STRAIGHT LINE

1. The locus of the mid-points of the perpendiculars drawn from points on the line, x = 2y to the line x = y is :

(1) $2x - 3y = 0$ (2) $7x - 5y = 0$
(3) $5x - 7y = 0$ (4) $3x - 2y = 0$

2. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is ________.

3. Let two points be A(1,–1) and B(0,2). If a point P(x',y') be such that the area of ΔPAB = 5 sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is

(1) 1 (2) 4
(3) 3 (4) –3

4. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point :

(1) (7, 6) (2) (–9, –6)
(3) (–9, –7) (4) (9, 7)

5. The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1, 2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is :

(1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{3\pi}{4}\right)$

(3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

6. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90°$, and $ar(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

(1) $2+\sqrt{5}$ (2) $1+\sqrt{5}$

(3) $1+2\sqrt{5}$ (4) $2\sqrt{5}-1$

7. If the perpendicular bisector of the line segment joining the points P (1, 4) and Q (k, 3) has y-intercept equal to –4, then a value of k is :-

(1) $\sqrt{15}$ (2) –2

(3) $\sqrt{14}$ (4) –4

8. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is _____

9. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line x=1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (2) $(3, -\sqrt{3})$

(3) $\left(4, -\frac{\sqrt{3}}{2}\right)$ (4) $(4, -\sqrt{3})$

10. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is :

(1) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (2) $\left(\frac{29}{5}, \frac{11}{5}\right)$

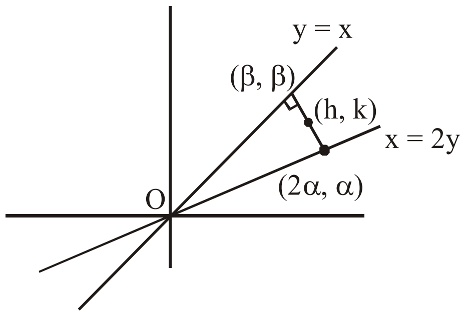
(3) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (4) $\left(\frac{29}{5}, \frac{8}{5}\right)$

SOLUTION

1. NTA Ans. (3)

Sol. $\frac{\alpha-\beta}{2\alpha-\beta}=-1$

$3\alpha = 2\beta$



$h = \frac{2\alpha+\beta}{2}$

$2h = \frac{7\alpha}{2}$

$k = \frac{\alpha+\beta}{2}$

$2k = \frac{5\alpha}{2}$

$\frac{h}{k}=\frac{7}{5}$

$5x = 7y$

2. NTA Ans. (5)

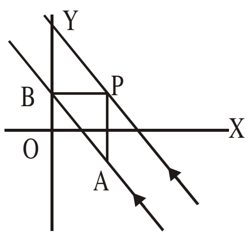
Sol. P is centroid of the triangle ABC

$\Rightarrow P \equiv \left(\frac{17}{6},\frac{8}{3}\right)$

$\Rightarrow PQ = 5$

3. NTA Ans. (3)

Sol. $\overleftrightarrow{AB}$: $3x + y - 2 = 0$



Also, $\frac{1}{2}\times\sqrt{10}\times h=5$

$\Rightarrow h=\sqrt{10}$

$\Rightarrow \frac{|4\lambda-2|}{\sqrt{10}}=\sqrt{10}\Rightarrow\lambda=3,-2$

4. NTA Ans. (2)

Sol. Centroid of $\Delta = (2, 2)$

line passing through intersection of

$x+3y-1=0$ and

$3x-y+1=0$, be given by

$(x+3y-1)+\lambda(3x-y+1)=0$

$\because$ It passes through (2, 2)

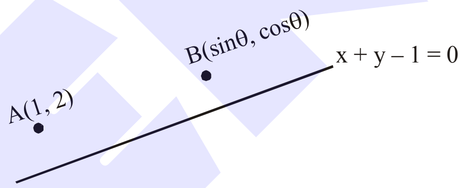
$\Rightarrow 7+5\lambda=0 \Rightarrow \lambda=-\frac{7}{5}$

$\therefore$ Required line is $8x-11y+6=0$

$\because$ (–9, –6) satisfies this equation.

