

LIMIT

1. $\lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ बराबर है _____ ।

2. $\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2} \right)^{\frac{1}{x^2}}$ बराबर है :

- (1) $\frac{1}{e}$ (2) e^2
 (3) e (4) $\frac{1}{e^2}$

3. यदि $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} = 820, (n \in \mathbb{N})$ है, तो n का मान है _____.

4. $\lim_{x \rightarrow 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{1/x}$ बराबर है -

- (1) 2 (2) e
 (3) 1 (4) e^2

5. माना $[t]$ महत्तम पूर्णांक $\leq t$ को दर्शाता है। यदि किसी $\lambda \in \mathbb{R} - \{0, 1\}$ के लिए $\lim_{x \rightarrow 0} \left| \frac{1 - x + |x|}{\lambda - x + [x]} \right| = L$ है, तो L का मान है -

- (1) 1 (2) 2
 (3) $\frac{1}{2}$ (4) 0

6. यदि $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^k$, तो k का मान है _____ ।

7. $\lim_{x \rightarrow a} \frac{(a + 2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a + x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}}$ ($a \neq 0$) बराबर है :

- (1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$
 (3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

8. माना $f : (0, \infty) \rightarrow (0, \infty)$ एक ऐसा अवकलनीय फलन है कि $f(1) = e$ तथा $\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$ है यदि

$f(x) = 1$, है, तो x का मान है

- (1) $2e$ (2) $\frac{1}{2e}$
 (3) e (4) $\frac{1}{e}$

9. यदि α , समीकरण $p(x) = x^2 - x - 2 = 0$ का धनात्मक मूल है, तो $\lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4}$ बराबर है :

- (1) $\frac{3}{\sqrt{2}}$ (2) $\frac{3}{2}$
 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

10. $\lim_{x \rightarrow 0} \frac{x \left(e^{(\sqrt{1+x^2+x^4}-1)/x} - 1 \right)}{\sqrt{1+x^2+x^4}-1}$

- (1) का अस्तित्व नहीं है। (2) \sqrt{e} के बराबर है।
 (3) 0 के बराबर है। (4) 1 के बराबर है।

11. $\lim_{x \rightarrow 1} \frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1) \sin(x-1)}$

- (1) का अस्तित्व नहीं है। (2) $\frac{1}{2}$ के बराबर है।
 (3) 1 के बराबर (4) $-\frac{1}{2}$ के बराबर है।

SOLUTION

1. NTA Ans. (36)

$$\begin{aligned} \text{Sol. } \lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}} &\Rightarrow \lim_{x \rightarrow 2} \frac{3^{2x} - 12 \cdot 3^x + 27}{3^{x/2} - 3} \\ &= \lim_{x \rightarrow 2} \frac{(3^x - 9)(3^x - 3)}{(3^{x/2} - 3)} \\ &= \lim_{x \rightarrow 2} \frac{(3^{x/2} + 3)(3^{x/2} - 3)(3^x - 3)}{(3^{x/2} - 3)} \\ &= 36 \end{aligned}$$

2. NTA Ans. (4)

$$\begin{aligned} \text{Sol. } \text{Required limit} &= e^{\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2} - 1 \right) \frac{1}{x^2}} \\ &= e^{\lim_{x \rightarrow 0} \left(\frac{-4}{7x^2 + 2} \right)} = \frac{1}{e^2} \end{aligned}$$

3. Official Ans. by NTA (40.00)

$$\begin{aligned} \text{Sol. } \lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^2 - n}{x - 1} &= 820 \\ \Rightarrow \lim_{x \rightarrow 1} \left(\frac{x-1}{x-1} + \frac{x^2-1}{x-1} + \dots + \frac{x^n-1}{x-1} \right) &= 820 \\ \Rightarrow 1 + 2 + \dots + n &= 820 \\ \Rightarrow n(n+1) &= 2 \times 820 \\ \Rightarrow n(n+1) &= 40 \times 41 \\ \text{Since } n \in \mathbb{N}, \text{ so } \boxed{n=40} \end{aligned}$$

4. Official Ans. by NTA (4)

$$\begin{aligned} \text{Sol. } \lim_{x \rightarrow 0} \left\{ \tan \left(\frac{\pi}{4} + x \right) \right\}^{1/x} \\ &= e^{\lim_{x \rightarrow 0} \frac{1}{x} \left\{ \tan \left(\frac{\pi}{4} + x \right) - 1 \right\}} \\ &= e^{\lim_{x \rightarrow 0} \left(\frac{1 + \tan x - 1 + \tan x}{x(1 - \tan x)} \right)} \\ &= e^{\lim_{x \rightarrow 0} \frac{2 \tan x}{x(1 - \tan x)}} \\ &= e^2 \end{aligned}$$

5. Official Ans. by NTA (2)

$$\text{Sol. LHL : } \lim_{x \rightarrow 0^-} \left| \frac{1-x-x}{\lambda-x-1} \right| = \left| \frac{1}{\lambda-1} \right|$$

$$\text{RHL : } \lim_{x \rightarrow 0^+} \left| \frac{1-x+x}{\lambda-x+1} \right| = \left| \frac{1}{\lambda} \right|$$

