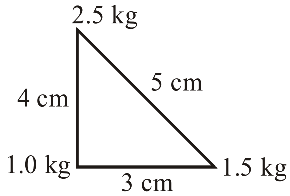
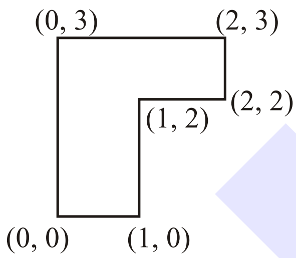
CENTRE OF MASS & COLLISION

1. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:



(1) 1.5 cm right and 1.2 cm above 1 kg mass
(2) 0.9 cm right and 2.0 cm above 1 kg mass
(3) 0.6 cm right and 2.0 cm above 1 kg mass
(4) 2.0 cm right and 0.9 cm above 1 kg mass

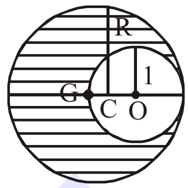
2. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are :



(1) (1.25m, 1.50m) (2) (1m, 1.75m)
(3) (0.75m, 0.75m) (4) (0.75m, 1.75m)

3. A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}\,ms^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\,ms^{-1}$. After collision, A moves with a velocity $\vec{v} = 4(\hat{i}+\hat{j})$. The energy of B after collision is written as $\frac{x}{10}J$. The value of x is ___________.

4. As shown in figure, when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e, on the surface of the cavity. R can be detemined by the equation :



(1) $(R^2 - R + 1)(2 - R) = 1$
(2) $(R^2 + R - 1)(2 - R) = 1$
(3) $(R^2 + R + 1)(2 - R) = 1$
(4) $(R^2 - R - 1)(2 - R) = 1$

5. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

(1) $\frac{1}{2}$ (2) $\sqrt{\frac{1}{2}}$

(3) $\sqrt{\frac{3}{4}}$ (4) $\sqrt{\frac{3}{2}}$

6. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is :

(1) $\frac{3}{4}mu^2$ (2) $\frac{1}{8}mu^2$

(3) $\sqrt{\frac{2}{3}}mu^2$ (4) $\frac{1}{3}mu^2$

7. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at :

(1) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (2) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

(3) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (4) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

8. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

(1) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$ (2) $2\sqrt{2}\frac{u^2}{g}$

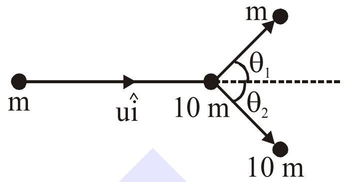
(3) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (4) $\frac{5}{8}\frac{u^2}{g}$

9. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by :

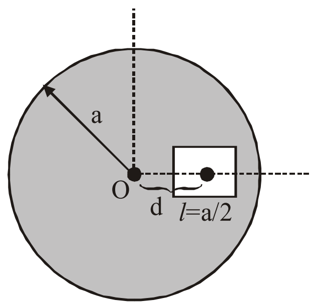
(1) $v = \sqrt{\frac{2}{3}}u$ (2) $v = \frac{1}{\sqrt{6}}u$

(3) $v = \frac{u}{\sqrt{3}}$ (4) $v = \frac{u}{\sqrt{2}}$

10. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is _______.



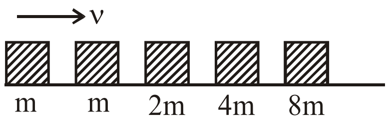
11. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is _____.



12. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10 m/s^2. Assume there is no rotational motion and loss of energy after the collision is negligable.]

(1) 21 J (2) 23 J (3) 19 J (4) 20 J

13. Blocks of masses m, 2m, 4m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to :



(1) 77 (2) 37 (3) 87 (4) 94

14. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where v(t) is its instantaneous velocity. The instantaneous acceleration of the satellite is:

(1) $-\frac{2bv^3}{M(t)}$ (2) $-\frac{bv^3}{2M(t)}$

(3) $-bv^3(t)$ (4) $-\frac{bv^3}{M(t)}$

15. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is.

16. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i} + \hat{j})ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i} + \sqrt{3}\hat{j})ms^{-1}$, the angle between $\vec{V}_1$ and $\vec{V}_2$ is :

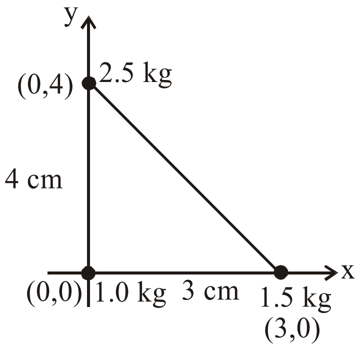
(1) 60° (2) 15° (3) –45° (4) 105°

17. The centre of mass of a solid hemisphere of radius 8 cm is X cm from the centre of the flat surface. Then value of x is ________ .

SOLUTION

1. NTA Ans. (2)

Sol.



Let 1 kg as origin and x-y axis as shown

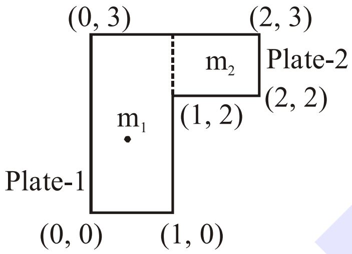
$$x_{cm} = \frac{1(0)+1.5(3)+2.5(0)}{5} = 0.9 \text{ cm}$$

$$y_{cm} = \frac{1(0)+1.5(0)+2.5(4)}{5} = 2 \text{ cm}$$

2. NTA Ans. (4)

Sol. $m_1 = 3\text{kg}$

$m_2 = 1\text{kg}$



Mass of plate-1 is assumed to be concentrated at (0.5, 1.5)

Mass of plate-2 is assumed to be concentrated at (1.5, 2.5).

$$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{3\times0.5+1\times1.5}{4} = 0.75$$

$$y_{cm} = \frac{m_1y_1+m_2y_2}{m_1+m_2} = \frac{3\times1.5+1\times2.5}{4} = 1.75$$

3. NTA Ans. (1.00)

Sol. By conservation of linear momentum :

$$(0.1)(3\hat{i})+(0.1)(5\hat{j}) = (0.1)(4)(\hat{i}+\hat{j})+(0.1)\vec{v}$$

$$\Rightarrow \vec{v} = -\hat{i}+\hat{j}$$

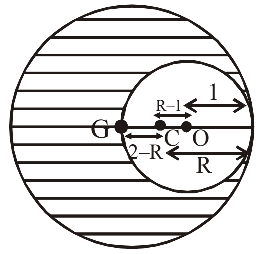
$\therefore$ Speed of B after collision $|\vec{v}| = \sqrt{2}$

Now, kinetic energy $= \frac{1}{2}mV^2 = \frac{1}{2}(0.1)(2) = \frac{1}{10}$

$\therefore$ x = 1

4. NTA Ans. (3)

Sol.



By concept of COM

$m_1R_1 = m_2R_2$

Remaining mass × (2–R) = cavity mass × (R–1)

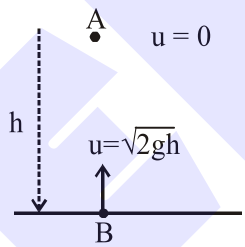
$$\left(\frac{4}{3}\pi R^3\rho - \frac{4}{3}\pi 1^3\rho\right)(2-R) = \frac{4}{3}\pi 1^3\rho\times(R-1)$$

$(R^3 - 1)(2 - R) = R - 1$

$(R^2 + R + 1)(2 - R) = 1$

5. NTA Ans. (4)

Sol.

