

INDEFINITE INTEGRATION

1. यदि $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$ है,

जहाँ c एक समाकलन अचर है, तो $\lambda f\left(\frac{\pi}{3}\right)$ का मान है :

(1) -2 (2) $-\frac{9}{8}$

(3) 2 (4) $\frac{9}{8}$

2. यदि $\int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)} = \lambda \tan \theta + 2 \log_e |f(\theta)| + C$

है, जहाँ C एक समाकलन-अचर है, तो क्रमित युग्म $(\lambda, f(\theta))$ बराबर है :

(1) (-1, 1 + tanθ) (2) (-1, 1 - tanθ)

(3) (1, 1 - tanθ) (4) (1, 1 + tanθ)

3. समाकल $\int \frac{dx}{\frac{8}{(x+4)^7} - \frac{6}{(x-3)^7}}$ बराबर है :

(जहाँ C एक समाकलन अचर है)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$ (2) $-\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$ (4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{13}{7}} + C$

4. यदि $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) + C$ है, जहाँ C एक समाकलन अचर है, तो क्रमित युग्म $(A(x), B(x))$ हो सकता है:

(1) $(x-1, \sqrt{x})$ (2) $(x+1, \sqrt{x})$

(3) $(x+1, -\sqrt{x})$ (4) $(x-1, -\sqrt{x})$

5. समाकलन $\int \left(\frac{x}{x \sin x + \cos x}\right)^2 dx$ होगा :

(जहाँ C, समाकलन अचर है)

(1) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$

(2) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$

(3) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

(4) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$

6. यदि $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x+e^{-x})} dx$

$= g(x)e^{(e^x+e^{-x})} + c$ है, जहाँ c एक समाकलन अचर है, तो $g(0)$ बराबर है :

(1) 2 (2) e^2

(3) e (4) 1

7. यदि $\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = A \log_e |B(\theta)|$

$+ C$, है, जहाँ C एक समाकलन अचर है, तो $\frac{B(\theta)}{A}$ हो सकता है:

(1) $\frac{2 \sin \theta + 1}{5(\sin \theta + 3)}$ (2) $\frac{2 \sin \theta + 1}{\sin \theta + 3}$

(3) $\frac{5(\sin \theta + 3)}{2 \sin \theta + 1}$ (4) $\frac{5(2 \sin \theta + 1)}{\sin \theta + 3}$

SOLUTION

1. NTA Ans. (1)

$$\begin{aligned} \text{Sol. } \int \frac{\cos x \, dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} &= \frac{-6}{-6} \int \frac{\cos x \, dx}{\sin^7 x \left(\frac{1}{\sin^6 x} + 1 \right)^{2/3}} \\ &= -\frac{1}{6} \times 3 \left(\frac{1}{\sin^6 x} + 1 \right)^{\frac{1}{3}} + c \\ &= -\frac{1}{2} \frac{(1 + \sin^6 x)^{\frac{1}{3}}}{\sin^2 x} + c \end{aligned}$$

$$\text{Hence, } \lambda = 3 \text{ and } f(x) = -\frac{1}{2\sin^2 x}$$

$$\text{so, } \lambda f\left(\frac{\pi}{3}\right) = -2$$

REMARK : Technically, this question should be marked as bonus. Because $f(x)$ and λ cannot be found uniquely.

For example, another such $f(x)$ and λ can be

$$-\frac{(1 + \sin^6 x)^{\frac{1}{6}}}{2\sin^2 x} \text{ and } 6 \text{ respectively.}$$

2. NTA Ans. (1)

$$\begin{aligned} \text{Sol. } I &= \int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)} \\ &= \int \frac{\sec^2 \theta \, d\theta}{\frac{2 \tan \theta}{1 - \tan^2 \theta} + \frac{1 + \tan^2 \theta}{1 - \tan^2 \theta}} = \int \frac{(1 - \tan^2 \theta) \sec^2 \theta \, d\theta}{(1 + \tan \theta)^2} \\ \tan \theta &= t \Rightarrow \sec^2 \theta \, d\theta = dt \\ I &= \int \frac{1 - t^2}{(1 + t)^2} dt = \int \frac{(1 - t)(1 + t)}{(1 + t)^2} dt \\ &= \int \frac{1}{1 + t} - \frac{t}{1 + t} dt \\ &= \ln|1 + t| - \int \left(\frac{1 + t}{1 + t} - \frac{1}{1 + t} \right) dt \end{aligned}$$

$$\begin{aligned} &= \ln|1 + t| - t + \ln|1 + t| = 2\ln|1 + t| - t + C \\ &= 2\ln|1 + \tan \theta| - \tan \theta + C \\ \lambda &= -1, f(\theta) = 1 + \tan \theta \end{aligned}$$

3. NTA Ans. (1)

$$\text{Sol. } I = \int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}} = \int \frac{dx}{\left(\frac{x+4}{x-3}\right)^{\frac{8}{7}}(x-3)^2}$$

$$\text{Let } \frac{x+4}{x-3} = t \Rightarrow \frac{dx}{(x-3)^2} = -\frac{1}{7} dt$$

$$\Rightarrow I = -\frac{1}{7} \int \frac{dt}{t^{8/7}} = -\frac{1}{7} \int t^{-8/7} dt$$

$$= t^{-1/7} + C = \left(\frac{x+4}{x-3}\right)^{-1/7} + C = \left(\frac{x-3}{x+4}\right)^{1/7} + C$$

4. Official Ans. by NTA (3)

$$\text{Sol. Put } x = \tan^2 \theta \Rightarrow dx = 2 \tan \theta \sec^2 \theta \, d\theta$$

$$\int \theta \cdot (2 \tan \theta \cdot \sec^2 \theta) \, d\theta$$

$$\downarrow \quad \downarrow$$

$$I \quad II \quad (\text{By parts})$$

$$= \theta \cdot \tan^2 \theta - \int \tan^2 \theta \, d\theta$$

$$= \theta \cdot \tan^2 \theta - \int (\sec^2 \theta - 1) \, d\theta$$

$$= \theta(1 + \tan^2 \theta) - \tan \theta + C$$

$$= \tan^{-1}(\sqrt{x})(1+x) - \sqrt{x} + C$$

