3D

1. यदि $(\alpha, 7, 1)$ से जाने वाली एक रेखा पर बिन्दु $(1, 0, 3)$ से डाले गये लम्ब का पाद $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ है, तो α बराबर है_____

2. यदि एक समतल P, तीन बिन्दुओं (2, 1, 0), (4, 1, 1) और (5, 0, 1) से होकर जाता है, तथा कोई और बिन्दु R (2, 1, 6) है, तो समतल P में R का प्रतिबिम्ब (image) है:

(1) (6, 5, –2) (2) (4, 3, 2)
(3) (3, 4, –2) (4) (6, 5, 2)

3. बिन्दु (1,2,3) का एक समतल में प्रतिबिम्ब (mirror image), $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$ है। निम्न में से कौन सा बिन्दु इस समतल पर स्थित है ?

(1) (–1, –1, –1) (2) (–1, –1, 1)
(3) (1, 1, 1) (4) (1, –1, 1)

4. रेखाओं $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ तथा

$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ के बीच की न्यूनतम दूरी है :

(1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$

(3) 3 (4) $2\sqrt{30}$

5. यदि समतल $23x - 10y - 2z + 48 = 0$ तथा रेखाओं $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ और $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda} (\lambda \in R)$ को अंतर्विष्ट करने वाले समतल के बीच की दूरी $\frac{k}{\sqrt{633}}$ है, तो k बराबर है_____.

6. यदि R में किन्हीं α तथा β के लिए, निम्न तीन समतलों
$x + 4y - 2z = 1$
$x + 7y - 5z = \beta$
$x + 5y + \alpha z = 5$
का प्रतिच्छेदन, R^3 में एक रेखा है, तो $\alpha + \beta$ का मान है :

(1) 10 (2) –10
(3) 2 (4) 0

7. बिन्दुओं (1, 2, 1), (2, 1, 2) से गुजरने वाला और रेखा $2x = 3y, z = 1$ के समान्तर समतल किस बिन्दु से भी गुजेगा:

(1) (0, 6, –2) (2) (–2, 0, 1)
(3) (0, –6, 2) (4) (2, 0, –1)

8. बिन्दु (3, 1,1) से होकर जाने वाले समतल में दो सरल रेखाएं स्थित है, जिनके दिक् अनुपात (direction ratios) क्रमशः 1, –2, 2 तथा 2, 3, –1 है। यदि यह समतल बिन्दु $(\alpha, -3, 5)$ से भी होकर जाता हे, तो α बराबर है –

(1) –10 (2) 5
(3) 10 (4) –5

9. बिन्दुओं (1, –2, 3) और (1, 1, 0) से होकर जाने वाली सरल रेखा पर बिन्दु (4, 2, 3) से डाले गए लम्ब का पाद समतल पर है, वह है –

(1) x + 2y – z = 1 (2) x – 2y + z = 1
(3) x – y – 2z = 1 (4) 2x + y – z = 1

10. वह समतल, जो बिन्दुओं (4, –2, 3) तथा (2, 4, –1) को मिलाने वाली सरल रेखा को लम्ब समद्विभाजित करता है, निम्न में से किस बिन्दु से भी होकर जाता है?

(1) (4, 0, –1) (2) (4, 0, 1)
(3) (0, 1, –1) (4) (0, –1, 1)

11. यदि समतल P का समीकरण, जो समतलों $x + 4y - z + 7 = 0$ तथा $3x + y + 5z = 8$ के प्रतिच्छेदन से गुजरता है, किसी $a, b \in R$ के लिये $ax + by + 6z = 15$ हो, तो समतल P से बिन्दु (3, 2, –1) की दूरी होगी

12. बिन्दु (1, – 2, 3) की समतल $x-y+z = 5$ से रेखा $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ के समांतर मापी गई दूरी है :

(1) 7 (2) 1
(3) $\frac{1}{7}$ (4) $\frac{7}{5}$

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13. यदि बिन्दु (1, 2, –3) का रेखा $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$ में प्रतिबिंब (a, b, c) है, तो $a + b + c$ बराबर है :

(1) –1 (2) 2

(3) 3 (4) 1

14. यदि किसी $\alpha \in R$ के लिए, रेखाएँ

$L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ तथा

$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ समतलीय हैं, तो रेखा L_2 जिस बिन्दु से होकर जाती है, वह है :

(1) (–2, 10, 2) (2) (10, 2, 2)

(3) (10, –2, –2) (4) (2, –10, –2)

15. रेखाओं $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ तथा $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ के बीच की न्यूनतम दूरी है :

(1) $\frac{1}{2}$ (2) 1

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

16. एक समतल P निदेशांक को क्रमशः A, B तथा C पर मिलाता है तथा त्रिभुज ΔABC का केन्द्रक (1, 1, 2) है, तो इस केन्द्रक से जाने वाली तथा समतल P के लम्बवत रेखा का समीकरण है :

