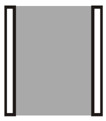
CAPACITOR

1. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression :



(1) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$ (2) $\frac{A\varepsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(3) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$ (4) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$

2. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capactior is parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) ____.

3. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be :

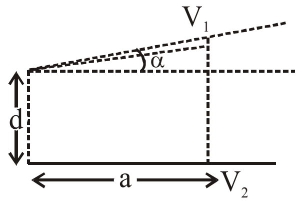
(1) 3.2 μF

(2) 8.4 μF

(3) 1.6 μF

(4) 4.2 μF

4. A capacitor is made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to :



(1) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$ (2) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$

(3) $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$ (4) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

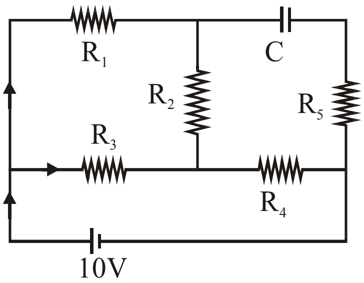
5. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$J then value of X to the nearest integer is________.

6. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is:

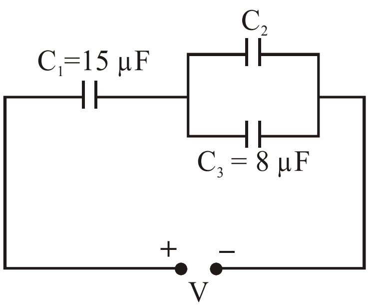
(1) 10 μF (2) 15 μF

(3) 20 μF (4) 30 μF

7. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2 Ω. The potential difference (in V) across the capacitor when it is fully charged is _______.



8. In the circuit shown in the figure, the total charge in 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is :



(1) 590 μC
(2) 450 μC
(3) 650 μC
(4) 160 μC

9. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is :

(1) $\frac{1}{6}CV_0^2$

(2) $\frac{1}{2}CV_0^2$

(3) $\frac{1}{3}CV_0^2$

(4) $\frac{1}{4}CV_0^2$

10. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is:

(1) $\frac{9}{2}CV^2$

(2) $\frac{25}{6}CV^2$

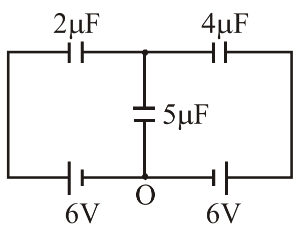
(3) zero

(4) $\frac{3}{2}CV^2$

11. A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant k = 4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will be energy stored in the capacitor be two times the initial energy stored?

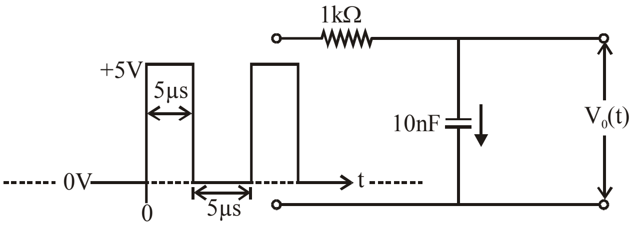
(1) $l / 4$
(2) $l / 2$
(3) $l / 3$
(4) $2l / 3$

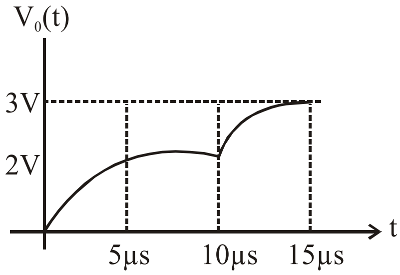
12. In the circuit shown, charge on the 5 μF capacitor is :

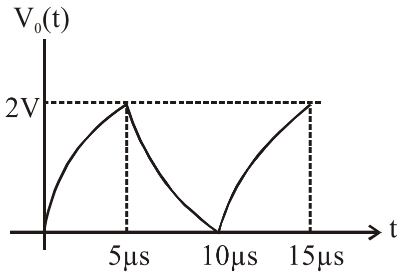


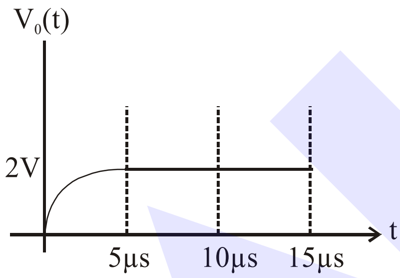
(1) 5.45 μC
(2) 16.36 μC
(3) 10.90 μC
(4) 18.00 μC