For existence of limit

$$\text{LHL} = \text{RHL}$$

$$\Rightarrow \frac{1}{|\lambda-1|} = \frac{1}{|\lambda|} \Rightarrow \lambda = \frac{1}{2}$$

$$\therefore L = \frac{1}{|\lambda|} = 2$$

6. Official Ans. by NTA (8)

$$\text{Sol. } \lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\left(1 - \cos \frac{x^2}{2} \right) \left(1 - \cos \frac{x^2}{4} \right)}{4 \left(\frac{x^2}{2} \right)^2 \cdot 16 \left(\frac{x^2}{4} \right)^2} = \frac{1}{8} \times \frac{1}{32} = 2^{-k}$$

$$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8.$$

7. Official Ans. by NTA (1)

Sol. Required limit

$$\begin{aligned}
 L &= \lim_{h \rightarrow 0} \frac{(a+2(a+h))^{1/3} - (3(a+h))^{1/3}}{(3a+a+h)^{1/3} - (4(a+h))^{1/3}} \\
 &= \lim_{h \rightarrow 0} \frac{(3a)^{1/3} \left(1 + \frac{2h}{3a}\right)^{1/3} - (3a)^{1/3} \left(1 + \frac{h}{a}\right)^{1/3}}{(4a)^{1/3} \left(1 + \frac{h}{4a}\right)^{1/3} - (4a)^{1/3} \left(1 + \frac{h}{a}\right)^{1/3}} \\
 &= \lim_{h \rightarrow 0} \left(\frac{3^{1/3}}{4^{1/3}}\right) \frac{\left(1 + \frac{2h}{9a}\right) - \left(1 + \frac{h}{3a}\right)}{\left(1 + \frac{h}{12a}\right) - \left(1 + \frac{h}{3a}\right)} \\
 &= \left(\frac{3}{4}\right)^{1/3} \frac{\left(\frac{2}{9} - \frac{1}{3}\right)}{\left(\frac{1}{12} - \frac{1}{3}\right)} = \left(\frac{3}{4}\right)^{1/3} \frac{(8-12)}{(3-12)} \\
 &= \left(\frac{3}{4}\right)^{1/3} \frac{(-4)}{(-9)} = \frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}} = \frac{4^{2/3}}{3^{5/3}} \\
 &= \frac{(8 \times 2)^{1/3}}{(27 \times 9)^{1/3}} = \frac{2}{3} \left(\frac{2}{9}\right)^{1/3}
 \end{aligned}$$

8. Official Ans. by NTA (4)

Sol. $L = \lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x}$

using L.H. rule

$$\begin{aligned}
 L &= \lim_{t \rightarrow x} \frac{2t f^2(x) - x^2 \cdot 2f'(t) \cdot f(t)}{1} \\
 \Rightarrow L &= 2xf(x) (f(x) - x f'(x)) = 0 \text{ (given)} \\
 \Rightarrow f(x) &= x f'(x) \Rightarrow \int \frac{f'(x) dx}{f(x)} = \int \frac{dx}{x} \\
 \Rightarrow \ln |f(x)| &= \ln |x| + C \\
 \therefore f(1) &= e, x > 0, f(x) > 0 \\
 \Rightarrow f(x) &= ex, \text{ if } f(x) = 1 \Rightarrow x = \frac{1}{e}
 \end{aligned}$$

9. Official Ans. by NTA (1)

Sol. $x^2 - x - 2 = 0$

roots are 2 & -1

$$\begin{aligned}
 \Rightarrow \lim_{x \rightarrow 2^+} \frac{\sqrt{1 - \cos(x^2 - x - 2)}}{(x - 2)} \\
 &= \lim_{x \rightarrow 2^+} \frac{\sqrt{2 \sin^2 \frac{(x^2 - x - 2)}{2}}}{(x - 2)} \\
 &= \lim_{x \rightarrow 2^+} \frac{\sqrt{2} \sin \left(\frac{(x-2)(x+1)}{2}\right)}{(x-2)} = \frac{3}{\sqrt{2}}
 \end{aligned}$$

10. Official Ans. by NTA (4)

Sol. $\lim_{x \rightarrow 0} \frac{x \left(e^{\frac{(\sqrt{1+x^2+x^4}-1)}{x}} - 1 \right)}{\sqrt{1+x^2+x^4}-1}$

$\therefore \lim_{x \rightarrow 0} \frac{\sqrt{1+x^2+x^4}-1}{x} \left(\frac{0}{0} \text{ from} \right)$

$$\lim_{x \rightarrow 0} \frac{(1+x^2+x^4)-1}{x(\sqrt{1+x^2+x^4}+1)}$$

$$\lim_{x \rightarrow 0} \frac{x(1+x^2)}{(\sqrt{1+x^2+x^4}+1)} = 0$$

So $\lim_{x \rightarrow 0} \frac{x \left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} - 1 \right)}{\sqrt{1+x^2+x^4}-1} \left(\frac{0}{0} \text{ from} \right)$

$$\lim_{x \rightarrow 0} \frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} = 1$$

11. **Official Ans. by NTA (1)**
Official Ans. by ALLEN
(Bonus-Answers musbe zero)

Sol.
$$\lim_{x \rightarrow 1} \frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1) \sin(x-1)} \left(\frac{0}{0} \right)$$

Apply L Hopital Rule

$$= \lim_{x \rightarrow 1} \frac{2(x-1) \cdot (x-1)^2 \cos(x-1)^4 - 0}{(x-1) \cdot \cos(x-1) + \sin(x-1)} \left(\frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 1} \frac{2(x-1)^3 \cdot \cos(x-1)^4}{(x-1) \left[\cos(x-1) + \frac{\sin(x-1)}{(x-1)} \right]}$$

$$= \lim_{x \rightarrow 1} \frac{2(x-1)^2 \cos(x-1)^4}{(x-1) \left[\cos(x-1) + \frac{\sin(x-1)}{(x-1)} \right]}$$

$$= \lim_{x \rightarrow 1} \frac{2(x-1)^2 \cos(x-1)^4}{\cos(x-1) + \frac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$= \frac{0}{1+1} = 0$$