DEFINITE INTEGRATION

1. (0, 2π) – {π} में समीकरण $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ को सन्तुष्ट करने वाले θ के न्यूनतम तथा अधिकतम मान क्रमश: θ_1 तथा θ_2 हैं, तो $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ बराबर हैं :

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}+\frac{1}{6}$

(3) $\frac{\pi}{9}$ (4) $\frac{\pi}{3}$

2. α का वह मान, जिसके लिए $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$ है, है:

(1) $\log_e\left(\frac{3}{2}\right)$ (2) $\log_e\left(\frac{4}{3}\right)$

(3) $\log_e 2$ (4) $\log_e\sqrt{2}$

3. यदि सभी x के लिए, $f(a + b + 1 - x) = f(x)$ है, जबकि a तथा b स्थिर (fixed) धन वास्तविक संख्याएँ हैं, तो $\frac{1}{a+b}\int_a^b x(f(x)+f(x+1))dx$ बराबर है:

(1) $\int_{a+1}^{b+1} f(x)dx$ (2) $\int_{a+1}^{b+1} f(x+1)dx$

(3) $\int_{a-1}^{b-1} f(x+1)dx$ (4) $\int_{a-1}^{b-1} f(x)dx$

4. यदि $I=\int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, है, तो :

(1) $\frac{1}{9}<I^2<\frac{1}{8}$ (2) $\frac{1}{16}<I^2<\frac{1}{9}$

(3) $\frac{1}{6}<I^2<\frac{1}{2}$ (4) $\frac{1}{8}<I^2<\frac{1}{4}$

5. $\lim_{x\to 0}\frac{\int_0^x t\sin(10t)dt}{x}$ बराबर हैं :

(1) 0 (2) $-\frac{1}{5}$

(3) $-\frac{1}{10}$ (4) $\frac{1}{10}$

6. $\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ का मान है :

(1) 2π (2) 4π

(3) $2\pi^2$ (4) π^2

7. यदि सभी वास्तविक त्रिकों (a, b, c) के लिए, $f(x) = a + bx + cx^2$ है, तो $\int_0^1 f(x)dx$ बराबर है :

(1) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

(2) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$

(3) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

(4) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$

8. समाकल $\int_0^2 ||x-1|-x|dx$ बराबर है _______.

9. यदि [t] महत्तम पूर्णांक ≤ t है, तो $\int_1^2 |2x-[3x]|dx$ का मान बराबर है –

10. $\int_{-\pi}^{\pi}|\pi-|x||dx$ का मान है:

(1) π^2 (2) $2\pi^2$

(3) $\sqrt{2}\pi^2$ (4) $\frac{\pi^2}{2}$

11. यदि समाकल $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ का मान $\frac{k}{6}$ है, तो k बराबर है :

(1) $2\sqrt{3}-\pi$ (2) $3\sqrt{2}+\pi$

(3) $3\sqrt{2}-\pi$ (4) $2\sqrt{3}+\pi$

12. माना $f(x) = |x - 2|$ तथा $g(x) = f(f(x))$, $x \in [0, 4]$ है। तब $\int_0^3 (g(x)-f(x))dx$ का मान होगा

(1) $\frac{3}{2}$ (2) 0

(3) $\frac{1}{2}$ (4) 1

13. माना $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx \ (x \geq 0)$ है। तब $f(3) - f(1)$ का मान होगा

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

14. समाकल

$\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x(2\sec^2 x \cdot \sin^2 3x + 3\tan x \cdot \sin 6x)dx$

का मान है :

(1) $\frac{9}{2}$ (2) $-\frac{1}{9}$

(3) $-\frac{1}{18}$ (4) $\frac{7}{18}$

15. माना $\{x\}$ तथा $[x]$, क्रमशः एक वास्तविक संख्या x के भिन्नात्मक भाग तथा महत्तम पूर्णांक $\leq x$, को दर्शाते हैं। यदि $\int_0^n \{x\}dx, \int_0^n [x]dx$ तथा $10(n^2 - n)$, $(n \in N, n > 1)$ एक गुणोत्तर श्रेढी के तीन क्रमागत पद हैं, तो n का मान है____