(1) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

(2) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

(3) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

(4) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

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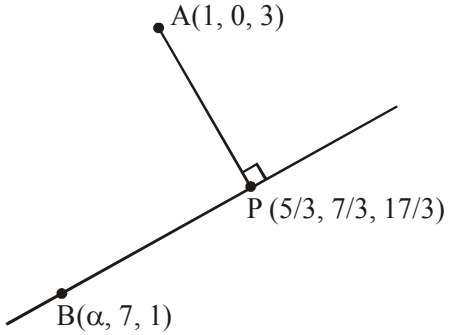
SOLUTION

1. NTA Ans. (4.00)

Sol. D.R. of BP = $<\frac{5}{3}-\alpha,\frac{7}{3}-7,\frac{17}{3}-1>$



D.R. of AP = $<\frac{5}{3}-1,\frac{7}{3}-0,\frac{17}{3}-3>$

BP $\perp^r$ AP

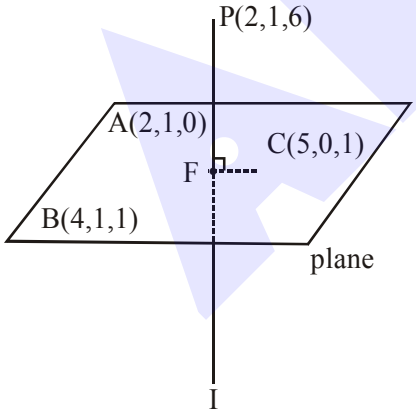
$\Rightarrow \alpha = 4$

2. NTA Ans. (1)

Sol. Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z = 3$



Let I and F are respectively image and foot of perpendicular of point P in the plane.

eq^n of line PI $\frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\lambda(\text{say})$

Let I $(\lambda + 2, \lambda + 1, -2\lambda + 6)$

$\Rightarrow F\left(2+\frac{\lambda}{2},1+\frac{\lambda}{2},-\lambda+6\right)$

F lies in the plane

$\Rightarrow 2+\frac{\lambda}{2}+1+\frac{\lambda}{2}+2\lambda-12-3=0$

$\Rightarrow \lambda = 4$

$\Rightarrow$ I (6, 5, –2)

3. NTA Ans. (4)

Sol. Point on plane $R\left(\frac{-2}{3},\frac{1}{3},\frac{4}{3}\right)$

Normal vector of plane is $\frac{10}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{10}{3}\hat{k}$

Equation of require plane is x + y + z = 1

Hence (1, –1, 1) lies on plane

(4) Option

4. NTA Ans. (2)

Sol. Shortest distance $=\frac{\left\|\begin{matrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{\sqrt{11\times 29-49}}=\frac{270}{\sqrt{270}}$

$=\sqrt{270}=3\sqrt{30}$

5. NTA Ans. (3)

Sol. If $\lambda = -7$, then planes will be parallel & distance between them will be $\frac{3}{\sqrt{633}} \Rightarrow k = 3$

But if $\lambda \neq -7$, then planes will be intersecting & distance between them will be 0

6. NTA Ans. (1)

Sol. For planes to intersect on a line

$\Rightarrow$ there should be infinite solution of the given system of equations

for infinite solutions

$$\Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

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$$\Delta_z = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 13 - \beta = 0 \Rightarrow \beta = 13$$

Also for $\alpha = -3$ and $b = 13$ $\Delta_x = \Delta_y = 0$

$\therefore \alpha + \beta = -3 + 13 = 10$

7. Official Ans. by NTA (2)

Sol. Two points on the line (L say) $\frac{x}{3} = \frac{y}{2}, z = 1$ are

(0, 0, 1) & (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2j), t \in R$ (–2, 0, 1) satisfies the line (for t = –1)

$\Rightarrow$ (–2, 0, 1) lies on given plane.

Answer of the question is (2)

We can check other options by finding eqution of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow 2(x - 1) - 3(y - 2) - 5(z - 1) = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

8. Official Ans. by NTA (2)

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y - 1) + 7(z - 1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0 \quad \ldots(1)$

Plane is also passing through $(\alpha, -3, 5)$ so this point satisfies the equation of plane so put in equation (1)

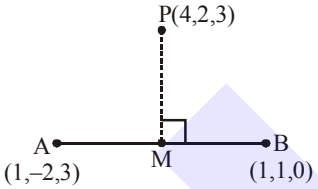
$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

9. Official Ans. by NTA (4)

Sol. Equation of AB = $\vec{r} = (\hat{i} + \hat{j}) + \lambda(3\hat{j} - 3\hat{k})$



Let coordinates of M = (1, (1 + 3λ), –3λ).