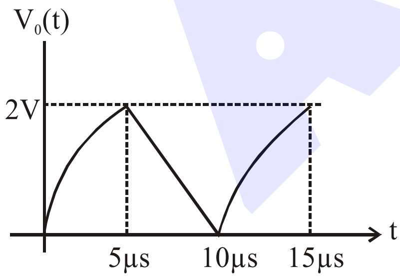
13. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_D(t)$, across the capacitor is correctly depicted by:



(1)


(2)


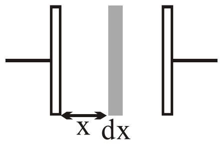
(3)


(4)


SOLUTION

Sol. As K is variable we take a plate element of Area A and thickness dx at distance x
Capacitance of element

$$dC=\frac{(A)K(1+\alpha x)\varepsilon_0}{dx}$$



Now all such elements are is series so equivalent capacitance

$$\frac{1}{C}=\int\frac{1}{dC}=\int_0^d\frac{dx}{AK\varepsilon_0(1+\alpha x)}$$

$$\frac{1}{C}=\frac{1}{\alpha AK\varepsilon_0}\ln\left(\frac{1+\alpha d}{1}\right)$$

$$=\frac{1}{C}=\frac{1}{\alpha AK\varepsilon_0}\left(\alpha d-\frac{(\alpha d)^2}{2}+\frac{(\alpha d)^3}{3}+....\right)$$

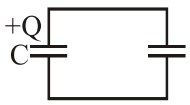
$$\Rightarrow\frac{1}{C}=\frac{\alpha d}{\alpha AK\varepsilon_0}\left(1-\frac{\alpha d}{2}+\frac{(\alpha d)^2}{3}+....\right)$$

$$\frac{1}{C}=\frac{d}{AK\varepsilon_0}\left(1-\frac{\alpha d}{2}\right)$$

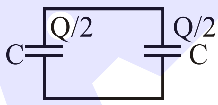
$$C=\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$$

2. NTA Ans. (6)

Sol.



$\Rightarrow$



$Q = CV$

$$\Delta Q_L=\frac{Q^2}{2C}-\left[\frac{(Q/2)^2}{2C}\times 2\right]=\frac{Q^2}{4C}$$

$$=\frac{1}{4}CV^2$$

$$=\frac{1}{4}\times 60\times 10^{-12}\times 4\times 10^2$$

$= 6nJ$

3. NTA Ans. (3)

Sol. $C_1 + C_2 = 10$ ....(i)

$$\frac{1}{2}C_2V^2=4\times\frac{1}{2}C_1V^2$$

$\therefore$ $C_2 = 4C_1$ ....(ii)

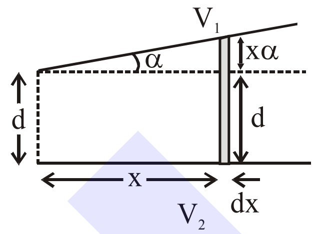
$\therefore$ $C_1 = 2$ & $C_2 = 8$

For series combination

$$C_{eq}=\frac{C_1C_2}{C_1+C_2}=1.6$$

4. NTA Ans. (4)

Sol. Assume small element dx at a distance x from left end



Capacitance for small element dx is

$$dC=\frac{\varepsilon_0 a\,dx}{d+x\alpha}$$

$$C=\int_0^a\frac{\varepsilon_0 a\,dx}{d+x\alpha}$$

$$=\frac{\varepsilon_0 a}{\alpha}\ln\left(\frac{1+\alpha a}{d}\right)\Bigg|_0^a \qquad \left(\ln(1+x)\approx x-\frac{x^2}{2}\right)$$

$$=\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$$

5. Official Ans. by NTA (36)
Official Ans. by ALLEN (4 Actual 4.033)

Sol. $u_i=\frac{1}{2}\times 5\times 10^{-6}(220)^2$

Final common potential

$$v=\frac{220\times 5+0\times 2.5}{5+2.5}=220\times\frac{2}{3}$$

$$u_f=\frac{1}{2}(5+2.5)\times 10^{-6}\left(220\times\frac{2}{3}\right)^2$$

$\Delta u = u_f - u_i$

$\Delta u = -403.33 \times 10^{-4}$

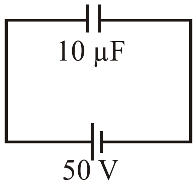
$$\Rightarrow -403.33\times 10^{-4}=\frac{X}{100}$$

