

CONTINUITY

1. यदि  $\left(-\frac{1}{3}, \frac{1}{3}\right)$  में  $f(x) = \begin{cases} \frac{1}{x} \log_e \left(\frac{1+3x}{1-2x}\right), & x \neq 0 \\ k, & x = 0 \end{cases}$

द्वारा परिभाषित फलन  $f$  संतत हैं, तो  $k$  बराबर है \_\_\_\_\_

2. माना  $[t]$  महत्तम पूर्णांक  $\leq t$  को दर्शाता है तथा  $\lim_{x \rightarrow 0} x \left[ \frac{4}{x} \right] = A$

है  $f(x) = [x^2] \sin(\pi x)$  असंतत है, जब  $x$  बराबर है :

- (1)  $\sqrt{A+5}$                       (2)  $\sqrt{A+1}$
- (3)  $\sqrt{A}$                               (4)  $\sqrt{A+21}$

3. यदि  $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}; & x < 0 \\ b; & x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{-\frac{1}{3}}}{x^{\frac{4}{3}}}; & x > 0 \end{cases}$

$x = 0$  पर संतत है, तो  $a + 2b$  का मान है :

- (1) -1                                      (2) 1
- (3) -2                                      (4) 0

4. माना  $f(x) = x \cdot \left[ \frac{x}{2} \right], -10 < x < 10$ , है जहाँ  $[t]$  महत्तम पूर्णांक फलन है, तो  $f$  के असंतत बिन्दुओं की संख्या है \_\_\_\_\_ ।

## SOLUTION

1. NTA Ans. (5.00)

$$\text{Sol. } k = \lim_{x \rightarrow 0} \left( \frac{\ln(1+3x)}{x} - \frac{\ln(1-2x)}{x} \right)$$

$$k = 3 + 2 = 5$$

2. NTA Ans. (2)

$$\text{Sol. } A = \lim_{x \rightarrow 0} x \left[ \frac{4}{x} \right] = \lim_{x \rightarrow 0} x \left( \frac{4}{x} \right) - x \left( \frac{4}{x} \right) = 4$$

$f(x) = [x^2]\sin(\pi x)$  will be discontinuous at nonintegers

$$\therefore x = \sqrt{A+1} \text{ i.e. } \sqrt{5}$$

3. NTA Ans. (4)

$$\text{Sol. } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} \left( \frac{\sin(a+2)x}{x} + \frac{\sin x}{x} \right) = a + 3$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} \frac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}}$$

$$= \lim_{x \rightarrow 0} \frac{(1+3x)^{1/3} - 1}{x} = 1$$

$$f(0) = b$$

for continuity at  $x = 0$

$$\lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow a + 3 = b = 1$$

$$\therefore a = -2, b = 1$$

$$\therefore a + 2b = 0$$

4. Official Ans. by NTA (8)

Sol.  $x \in (-10, 10)$ 

$$\frac{x}{2} \in (-5, 5) \rightarrow 9 \text{ integers}$$

check continuity at  $x = 0$

$$\left. \begin{array}{l} f(0) = 0 \\ f(0^+) = 0 \\ f(0^-) = 0 \end{array} \right\} \text{continuous at } x = 0$$

function will be discontinuous when

$$\frac{x}{2} = \pm 4, \pm 3, \pm 2, \pm 1$$

8 points of discontinuity