$\overrightarrow{PM} = -3\hat{i} + (3\lambda - 1)\hat{j} - 3(\lambda + 1)\hat{k}$

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$

$\because \overrightarrow{PM} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{PM} . \overrightarrow{AB} = 0$

$\Rightarrow 3(3\lambda - 1) + 9(\lambda + 1) = 0$

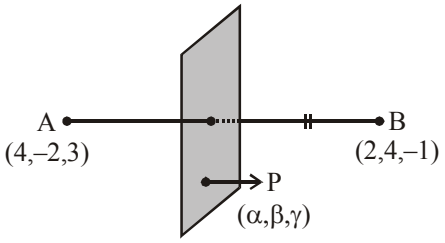
$\Rightarrow \lambda = -\frac{1}{3}$

$\therefore$ M = (1, 0, 1)

Clearly M lies on 2x + y – z = 1.

10. Official Ans. by NTA (1)

Sol. PA = PB



$\Rightarrow PA^2 = PB^2$

$\Rightarrow (\alpha - 4)^2 + (\beta + 2)^2 + (\gamma - 3)^2$

$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$

$\Rightarrow -4\alpha + 12\beta - 8\gamma = -8$

$\Rightarrow 2x - 6y + 4z = 4$

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11. Official Ans. by NTA (3)

Sol. $D_1 = \begin{vmatrix} -7 & 4 & -1 \\ 8 & 1 & 5 \\ 15 & b & 6 \end{vmatrix} = 0 \Rightarrow b = -3$

$$D = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 1 & 5 \\ a & b & 6 \end{vmatrix} = 0 \Rightarrow 21a - 8b - 66 = 0 \quad .... (1)$$

P : 2x – 3y + 6z = 15

so required distance $= \frac{21}{7} = 3$

12. Official Ans. by NTA (2)

Sol. equation of line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ passes through (1, –2, 3) is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$$

x = 2r + 1

y = 3r –2,

z = –6r + 3

So 2r + 1 – 3r + 2– 6r + 3 = 5

$\Rightarrow$ –7r + 1 = 0

$r = \frac{1}{7}$

$x = \frac{9}{7},\ y = \frac{-11}{7},,\ z = \frac{15}{7}$

$$\text{Distance is} = \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(2-\frac{11}{7}\right)^2 + \left(3-\frac{15}{7}\right)^2}$$

$$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$$

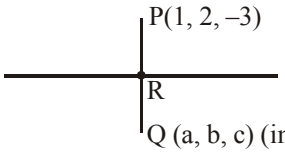
$$= \frac{1}{7}\sqrt{4+9+36}$$

$$= \frac{1}{7}\sqrt{49} = 1$$

13. Official Ans. by NTA (2)

Sol. Line is $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$: Let point R is

$(2\lambda - 1, -2\lambda + 3, -\lambda)$



Direction ratio of PQ≡$(2\lambda - 2, -2\lambda + 1, 3 - \lambda)$

PQ is $\perp^r$ to line

$\Rightarrow$ 2 (2λ – 2) – 2 (–2λ + 1)– 1(3 – λ)= 0

4λ – 4 + 4λ – 2 – 3 + λ = 0

9λ = 9 $\Rightarrow$ λ = 1

$\Rightarrow$ Point R is (1, 1, –1)

$$\frac{a+1}{2} = 1 \quad \left| \quad \frac{b+2}{2} = 1 \quad \right| \quad \frac{c-3}{2} = -1$$

a = 1 b = 0 c = 1

$\Rightarrow$ a + b + c = 2

14. Official Ans. by NTA (4)

Sol. $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point A(–1, 2, 1) B(–2, –1, –1)

$\because$ L_1 and L_2 are coplanar

$$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$$

α = –4

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options (2, –10, –2) lies on L_2

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15. Official Ans. by NTA (4)

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$$x = \frac{-2z-4}{3} \quad ...(3)$$

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$$\frac{3x+4}{-2} = -3y+1 = z$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\frac{1}{3}}{-\frac{1}{3}} = \frac{z-0}{1}$$

now shortest distance between skew lines

$$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\left(\frac{1}{3}\right)}{-\frac{1}{3}} = \frac{z-0}{1}$$

$$\text{S.D.} = \left|\frac{\left(\vec{b}-\vec{a}\right).\left(\vec{c}\times\vec{d}\right)}{\left|\vec{c}\times\vec{d}\right|}\right|$$

where $\vec{a} = (1,-1,0)$

$$\vec{b} = \left(-\frac{4}{3}, \frac{1}{3}, 0\right)$$

$$\vec{c} = (0,-1,1)$$

$$\vec{d} = \left(-\frac{2}{3}, -\frac{1}{3}, 1\right)$$

$$\Rightarrow \text{S.D} = \frac{1}{\sqrt{3}}$$

16. Official Ans. by NTA (2)

Sol. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$A \equiv (a, 0, 0), B \equiv (0, b, 0), C \equiv (0, 0, c)$

$$\text{Centroid} \equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2)$$

$a = 3, b = 3, c = 6$

Plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

line $\perp$ to the plane (DR of line = $2\hat{i} + 2\hat{j} + \hat{k}$)

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

ALLEN

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