theta)}{4\pi} = \frac{(1 - \cos\theta)}{2} = \frac{1 - \frac{\sqrt{7}}{4}}{2}$$

$$= \frac{0.33}{2} = 0.17$$

$\therefore$ Correct answer (1)

9. Official Ans. by NTA (1)
Official Ans. by ALLEN (4)

Sol. $f = \frac{-8}{2} = -4\text{cm}$

u = –10 cm
v = ?

as $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \left(\frac{1}{-10}\right) = \frac{1}{-4}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{4}$

$\frac{1}{v} = \frac{4-10}{40}$

$v = \frac{40}{-6}$

$v = \frac{-20}{3}$

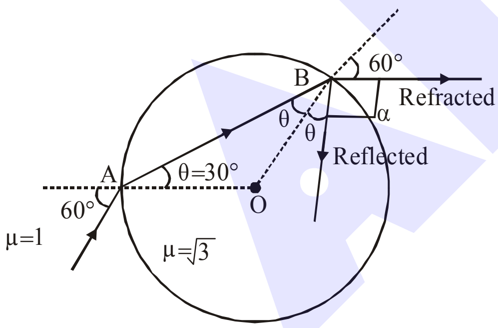
$m = \frac{-v}{u}$

$m = \frac{-\left(-\frac{20}{3}\right)}{-10} \Rightarrow m = \frac{-2}{3}$

or image will be real, inverted and unmagnified.

10. Official Ans. by NTA (90.00)

Sol.



By Snell's law at A :

$1 \times \sin 60° = \sqrt{3} \times \sin\theta$

$\frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

So at B :

θ + 60° + α = 180°

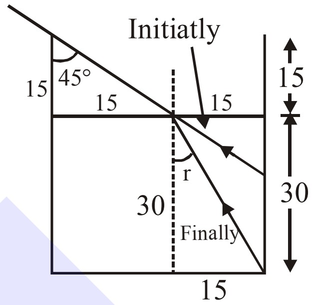
30° + 60° + α = 180°

α = 90°

11. Official Ans. by NTA (158)

Sol. $\tan r = \frac{15}{30} = \frac{1}{2}$

$\sin r = \frac{1}{\sqrt{5}}$



1sin45° = μsin r

$\frac{1}{\sqrt{2}} = \mu\left(\frac{1}{\sqrt{5}}\right)$

$\boxed{\mu = \sqrt{\frac{5}{2}} = 1.581}$

$\frac{N}{100} = \mu$

N = 100 μ

N = 158.11

So integer value of N = 15800

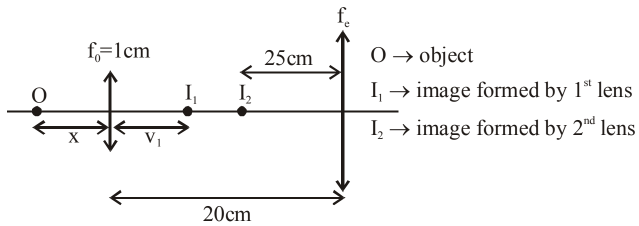
12. Official Ans. by NTA (1)

Sol. $\left|\left(\frac{dv}{dt}\right)\right| = \left|\frac{v^2}{4^2}\right|\left|\frac{du}{dt}\right|$

$= \left(\frac{10}{30}\right)2 \times 9 = 1\text{m/s}$

13. Official Ans. by NTA (5)
Official Ans. by ALLEN (4.48)

Sol.