16. $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ का मान है :

(1) π (2) $\frac{3\pi}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

17. यदि $I_1 = \int_0^1 (1-x^{50})^{100} dx$ तथा $I_2 = \int_0^1 (1-x^{50})^{101} dx$ है जिन के लिए $I_2 = \alpha I_1$ है, तो α बराबर है :

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$

(3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

18. समाकल $\int_1^2 e^x . x^x (2+\log_e x)dx$ बराबर है :

(1) $e(4e + 1)$ (2) $e(2e - 1)$

(3) $4e^2 - 1$ (4) $e(4e - 1)$

SOLUTION

1. NTA Ans. (4)

Sol. $2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0$

$3\sin^2\theta - 5\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}, 2$ (Rejected)

$$\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

2. NTA Ans. (3)

Sol. $$4\alpha\left[\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right] = 5$$

$$\Rightarrow 4\alpha\left(\left[\frac{e^{\alpha x}}{\alpha}\right]_{-1}^{0} + \left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{2}\right) = 5$$

$\Rightarrow 4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Let $e^{-\alpha} = t$, $4t^2 + 4t - 3 = 0$, $t = \frac{1}{2}, \frac{-3}{2}$

(Rejected)

$e^{-\alpha} = \frac{1}{2}$; $\alpha = \ell n2$

3. NTA Ans. (1)

ALLEN Ans. (1 OR 3)

Note: In this Question, both options (1) as well as (3) are correct, but NTA accepte only option (1).

Sol. $f(x + 1) = f(a + b - x)$

$$I = \frac{1}{(a+b)}\int_a^b x\left(f(x) + f(x+1)\right)dx \quad .....(1)$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(x+1) + f(x)\right)dx \quad ...(2)$$

from (1) and (2)

$$2I = \int_a^b \left(f(x) + f(x+1)\right)dx$$

$$2I = \int_a^b f\left(a+b-x\right)dx + \int_a^b f(x+1)dx$$

$$2I = 2\int_a^b f(x+1)dx \Rightarrow I = \int_a^b f(x+1)dx$$

$$= \int_{a+1}^{b+1} f(x)dx$$

OR

$$I = \frac{1}{(a+b)}\int_a^b x\left(f(x) + f(x+1)\right)dx \quad ...(1)$$

$$= \frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(a+b-x) + f(a+b+1-x)\right)dx$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(x+1) + f(x)\right)dx \quad ...(2)$$

equation (1) + (2)

$$2I = \frac{1}{(a+b)}\int_a^b (a+b)\left(f(x+1) + f(x)\right)dx$$

$$I = \frac{1}{2}\left[\int_a^b f(x+1)dx + \int_a^b f(x)dx\right]$$

$$= \frac{1}{2}\left[\int_a^b f(a+b+1-x)dx + \int_a^b f(x)dx\right]$$

$$= \frac{1}{2}\left[\int_a^b f(x)dx + \int_a^b f(x)dx\right]$$

$$I = \int_a^b f(x)dx$$

Let $x = T + 1$

$$= \int_{a-1}^{b-1} f(T+1)dT$$

$$I = \int_{a-1}^{b-1} f(x+1)dx$$

4. NTA Ans. (1)

Sol. $f(x)=\dfrac{1}{\sqrt{2x^3-9x^2+12x+4}}$

$$f'(x)=\frac{-6(x-1)(x-2)}{2\left(2x^3-9x^2+12x+4\right)^{3/2}}$$

$\therefore$ $f(x)$ is decreasing in (1,2)

$$f(1)=\frac{1}{3};\ f(2)=\frac{1}{\sqrt{8}}$$

$$\frac{1}{3}<I<\frac{1}{\sqrt{8}} \Rightarrow I^2\in\left(\frac{1}{9},\frac{1}{8}\right)$$