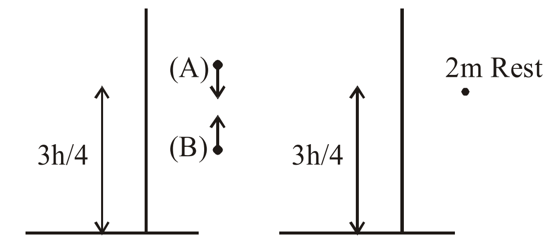


Particles will collide after time $t_0 = \frac{h}{\sqrt{2gh}}$

at collision, $v_A = gt_0$ $\quad v_B = u_B - gt_0$

$\Rightarrow v_A = -v_B$

Before collision After collision



Time taken by combined mass to reach the ground

$$\text{time} = \sqrt{\frac{2\times3h/4}{g}} = \sqrt{\frac{3h}{2g}}$$

6. NTA Ans. (2)

Sol. From momentum conservation

$$mu\hat{i} + mu\left(\frac{\hat{i}+\hat{j}}{2}\right) = (m+m)\vec{v}$$

$$\Rightarrow \vec{v} = \frac{3}{4}u\hat{i} + \frac{u}{4}\hat{j}$$

$$\Rightarrow |v| = \frac{u}{4}\sqrt{10}$$

Final kinetic energy = $\frac{1}{2}2m\left(\frac{u}{4}\sqrt{10}\right)^2 = \frac{5}{8}mu^2$

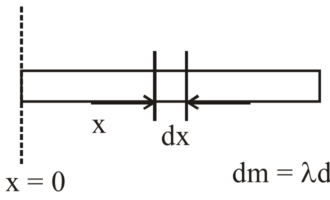
Initial kinetic energy

$$= \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{6}{8}mu^2$$

Loss in K.E. = $k_i - k_f = \frac{1}{8}mu^2$

7. NTA Ans. (4)

Sol.



dm = λdx

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int(\lambda dx)x}{\int dm}$$

$$= \frac{\int_0^L\left(a + \frac{bx^2}{L^2}\right)x\,dx}{\int_0^L\left(a + \frac{bx^2}{L^2}\right)dx}$$

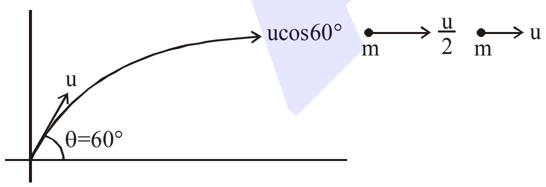
$$= \frac{\frac{aL^2}{2} + \frac{b}{L^2}\cdot\frac{L^4}{4}}{aL + \frac{b}{L^2}\cdot\frac{L^3}{3}}$$

$$= \frac{\left(\frac{4a+2b}{8}\right)L}{\frac{(3a+b)}{3}} = \frac{3}{4}\frac{(2a+b)L}{(3a+b)}$$

∴ correct answer 4

8. NTA Ans. (3)

Sol.



By momentum conservation,

$$\frac{mu}{2} + mu = 2mv'$$

$$v' = \frac{3v}{4}$$

Range after collision = $\frac{3v}{4}\sqrt{\frac{2H}{g}}$

$$= \frac{3v}{4}\sqrt{\frac{2\cdot u^2\sin^2 60°}{g\,2g}}$$

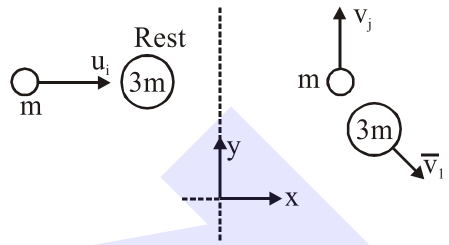
$$= \frac{3}{4}\frac{\sqrt{3}}{2}\cdot\frac{u^2}{g} = \frac{3\sqrt{3}u^2}{8g}$$

∴ Correct answer (3)

9. Official Ans. by NTA (4)

Before collision After collision

Sol.