for first lens = $\frac{1}{v_1} - \frac{1}{-x} = \frac{1}{1} \Rightarrow v_1 = \frac{x}{x-1}$

also magnification $|m_1| = \left|\frac{v_1}{u_1}\right| = \frac{1}{x-1}$

for 2^{nd} lens this is acting as object

so $u_2 = -(20 - v_1) = -\left(20 - \frac{x}{x-1}\right)$

and $v_2 = -25cm$

angular magnification $|m_A| = \left|\frac{D}{u_2}\right| = \frac{25}{|u_2|}$

Total magnification $m = m_1 m_A = 100$

$$\left(\frac{1}{x-1}\right)\left(\frac{25}{20-\frac{x}{x-1}}\right) = 100$$

$$\frac{25}{20(x-1)-x} = 100 \Rightarrow 1 = 80(x-1) - 4x$$

$$\Rightarrow 76x = 81 \Rightarrow x = \frac{81}{76}$$

$$\Rightarrow u_2 = -\left(20 - \frac{81/76}{81/76-1}\right) = \frac{-19}{5}$$

now by lens formula

$$\frac{1}{-25} - \frac{1}{-19/5} = \frac{1}{f_e} \Rightarrow f_e = \frac{25\times 19}{106} \approx 4.48cm$$

14. Official Ans. by NTA (5)
Official Ans. by ALLEN (476)

Sol. Using displacement method

$$f = \frac{D^2 - d^2}{4D}$$

Here, D = 100 cm
d = 40 cm

$$f = \frac{100^2 - 40^2}{4(100)} = 21 \text{ cm}$$

$$P = \frac{1}{f} = \frac{100}{21} D \qquad \frac{N}{100} = \frac{100}{21} \qquad N = 47$$

15. Official Ans. by NTA (4)

Sol. $v = \frac{uf}{u+f}$

Case-I

If $v = u$

$\Rightarrow f + u = f$

$\Rightarrow u = 0$

Case-II

If $u = \infty$

then $v = f$

Only option (4) satisfies this condition.

16. Official Ans. by NTA (50.00)

Sol. Final image at ∞

$\Rightarrow$ obj. for eye piece at 5cm

$\Rightarrow$ image for objective at 5 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \Rightarrow \frac{1}{5} + \frac{1}{x} = 1$$

$$\frac{1}{x} = 1 - \frac{1}{5} = \frac{4}{5} \Rightarrow x = \frac{5}{4}$$

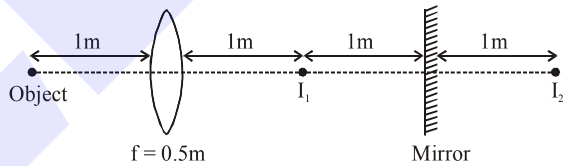
17. Official Ans. by NTA (5.00)

Sol. $\delta_{min} = (\mu - 1) A$
$= (1.5 - 1)1$
$= 0.5$

$\delta_{min} = \frac{5}{10}$
$N = 5$

18. Official Ans. by NTA (1,4)
Official Ans. by ALLEN (3)

Sol.



Object is at 2f. So image will also be at '2f'. (I_1).

Image of I_1 will be 1m behind mirror.

i.e. $\Rightarrow I_2$

Now I_2 will be object for lens.

$\therefore u \Rightarrow -3m$

$f \Rightarrow +0.5$ m

$$\frac{1}{v} \Rightarrow \frac{1}{f} + \frac{1}{u} \qquad \Rightarrow \frac{1}{+0.5} + \frac{1}{-3}$$

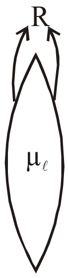
$$v \Rightarrow \frac{3}{5} \Rightarrow 0.6m$$

So total distance from mirror $\Rightarrow 2 + 0.6$

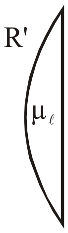
$\Rightarrow 2.6$ m and real image

Ans. (3)

19. Official Ans. by NTA (4)

Sol. 

$R_1 = R_2 = R$
Power (P)
Refractive index is assume (μ_ℓ)



$$P = \frac{1}{f} = (\mu_\ell - 1)\left(\frac{2}{R}\right) \quad \text{....(i)}$$

$$P' = \frac{1}{f'} = (\mu_\ell - 1)\left(\frac{1}{R'}\right) \quad \text{....(ii)}$$

$$P' = \frac{3}{2}P$$

$$(\mu_\ell - 1)\left(\frac{1}{R'}\right) = \mu\frac{3}{2}(\mu_\ell - 1)\left(\frac{2}{R}\right)$$

$$\therefore \quad R' = \frac{R}{3}$$