

SOLUTION

1. Official Ans. by NTA (1)

$$\text{Sol. } f(x) = \sin\left(\frac{|x|+5}{x^2+1}\right)$$

For domain :

$$-1 \leq \frac{|x|+5}{x^2+1} \leq 1$$

Since $|x| + 5$ & $x^2 + 1$ is always positive

$$\text{So } \frac{|x|+5}{x^2+1} \geq 0 \quad \forall x \in \mathbb{R}$$

So for domain :

$$\frac{|x|+5}{x^2+1} \leq 1$$

$$\Rightarrow |x| + 5 \leq x^2 + 1$$

$$\Rightarrow 0 \leq x^2 - |x| - 4$$

$$\Rightarrow 0 \leq \left(|x| - \frac{1+\sqrt{17}}{2}\right) \left(|x| - \frac{1-\sqrt{17}}{2}\right)$$

$$\Rightarrow |x| \geq \frac{1+\sqrt{17}}{2} \text{ or } |x| \leq \frac{1-\sqrt{17}}{2} \quad (\text{Rejected})$$

$$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$$

$$\text{So, } a = \frac{1+\sqrt{17}}{2}$$

2. Official Ans. by NTA (3)

$$\begin{aligned} \text{Sol. } 2\pi - \left(\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right) \\ = 2\pi - \left(\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right) \\ = 2\pi - \left(\tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right) \\ = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2} \end{aligned}$$

3. Official Ans. by NTA (4)

$$\begin{aligned} \text{Sol. } S &= \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \dots \\ S &= \tan^{-1}\left(\frac{2-1}{1+1 \cdot 2}\right) + \tan^{-1}\left(\frac{3-2}{1+2 \cdot 3}\right) + \tan^{-1} \\ &\quad \left(\frac{4-3}{1+3 \cdot 4}\right) + \dots + \tan^{-1}\left(\frac{11-10}{1+10 \cdot 11}\right) \\ S &= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + \\ &\quad (\tan^{-1} 4 - \tan^{-1} 3) + \dots + (\tan^{-1} 11) - \tan^{-1}(10) \\ S &= \tan^{-1} 11 - \tan^{-1} 1 = \tan^{-1}\left(\frac{11-1}{1+11}\right) \\ \tan(S) &= \frac{11-1}{1+11 \cdot 1} = \frac{10}{12} = \frac{5}{6} \end{aligned}$$