From momentum conservation

$\vec{P}_i = \vec{P}_f$

m(ui) + 3m(0) = mvj + 3m $\bar{v}_1$

mui – mvj = 3m $\bar{v}_1$

$$\bar{v}_1 = \frac{ui - vj}{3}$$

or $|v_1| = \frac{\sqrt{u^2 + v^2}}{3}$

or $v_1^2 = \frac{u^2 + v^2}{9}$ ....(1)

As collision is perfectly elastic hence

$k_i = k_j$

$$\frac{1}{2}mu^2 + \frac{1}{2}3m0^2 = \frac{1}{2}mv^2 + \frac{1}{2}3mv_1^2$$

⇒ $u^2 = v^2 + 3v_1^2$

$$u^2 = v^2 + 3\frac{(u^2 + v^2)}{9}$$

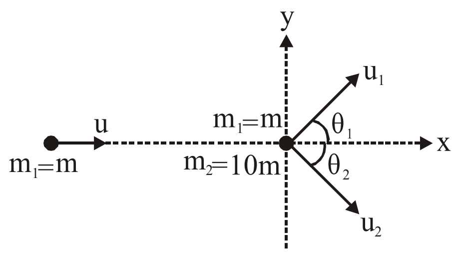
⇒ $3u^2 = 3v^2 + u^2 + v^2$

⇒ $2u^2 = 4v^2$

$$v = \frac{u}{\sqrt{2}}$$

10. Official Ans. by NTA (10.00)

Sol.



By momentum conservation along y :
$m_1u_1\sin\theta_1 = m_2u_2\sin\theta_2$
i.e. $mu_1\sin\theta_1 = 10mu_2\sin\theta_2$

$$\Rightarrow \boxed{u_1\sin\theta_1 = 10u_2\sin\theta_2} \quad .....(i)$$

$$kf_{m_1} = \frac{1}{2}ki_{m_1} \quad \text{i.e.} \quad \frac{1}{2}mu_1^2 = \frac{1}{2}\times\frac{1}{2}mu^2$$

$$\text{i.e.} \boxed{u_1 = \frac{u}{\sqrt{2}}} \quad .....(ii)$$

Also collision is elastic : $k_i = k_f$

$$\frac{1}{2}mu^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}.10m.u_2^2$$

$$\frac{1}{2}mu^2 = \frac{1}{2}\times\frac{1}{2}mu^2 + \frac{1}{2}\times 10m.u_2^2$$

$$\frac{1}{4}mu^2 = \frac{1}{2}\times 10\times mu_2^2$$

$$\boxed{u_2 = \frac{u}{\sqrt{20}}} \quad ....(iii)$$

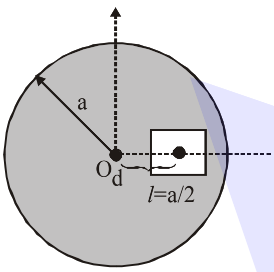
Putting (ii) & (iii) in (i)

$$\frac{u}{\sqrt{2}}\sin\theta_1 = 10.\frac{u}{\sqrt{20}}\sin\theta_2$$

$$\boxed{\sin\theta_1 = \sqrt{10}\sin\theta_2} \rightarrow \text{Hence } n = 10$$

11. Official Ans. by NTA (23.00)

Sol.



$$X_{com} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

where :
- m_1 = mass of complete disc
- m_2 = removed mass
- Let σ = surface mass density of disc material

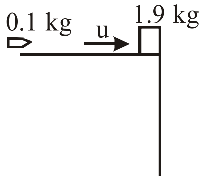
$$\text{wrt 'O'} : X_{com} = \frac{\sigma\pi a^2(O) - \sigma.\frac{a^2}{4}.d}{\sigma\pi a^2 - \sigma\frac{a^2}{4}} = \frac{-\frac{a^2}{4}d}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-d}{4\pi - 1} = -\frac{a}{2(4\pi - 1)}$$

So, X = 2(4π–1) = (8π–2) = 23.12
So, nearest integer value of X = 23

12. Official Ans. by NTA (1)

Sol.

