STATISTICS

1. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to_____

2. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to ________.

3. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is

(1) 3.99 (2) 3.98
(3) 4.02 (4) 4.01

4. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 resepectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to

(1) –20 (2) 10
(3) –10 (4) –5

5. Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, ..., x_{10} - 3$, then the ordered pair (μ, λ) is equal to :

(1) (6, 6) (2) (3, 6)
(3) (6, 3) (4) (3, 3)

6. Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y = \{ax + b: x \in X \text{ and } a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to :

(1) –7 (2) 7
(3) 9 (4) –27

7. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, ... b_{11}$ is 90, then the common difference of this A.P. is_______.

8. For the frequency distribution :

Variate (x) : x_1 x_2 x_3 .....x_{15}
Frequency (f) : f_1 f_2 f_3 ....f_{15}

where $0 < x_1 < x_2 < x_3 < .... < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :

(1) 2 (2) 1
(3) 4 (4) 6

9. Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \neq p \in R$, then the standard deviation of these observations is :

(1) $\sqrt{\frac{3}{5}}$ (2) $\frac{7}{10}$
(3) $\frac{9}{10}$ (4) $\frac{4}{5}$

10. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(1) 7 (2) 3
(3) 5 (4) 9

11. If the variance of the following frequency distribution:

Class :	10–20	20–30	30–40
Frequency :	2	x	2

is 50, then x is equal to ____

12. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is :

(1) 2 (2) 4
(3) 3 (4) 1

13. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

(1) $2x^2 - 20x + 19 = 0$
(2) $x^2 - 10x + 19 = 0$
(3) $x^2 - 10x + 18 = 0$
(4) $x^2 - 20x + 18 = 0$

14. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, (n,a > 1) then the standard deviation of n observations $x_1, x_2, ..., x_n$ is

(1) $n\sqrt{a-1}$

(2) $\sqrt{a-1}$

(3) a – 1

(4) $\sqrt{n(a-1)}$

15. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ____________.

SOLUTION

1. NTA Ans. (54.00)

Sol. $\frac{3+7+9+12+13+20+x+y}{8}=10$

$x + y = 16$

$\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2=25$

$3^2 + 7^2 + 9^2 + 12^2 + 13^2 + 20^2 + x^2 + y^2 = 1000$

$x^2 + y^2 = 148$

$xy = 54$

2. NTA Ans. (18)

Sol. Variance of first 'n' natural numbers $=\frac{n^2-1}{12}=10$

$\Rightarrow n = 11$

and variance of first 'm' even natural numbers

$= 4\left(\frac{m^2-1}{12}\right)\Rightarrow\frac{m^2-1}{3}=16\Rightarrow m=7$

$m + n = 18$

3. NTA Ans. (1)

Sol. $\frac{\sum x_i}{20}=10 \Rightarrow \Sigma x_i = 200$

...(i)

$\frac{\sum x_i^2}{20}-100=4 \Rightarrow \Sigma x_i^2 = 2080$

...(ii)

Actual mean $= \frac{200-9+11}{20}=\frac{202}{20}$

Variance $= \frac{2080-81+121}{20}-\left(\frac{202}{20}\right)^2=3.99$

(1) Option

4. NTA Ans. (1)

Sol. $20p - q = 10$ ...(i)

and $2|p| = 1 \Rightarrow p=\pm\frac{1}{2}$ ...(ii)

so, $p=-\frac{1}{2}$ and $q=-20$

5. NTA Ans. (4)

Sol. $\sum_{i=1}^{10}(x_i-5)=10$

$\Rightarrow$ Mean of observation $x_i-5=\frac{1}{10}\sum_{i=1}^{3}(x_i-5)=1$

$\Rightarrow \mu$ = mean of observation $(x_i - 3)$

= (mean of observation $(x_i - 5)$) + 2

= 1 + 2 = 3

Variance of observation

$x_i-5=\frac{1}{10}\sum_{i=1}^{10}(x_i-5)^2$

$-$ (Mean of $(x_i - 5))^2 = 3$

$\Rightarrow \lambda$ = variance of observation $(x_i - 3)$

= variance of observation $(x_i - 5) = 3$

$\therefore (\mu, \lambda) = (3, 3)$

6. Official Ans. by NTA (1)

Sol. σ^2 = variance

μ = mean

$\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$

$\mu = 17$

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17}=17$

$\Rightarrow 9a + b = 17$ ....(1)