5. Official Ans. by NTA (4)

Sol. Given that both points (1, 2) & $(\sin\theta, \cos\theta)$ lie on same side of the line $x + y - 1 = 0$



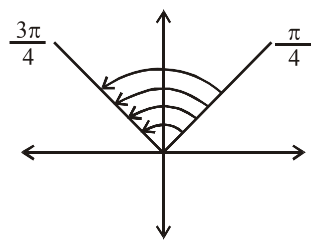
So, $\begin{pmatrix}\text{Put (1,2) in}\\ \text{given line}\end{pmatrix}\begin{pmatrix}\text{Put } (\sin\theta,\cos\theta \text{ in}\\ \text{given line}\end{pmatrix}>0$

$\Rightarrow (1 + 2 - 1) (\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \quad \{\div \text{by} \sqrt{2}\}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta>\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$

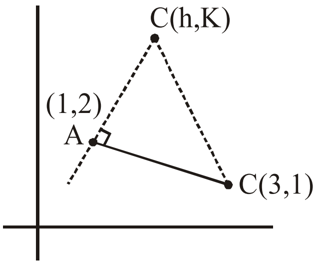


$\Rightarrow \frac{\pi}{4}<\theta+\frac{\pi}{4}<\frac{3\pi}{4}$

$\Rightarrow \boxed{0<\theta<\frac{\pi}{2}}$

6. Official Ans. by NTA (3)

Sol.



$$\left(\frac{K-2}{h-1}\right)\left(\frac{1-2}{3-1}\right)=-1 \Rightarrow K = 2h \quad ...(1)$$

$$\sqrt{5}\,|h-1| = 10$$

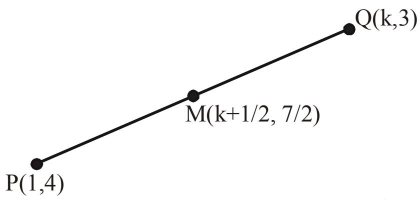
$$\because\ [\Delta ABC] = 5\sqrt{5}$$

$$\Rightarrow \frac{1}{2}\left(\sqrt{5}\right)\sqrt{(h-1)^2+(K-2)^2} = 5\sqrt{5} \quad ....(2)$$

$$\Rightarrow\ h = 2\sqrt{5}+1\ \ (h>0)$$

7. Official Ans. by NTA (4)

Sol.



Slope = $m = \frac{1}{1-k}$

Equation of $\perp^r$ bisector is

$y + 4 = (k-1)(x-0)$

$\Rightarrow y + 4 = x(k-1)$

$$\Rightarrow \frac{7}{2}+4 = \frac{k+1}{2}(k-1)$$

$$\Rightarrow \frac{15}{2} = \frac{k^2-1}{2} \Rightarrow k^2 = 16 \Rightarrow k = 4, -4$$

8. Official Ans. by NTA (30)

Sol. Apply distance between parallel line formula

$4x - 2y + \alpha = 0$

$4x - 2y + 6 = 0$

$$\left|\frac{\alpha-6}{255}\right| = \frac{1}{55}$$

$|\alpha - 6| = 2 \Rightarrow \alpha = 8, 4$

sum = 12

again

$6x - 3y + \beta = 0$

$6x - 3y + 9 = 0$

$$\left|\frac{\beta-9}{3\sqrt{5}}\right| = \frac{2}{\sqrt{5}}$$

$|\beta - 9| = 6 \Rightarrow \beta = 15, 3$

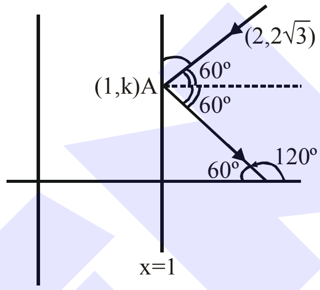
sum = 18

sum of all values of α and β is = 30

9. Official Ans. by NTA (2)

Sol. For point A

$$\tan 60^\circ = \frac{2\sqrt{3}-k}{2-1}$$



$$\sqrt{3} = 2\sqrt{3}-k$$

$$\therefore\ k = \sqrt{3}$$

so point $A\left(1,\sqrt{3}\right)$

Now slope of line AB is $m_{AB} = \tan 120^\circ$

$m\, m_{AB} = -\sqrt{3}$

Now equation of line AB is

$$y-\sqrt{3} = -\sqrt{3}(x-1)$$

$$\sqrt{3}x+y = 2\sqrt{3}$$

Now satisfy options

10. Official Ans. by NTA (3)

Sol. $L : \frac{x}{3}+\frac{y}{1} = 1 \Rightarrow x + 3y - 3 = 0$

Image of point (–1, –4)

$$\frac{x+1}{1} = \frac{y+4}{3} = -2\left(\frac{-1-12-3}{10}\right)$$

$$\frac{x+1}{1} = \frac{y+4}{3} = \frac{16}{5}$$

$$(x,y) \equiv \left(\frac{11}{5}, \frac{28}{5}\right)$$