(1) Option

5. NTA Ans. (1)

Sol. Using L.H. Rule

$$\lim_{x\to 0}\frac{x\sin(10x)}{1}=0$$

(1) Option

6. NTA Ans. (4)

Sol. $I=\displaystyle\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ ....(1)

$$=\left[\int_0^{\pi}\frac{x\sin^8 x}{\sin^8 x\cos^8 x}dx+\int_0^{\pi}\frac{(2\pi-x)\sin^8 x}{\sin^8 x+\cos^8 x}dx\right]$$

$$=2\pi\int_0^{\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

$$I=2\pi\left[\int_0^{\pi/2}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx+\int_0^{\pi/2}\frac{\cos^8 xdx}{\sin^8 x+\cos^8 x}dx\right]$$

$$=2\pi\int_0^{\pi/2}1dx=2\pi.\frac{\pi}{2}=\pi^2$$

7. NTA Ans. (3)

Sol. $f(x) = a + bx + cx^2$

$$\int_0^1 f(x)dx=\left[ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right]_0^1$$

$$=a+\frac{b}{2}+\frac{c}{3}=\frac{1}{6}[6a+3b+c]$$

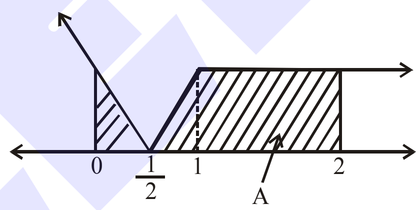
$$=\frac{1}{6}\left[f(0)+f(1)+4f\left(\frac{1}{2}\right)\right]$$

8. Official Ans. by NTA (1.50)

Sol. $\displaystyle\int_0^2 ||x-1|-x|dx$

Let f(x) ||x – 1|–x|

$$=\begin{cases}1, & x\ge 1\\ |1-2x|, & x\le 1\end{cases}$$



$$A=\frac{1}{2}+1=\frac{3}{2}$$

OR

$$\int_0^{1/2}(1-2x)dx+\int_{1/2}^{1}(2x-1)+\int_0^{2}1dx$$

$$=\left[x-x^2\right]_0^{\frac{1}{2}}+\left[x^2-x\right]_{1/2}^{1}+\left[x\right]_1^2$$

$=\boxed{3/2}$

9. Official Ans. by NTA (1.0)

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$;

Now $\displaystyle\int_1^2 |2x-[3x]|dx$

$$=\int_1^{4/3}(3-2x)dx+\int_{4/3}^{5/3}(4-2x)\,dx+\int_{5/3}^{2}(5-2x)dx$$

$$=\frac{2}{9}+\frac{3}{9}+\frac{4}{9}=1$$

10. Official Ans. by NTA (1)

Sol. $\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx = 2\int_{0}^{\pi} |\pi - x| dx$

$= 2\int_{0}^{\pi} (\pi - x) dx$

$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$

11. Official Ans. by NTA (1)

Sol. $\int_0^{1/2} \frac{((x^2-1)+1)}{(1-x^2)^{3/2}} dx$

$\int_0^{1/2} \frac{dx}{(1-x^2)^{3/2}} - \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$

$\int_0^{1/2} \frac{x^{-3}}{(x^{-2}-1)^{3/2}} dx - (\sin^{-1} x)_0^{1/2}$

Let $x^{-2} - 1 = t^2 \Rightarrow x^{-3}\, dx = -\, t\,dt$

$\int_{\infty}^{\sqrt{3}} \frac{-t\,dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt{3}}^{\infty} \frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{k}{6}$

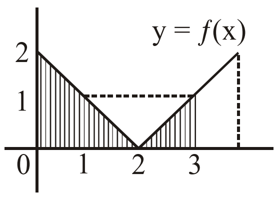
$k = 2\sqrt{3} - \pi$

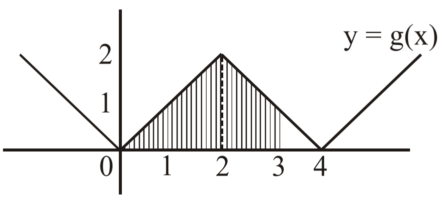
12. Official Ans. by NTA (4)

Sol. $\int_0^3 g(x) - f(x) = \int_0^3 \left||x-2| - 2\right| dx - \int_0^3 |x-2| dx$

