TRIGONOMETRIC EQUATION

1. The number of distinct solutions of the equation $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is ______.

2. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval :

(1) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (2) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

(3) $\left(-\frac{5}{4}, -1\right)$ (4) $\left[-1, -\frac{1}{2}\right]$

3. Let a, b, c $\in$ R be such that $a^2 + b^2 + c^2 = 1$. If $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is :

(1) $\frac{\pi}{2}$ (2) 0

(3) $\frac{\pi}{9}$ (4) $\frac{2\pi}{3}$

SOLUTION

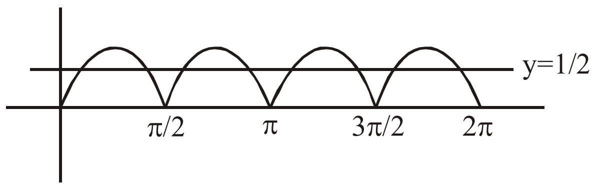
1. NTA Ans. (8.00)

Sol. $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|;\ x \in [0, 2\pi]$

$\Rightarrow \log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$

$\Rightarrow \log_{1/2}(|\sin x \cos x|) = 2$

$$\Rightarrow |\sin x\cos x| = \frac{1}{4} \quad \Rightarrow |\sin 2x| = \frac{1}{2}$$



$\Rightarrow$ 8 solutions

2. Official Ans. by NTA (4)

Sol. $\lambda = -(\sin^4\theta + \cos^4\theta)$

$\lambda = -(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$$\lambda = \frac{\sin^2 2\theta}{2} - 1$$

$$\frac{\sin^2 2\theta}{2} \in \left[0,\ \frac{1}{2}\right]$$

$$\lambda \in \left[-1,\ -\frac{1}{2}\right]$$

3. Official Ans. by NTA (1)

Sol. $$\cos\phi = \frac{\bar{p}.\bar{q}}{|\bar{p}||\bar{q}|} = \frac{ab+bc+ca}{a^2+b^2+c^2} = \frac{\Sigma ab}{1}$$

$$= abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$= \frac{abc}{\lambda}\left(\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta+\frac{4\pi}{3}\right)\right)$$

$$= \frac{abc}{\lambda}\left(\cos + 2\cos(\theta+\pi)\cos\frac{\pi}{3}\right)$$

$$= \frac{abc}{\lambda}(\cos\theta - \cos\theta) = 0$$

$$\phi = \frac{\pi}{2}$$