$X = -4.03$

or magnitude or value of X is approximate 4

6. Official Ans. by NTA (2)

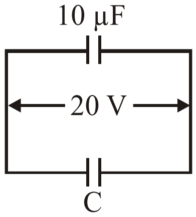
Sol. Initially



- Charge on capacitor 10 µF

$Q = CV = (10\ \mu F)\ (50V)$

$Q = 500\ \mu C$•



- Final Charge on 10 µF capacitor

$Q = CV = (10\ \mu F)\ (20V)$

$Q = 200\ \mu C$

- From charge conservation,

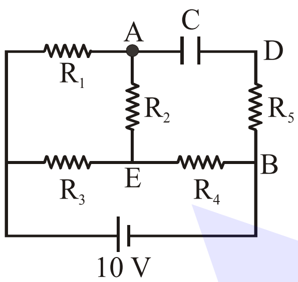
Charge on unknown capacitor

$C = 500\ \mu C - 200\ \mu C = 300\ \mu C$

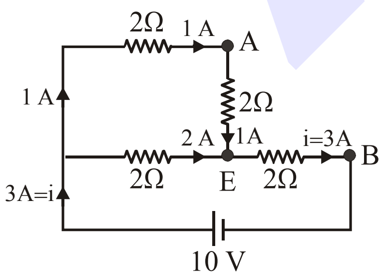
$$\Rightarrow \text{Capacitance (C)} = \frac{Q}{V} = \frac{300\ \mu C}{20\ V} = 15\ \mu F$$

7. Official Ans. by NTA (8.00)

Sol.



- R_1 to $R_5 \rightarrow$ each 2Ω
- Cap. is fully charged
- So no current is there in branch ADB
- Effective circuit of current flow :



$$R_{eq} = \left(\frac{4 \times 2}{4+2}\right) + 2$$

$$R_{eq} = \frac{4}{3} + 2 = \frac{10}{3}\Omega$$

$$i = \frac{10}{10/3} = 3A$$

So potential different across AEB

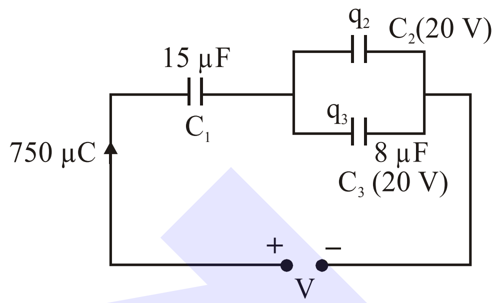
$\Rightarrow 2 \times 1 + 2 \times 3 = 8V$

Hence potential difference across

Capacitor $= \Delta V = V_{AEB} = 8V$

8. Official Ans. by NTA (1)

Sol.



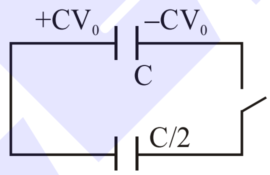
$q_3 = 20 \times 8 = 160\ \mu C$

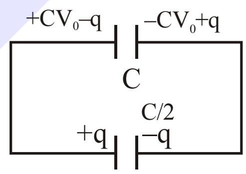
$\therefore\ q_2 = 750 - 160 = 590\ \mu C$

9. Official Ans. by NTA (4)

Official Ans. by ALLEN (1)

Sol.





$$\frac{CV_0 - q}{C} = \frac{q}{C/2} = \frac{2q}{C}$$

$$V_0 = \frac{3q}{C} \quad \Rightarrow q = \frac{CV_0}{3}$$

$$U_i = \frac{1}{2}CV_0^2$$

$$U_f = \frac{\left(\frac{2CV_0}{3}\right)^2}{2C} + \frac{\left(\frac{CV_0}{3}\right)^2}{2\left(\frac{C}{2}\right)}$$