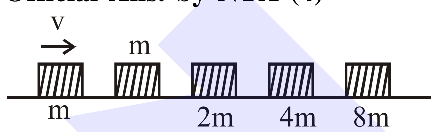


$p_i = p_f \Rightarrow 0.1 \times 20 = 2v$
$\therefore$ v = 1 m/s

$$KE_f = mgh + \frac{1}{2}mv^2 = 213$$

13. Official Ans. by NTA (4)

Sol.



All collisions are perfectly inelastic, so after the final collision, all blocks are moving together. So let the final velocity be v', so on applying momentum conservation:
mv = 16m v' ⇒ v' = v/16

$$\text{Now initial energy } E_i = \frac{1}{2}mv^2$$

$$\text{Final energy} : E_f = \frac{1}{2}\times 16m\times\left(\frac{v}{16}\right)^2$$

$$\Rightarrow E_f = \frac{1}{2}m\frac{v^2}{16}$$

$$\text{Energy loss} : E_i - E_f \quad \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$$

$$\Rightarrow \frac{1}{2}mv^2\left[1 - \frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$$

$$\%p = \frac{\text{Energy loss}}{\text{Original energy}}\times 100$$

$$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2}\times 100 = 93.75\%$$

⇒ Value of P is close to 94.

14. Official Ans. by NTA (4)

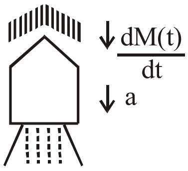
Sol. $$\frac{dm(t)}{dt} = bv^2$$

$$F_{thast} = v\frac{dm}{dt}$$

$$\text{Force on statellile} = -\vec{v}\frac{dm(t)}{dt}$$

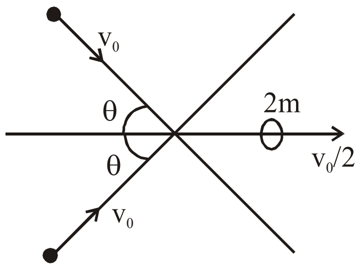
$M(t)\,a = -\,v\,(bv^2)$

$$\boxed{a = a\frac{bv^3}{M(t)}}$$



15. Official Ans. by NTA (120.00)

Sol.



Momentum conservation along x

$$2mv_0\cos\theta = 2m\frac{v_0}{2}$$

$$\cos\theta = \frac{1}{2}$$

$\theta = 60$

Angle is $2\theta = 120$

Ans. 120.00

16. Official Ans. by NTA (4)

Sol. $\vec{v}_{01} = (\sqrt{3}\hat{i} + \hat{j})\ m/s$

$\vec{v}_{02} = \vec{0}$

$m_1 = 2m_2$

After collision, $\vec{v}_1 = (\hat{i} + \sqrt{3}\hat{j})\ m/s$

$\vec{v}_2 = ?$

Applying conservation of linear momentum,

$$m_1\vec{v}_{01} + m_2\vec{v}_{02} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$2m_2(\sqrt{3}\hat{i} + \hat{j}) + 0 = 2m_2(\hat{i} + \sqrt{3}\hat{j}) + m_2\vec{v}_2$$

$$\vec{v}_2 = 2(\sqrt{3}\hat{i} + \hat{j}) - 2(\hat{i} + \sqrt{3}\hat{j})$$

$$= 2(\sqrt{3}\hat{i} - \hat{j}) + 2(\hat{i} - \sqrt{3}\hat{j})$$

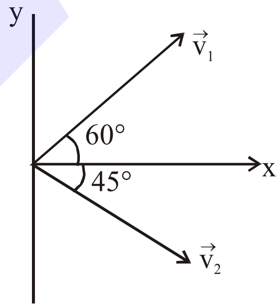
$$\vec{v}_2 = 2(\sqrt{3} - 1)(\hat{i} - \hat{j})$$

for angle between $\vec{v}_1$ & $\vec{v}_2$,

$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{v_1 v_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2 \times 2\sqrt{2}(\sqrt{3}-1)}$$

$$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = 105°$$

or



17. Official Ans. by NTA (3.00)

Sol. $x = \frac{3R}{8} = 3cm$

$x = 3$