$\sigma^2 = 216$

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17}=216$

$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17}=216$

$\Rightarrow a^2 81 - 18 \times 9a^2 + a^2\, 3 \times (35) = 216$

$\Rightarrow a^2 = \frac{216}{24} = 9 \Rightarrow a = 3\ (a > 0)$

$\Rightarrow$ From (1), $b = -10$

So, $a + b = -7$

7. Official Ans. by NTA (3.00)

Sol. Let a be the first term and d be the common difference of the given A.P. Where d > 0

$$\overline{X} = a + \frac{0 + d + 2d + ... + 10d}{11}$$

$= a + 5d$

$$\Rightarrow \text{varience} = \frac{\Sigma(\overline{X} - x_i)^2}{11}$$

$\Rightarrow 90 \times 11 = (25d^2 + 16d^2 + 9d^2 + 4d^2) \times 2$

$\Rightarrow d = \pm 3 \Rightarrow d = 3$

8. Official Ans. by NTA (4)

Sol. $\because \sigma^2 \le \frac{1}{4}(M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$\therefore \ \sigma^2 < \frac{1}{4}(10 - 0)^2$

$\Rightarrow 0 < \sigma < 5$

$\therefore \ \sigma \ne 6.$

9. Official Ans. by NTA (3)

Sol. $\text{Variance} = \frac{\Sigma(x_i - p)^2}{n} - \left(\frac{\Sigma(x_i - p)}{n}\right)^2$

$= \frac{9}{10} - \left(\frac{3}{10}\right)^2 = \frac{81}{100}$

S.D. $= \frac{9}{10}$

10. Official Ans. by NTA (1)

Sol. $\overline{x} = 10$

$\Rightarrow \overline{x} = \frac{63 + a + b}{8} = 10 \Rightarrow a + b = 17$ ...(1)

Since, variance is independent of origin.
So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \frac{79 + (a - 10)^2 + (b - 10)^2}{8} - (10 - 10)^2$

$\Rightarrow a^2 + b^2 - 20(a + b) = -171$

$\Rightarrow a^2 + b^2 = 169$ ...(2)

From (i) & (ii) ; a = 12 & b = 5

11. Official Ans. by NTA (4)

Sol. $\because$ Variance is independent of shifting of origin

$\Rightarrow$ x_i : 15 25 35 or –10 0 10

f_i : 2 x 2 2 x 2

$\Rightarrow$ Variance $(\sigma^2) = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - (\overline{x})^2$

$\Rightarrow 50 = \frac{200 + 0 + 200}{x + 4} - 0 \quad \{\overline{x} = 0\}$

$\Rightarrow 200 + 50x = 200 + 200$

$\Rightarrow x = 4$

12. Official Ans. by NTA (1)

Sol. $\overline{x} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$

$x + y = 14$
.....(i)

$(\sigma)^2 = \frac{\sum (x_i)^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$16 = \frac{4 + 16 + 100 + 144 + 196 + x^2 + y^2}{7} - 8^2$

$16 + 64 = \frac{460 + x^2 + y^2}{7}$

$560 = 460 + x^2 + y^2$

$x^2 + y^2 = 100$ ....(ii)

Clearly by (i) and (ii), $|x - y| = 2$

Ans. 1

13. Official Ans. by NTA (2)

Sol. Mean = 5

$\frac{3 + 5 + 7 + a + b}{5} = 5$

$a + b = 10$ ....(i)

S.d. $= 2 \Rightarrow \sqrt{\frac{\sum_{i=1}^{5} (x_i - \overline{x})^2}{5}} = 2$

$(3–5)^2 + (5–5)^2 + (7–5)^2 + (a–5)^2 + (b–5)^2 = 20$

$\Rightarrow 4 + 0 + 4 + (a - 5)^2 + (b - 5)^2 = 20$

$a^2 + b^2 - 10(a + b) + 50 = 12$

$(a + b)^2 - 2ab - 100 + 50 = 12$

$ab = 19$ ....(ii)

Equation is $x^2 - 10x + 19 = 0$

14. Official Ans. by NTA (2)

Sol. S.D $= \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - a)}{n} - \left(\dfrac{\sum_{i=1}^{n}(x_i - a)}{n}\right)^2}$

$= \sqrt{\dfrac{na}{n} - \left(\dfrac{n}{n}\right)^2}$

$\{$Given $\sum_{i=1}^{n}(x_i - a) = n \quad \sum_{i=1}^{n}(x_i - a)^2 = na \}$

$= \sqrt{a-1}$

15. Official Ans. by NTA (6.00)

Sol.

x	0	2	4	8		2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum_{r=1}^{n} 2^r \ {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$$

$$\text{Mean} = \frac{(1+2)^n - {}^nC_0}{2^n} = \frac{728}{2^n}$$

$$\Rightarrow \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$$\Rightarrow 3^n = 729 \Rightarrow n = 6$$