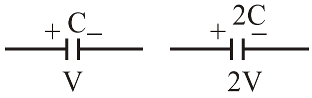
$$= \frac{1}{2}CV_0^2\left[\frac{4}{9} + \frac{2}{9}\right] = \frac{1}{2}CV_0^2\left(\frac{2}{3}\right)$$

$$\text{Heat loss} = \frac{1}{2}CV_0^2 - \left(\frac{2}{3}\right)\left(\frac{1}{2}CV_0^2\right)$$

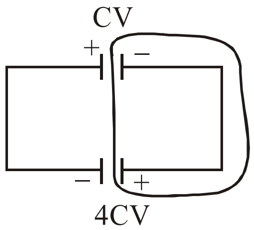
$$= \frac{1}{6}CV_0^2$$

10. Official Ans. by NTA (4)

Sol.



$Q_1 = CV$ $\quad Q_2 = 2C \times 2V = 4CV$



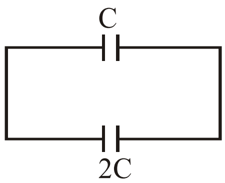
$\Rightarrow$ By conservation of charge

$q_i = q_f$

$Q_1 + Q_2 = q_1 + q_2$

$4CV - CV = (C + 2C)\, V_C$

$$V_C = \frac{3CV}{3C} \Rightarrow V$$

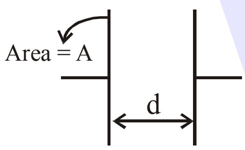


$$\Rightarrow \frac{1}{2}\times(3C)\times V_C^2$$

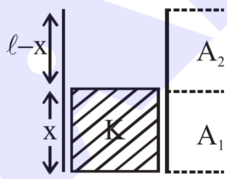
$$= \frac{1}{2} \times 3C \times V^2 = \frac{3}{2}CV^2$$

11. Official Ans. by NTA (3)

Sol.



Before inserting slab



After inserting dielectric slab

$$C_i = \frac{\varepsilon_0 A}{d} \qquad C_f = C_1 + C_2$$

$$C_i = \frac{\varepsilon_0 \ell w}{d} \qquad C_f = \frac{K\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$$

$$C_f = \frac{K\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(\ell - x)}{d}$$

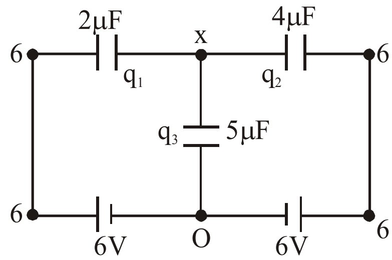
$$C_f = 2C_i \Rightarrow \frac{K\varepsilon_0 wx}{x} + \frac{\varepsilon_0 w(\ell - x)}{d} = \frac{2\varepsilon_0 \ell w}{d}$$

$$4x + \ell - x = 2\ell$$

$$\boxed{x = \frac{\ell}{3}}$$

12. Official Ans. by NTA (2)

Sol.



Let potential of point O $v_0 = 0$

Now, using junction analysis

We can say, $q_1 + q_2 + q_3 = 0$

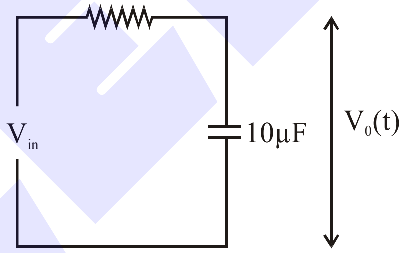
$2(x - 6) + 4(x - 6) + 5(x) = 0$

$$x = \frac{36}{11} \qquad q_3 = \frac{36(5)}{11} = \frac{180}{11}$$

$q_3 = 16.36\ \mu C$

13. Official Ans. by NTA (1)

Sol.



$$V_0(t) = V_{in}\left(1 - e^{-\frac{t}{RC}}\right)$$

at $t = 5\mu s$

$$V_0(t) = 5\left(1 - e^{-\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right)$$

$= 5\,(1 - e^{-0.5}) = 2V$

Now $V_{in} = 0$ means discharging

$$V_0(t) = 2e^{-\frac{t}{RC}} = 2e^{-0.5}$$

$= 1.21$ V

Now for next 5 μs

$$V_0(t) = 5 - 3.79e^{-\frac{t}{RC}}$$

after 5 μs again

$V_0(t) = 2.79$ Volt $\approx 3V$

Most appropriate Ans. (1)

$$U\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$