TANGENT & NORMAL

1. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2,2) is

(1) $4\sqrt{2}$ (2) $2\sqrt{2}$
(3) 2 (4) $\sqrt{2}$

2. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then |m| is equal to

3. If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then :

(1) $b = a$ (2) $b = \frac{\pi}{2} + a$
(3) $|b - a| = 1$ (4) $|a+b| = 1$

4. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :

(1) $y = 4x + 2$ (2) $x + 4y = 8$
(3) $y + 4x = 2$ (4) $2y + x = 4$

5. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is :

(1) 9 (2) 18
(3) 10 (4) 20

6. If the tangent of the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is ________.

7. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to________.

8. The position of a moving car at time t is given by $f(t) = at^2 + bt + c, t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :

(1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$
(3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

SOLUTION

1. NTA Ans. (2)

Sol. $x^2 + 2xy - 3y^2 = 0$

m_N = slope of normal drawn to curve at (2,2) is –1

L : x + y = 4.

perpendicular distance of L from (0,0)

$$= \frac{|0+0-4|}{\sqrt{2}} = 2\sqrt{2}$$

(2) Option

2. NTA Ans. (4.00)

Sol. Let $P(\alpha,\beta)$

so, $\beta^2 - 3\alpha^2 + \beta + 10 = 0$ ...(i)

Now, $2yy' - 6x + y' = 0$

$$\Rightarrow m = \frac{6\alpha}{2\beta+1} \quad ....(ii)$$

Also, $\dfrac{\beta - \frac{3}{2}}{\alpha} = -\dfrac{1}{m}$

$$\Rightarrow \frac{2\beta-3}{2\alpha} = -\frac{(2\beta+1)}{6\alpha} \text{ (from (ii))}$$

$\Rightarrow \beta = 1 \Rightarrow \alpha^2 = 4$ (from (1))

Hence, $|m| = \dfrac{12}{3} = 4.00$

3. Official Ans. by NTA (3)

Sol. Slope of tangent to the curve $y = x + \sin y$

at (a, b) is $\dfrac{2 - \frac{3}{2}}{\frac{1}{2} - 0} = 1$

$$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a} = 1$$

$$\frac{dy}{dx} = 1 + \cos y.\frac{dy}{dx} \text{ (from equation of curve)}$$

$$\left.\frac{dy}{dx}\right]_{x=a} = 1 + \cos b.\left.\frac{dy}{dx}\right]_{x=a}$$

$\Rightarrow \cos b = 0$

$\Rightarrow \sin b = \pm 1$

Now, from curve y = x + sin y

b = a + sin b

$\Rightarrow |b - a| = |\sin b| = 1$

4. Official Ans. by NTA (2)

Sol. Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $\boxed{x = 0}$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1$

$\boxed{y = 2}$

So we have to find the normal at (0, 2)

Now $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$$

$y = e^{2y\ln(1+x)} + (1-x^2)$ ...(1)

Now differentiate w.r.t. x

$$y' = e^{2y\ln(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right)+\ln(1+x).2y'\right]-2x$$

Put x = 0 & y = 2

$$y' = e^{2\times 2 \ln 1}\left[2\times 2\left(\frac{1}{1+0}\right)+\ln(1+0).2y'\right]-2\times 0$$

$y' = e^0[4 + 0] - 0$

$y' = 4$ = slope of tangent to the curve

so slope of normal to the curve $= -\frac{1}{4}\{m_1m_2=-1\}$

Hence equation of normal at (0, 2) is

$$y-2=-\frac{1}{4}(x-0)$$

$\Rightarrow 4y - 8 = -x$

$\Rightarrow \boxed{x+4y=8}$

5. Official Ans. by NTA (1)

Sol. $\frac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a\frac{da}{dt}=3.6$

$$a\frac{da}{dt}=0.3$$

$$\frac{dv}{dt}=\frac{d}{dt}(a^3)=3a\left(a\frac{da}{dt}\right)$$

$= 3 \times 10 \times 0.3 = 9$

6. Official Ans. by NTA (4)

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$$m=\left(\frac{dy}{dx}\right)_{(c,e^c)}=e^c$$

$\Rightarrow$ Tangent at (c, e^c)

$y - e^c = e^c (x - c)$

it intersect x-axis

Put $y = 0 \Rightarrow x = c - 1$

......(1)

Now $y^2 = 4x \Rightarrow \frac{dy}{dx}=\frac{2}{y}\Rightarrow\left(\frac{dy}{dx}\right)_{(1,2)}=1$

$\Rightarrow$ Slope of normal = –1

Equation of normal $y - 2 = -1(x - 1)$

$x + y = 3$ it intersect x-axis

Put $y = 0 \Rightarrow x = 3$

......(2)

Points are same

$\Rightarrow x = c - 1 = 3$

$\Rightarrow c = 4$

7. Official Ans. by NTA (0.50)

Sol. $y = x^2 - 3x + 2$

At x-axis $y = 0 = x^2 - 3x + 2$

$x = 1, 2$

$\frac{dy}{dx} = 2x - 3$

A(1, 0) B(2, 0)

$\left(\frac{dy}{dx}\right)_{x=1}=-1$ and $\left(\frac{dy}{dx}\right)_{x=2}=1$

$x + y = a \Rightarrow \frac{dy}{dx} = -1$ So A(1, 0) lies on it

$\Rightarrow 1 + 0 = a \Rightarrow \boxed{a=1}$

$x - y = b \Rightarrow \frac{dy}{dx}=1$ So B(2, 0) lies on it

$2 - 0 = b \Rightarrow \boxed{b=2}$

$\frac{a}{b} = 0.50$

8. Official Ans. by NTA (4)

Sol. $$\frac{f(t_2)-f(t_1)}{t_2-t_1}=2at+b$$

$$\frac{a(t_2^2-t_1^2)+b(t_2-t_1)}{t_2-t_1}=2at+b$$

$\Rightarrow a(t_2 + t_1) + b = 2at + b$

$$\Rightarrow t=\frac{t_1+t_2}{2}$$