$= \left(\frac{1}{2}\times 2\times 2 + 1 + \frac{1}{2}\times 1\times 1\right) - \left(\frac{1}{2}\times 2\times 2 + \frac{1}{2}\times 1\times 1\right)$

$= \left(2 + 1 + \frac{1}{2}\right) - \left(2 + \frac{1}{2}\right) = 1$





13. Official Ans. by NTA (4)

Sol. $f(x) = \int_1^3 \frac{\sqrt{x}\, dx}{(1+x)^2} = \int_1^{\sqrt{3}} \frac{t.2t\, dt}{(1+t^2)^2}$ $\left(\text{put } \sqrt{x} = t\right)$

$= \left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}} + (\tan^{-1} t)_1^{\sqrt{3}}$ [Appling by parts]

$= -\left(\frac{\sqrt{3}}{4} - \frac{1}{2}\right) + \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{1}{2} - \frac{\sqrt{3}}{4} + \frac{\pi}{12}$

14. Official Ans. by NTA (3)

Sol. $I = \int_{\pi/6}^{\pi/3} ((2\tan^3 x \cdot \sec^2 x \cdot \sin^4 3x) + (3\tan^4 x \cdot \sin^3 3x$

$\cdot \cos 3x))dx$

$\Rightarrow I = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d((\sin 3x)^4 (\tan x)^4)$

$\Rightarrow I = ((\sin 3x)^4 (\tan x)^4)_{\pi/6}^{\pi/3}$

$\Rightarrow I = -\frac{1}{18}$

15. Official Ans. by NTA (21)

Sol. $\int_0^n \{x\}dx = n\int_0^1 \{x\}dx = n\int_0^1 x\, dx = \frac{n}{2}$

$\int_0^n [x]dx = \int_0^n (x - \{x\})dx = \frac{n^2}{2} - \frac{n}{2}$

$\Rightarrow \left(\frac{n^2 - n}{2}\right)^2 = \frac{n}{2} \cdot 10 \cdot n(n-1)$ (where $n > 1$)

$\Rightarrow \frac{n-1}{4} = 5 \Rightarrow n = 21$

16. Official Ans. by NTA (4)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ ....(1)

Apply King property

$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}} dx = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}} dx$ ...(2)

Add (1) & (2)

$2I = \int_{-\pi/2}^{\pi/2} dx = \pi$

$I = \frac{\pi}{2}$

17. Official Ans. by NTA (1)

Sol. $I_1 = \int_0^1 \left(1-x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$

and $I_1 = \lambda I_2$

$I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$

$I_2 = \int_0^1 \left(1-x^{50}\right)\left(1-x^{50}\right)^{100} dx$

$I_2 = \int_0^1 \left(1-x^{50}\right) dx - \int_0^1 x^{50}.\left(1-x^{50}\right)^{100} dx$

$I_2 = I_1 - \int_0^1 \underbrace{x}_{I}.\underbrace{x^{49}.\left(1-x^{50}\right)^{100} dx}_{II}$

Now apply IBP

$I_2 = I_1 - \left[x\int x^{49}.\left(1-x^{50}\right)^{100} dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.\left(1-x^{50}\right)^{100} dx \right]$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1 \left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101} dx \right]$

$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050} I_2$

$I_2 + \frac{1}{5050} I_2 = I_1 \Rightarrow \frac{5051}{5050} I_2 = I_1$

$\therefore \ \alpha = \frac{5050}{5051}$

$I_2 = \frac{5050}{5051} I_1$

$\because I_2 = \alpha.I_1$

18. Official Ans. by NTA (4)

Sol. $\int_1^2 e^x.x^x \left(2+\log_e x\right) dx$

$\int_1^2 e^x \left(2x^x + x^x \log_e x\right) dx$

$\int_1^2 e^x \left(\underbrace{x^x}_{f(x)} + \underbrace{x^x\left(1+\log_e x\right)}_{f'(x)} \right) dx$

$\left(e^x.x^x\right)_1^2 = 4e^2 - e$