



14. If the sum of first 11 terms of an A.P.,  $a_1, a_2, a_3, \dots$  is 0 ( $a_1 \neq 0$ ), then the sum of the A.P.,  $a_1, a_3, a_5, \dots, a_{23}$  is  $ka_1$ , where  $k$  is equal to :
- (1)  $\frac{121}{10}$  (2)  $-\frac{72}{5}$   
 (3)  $\frac{72}{5}$  (4)  $-\frac{121}{10}$
15. Let  $S$  be the sum of the first 9 terms of the series:  $\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\} + \{x^4 + (k+6)a\} + \dots$  where  $a \neq 0$  and  $x \neq 1$ .  
 If  $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$ , then  $k$  is equal to :
- (1)  $-5$  (2)  $1$   
 (3)  $-3$  (4)  $3$
16. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :
- (1)  $\frac{1}{4}$  (2)  $\frac{1}{5}$   
 (3)  $\frac{1}{7}$  (4)  $\frac{1}{6}$
17. If the sum of the series  $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$  upto  $n^{\text{th}}$  term is 488 and the  $n^{\text{th}}$  term is negative, then :
- (1)  $n^{\text{th}}$  term is  $-4\frac{2}{5}$  (2)  $n = 41$   
 (3)  $n^{\text{th}}$  term is  $-4$  (4)  $n = 60$
18. If  $m$  arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4<sup>th</sup> A.M. is equal to 2<sup>nd</sup> G.M., then  $m$  is equal to \_\_\_\_\_.
19. If  $1 + (1-2^2 \cdot 1) + (1-4^2 \cdot 3) + (1-6^2 \cdot 5) + \dots + (1-20^2 \cdot 19) = \alpha - 220\beta$ , then an ordered pair  $(\alpha, \beta)$  is equal to:
- (1) (10, 97) (2) (11, 103)  
 (3) (10, 103) (4) (11, 97)
20. Let  $a_1, a_2, \dots, a_n$  be a given A.P. whose common difference is an integer and  $S_n = a_1 + a_2 + \dots + a_n$ . If  $a_1 = 1$ ,  $a_n = 300$  and  $15 \leq n \leq 50$ , then the ordered pair  $(S_{n-4}, a_{n-4})$  is equal to :
- (1) (2480, 249) (2) (2490, 249)  
 (3) (2490, 248) (4) (2480, 248)
21. The minimum value of  $2^{\sin x} + 2^{\cos x}$  is :-
- (1)  $2^{1-\frac{1}{\sqrt{2}}}$  (2)  $2^{-1+\sqrt{2}}$   
 (3)  $2^{1-\sqrt{2}}$  (4)  $2^{-1+\frac{1}{\sqrt{2}}}$
22. If  $3^{2 \sin 2\alpha} - 1$ , 14 and  $3^{4-2 \sin 2\alpha}$  are the first three terms of an A.P. for some  $\alpha$ , then the sixth term of this A.P. is :
- (1) 66 (2) 65  
 (3) 81 (4) 78
23. If  $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$ , then  $S$  is equal to :
- (1)  $\frac{3^{11}}{2} + 2^{10}$  (2)  $3^{11} - 2^{12}$   
 (3)  $3^{11}$  (4)  $2 \cdot 3^{11}$
24. If the sum of the first 20 terms of the series  $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$  is 460, then  $x$  is equal to:
- (1)  $7^{46/21}$  (2)  $7^{1/2}$   
 (3)  $e^2$  (4)  $7^2$
25. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is :
- (1)  $\frac{2}{13}(3^{50} - 1)$  (2)  $\frac{1}{26}(3^{50} - 1)$   
 (3)  $\frac{1}{13}(3^{50} - 1)$  (4)  $\frac{1}{26}(3^{49} - 1)$



## SOLUTION

## 1. NTA Ans. (1)

Sol. Sum of the 40 terms of

$$\begin{aligned} & 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 \dots \\ & = (3 + 8 + 13 + \dots \text{upto } 20 \text{ term}) \\ & \quad + [4 + 9 + 15 + \dots \text{upto } 20 \text{ terms}] \\ & = 10 [\{6 + 19 \times 5\} + \{8 + 19 \times 5\}] \\ & = 10 \times 204 = 20 \times 102 \end{aligned}$$

## 2. NTA Ans. (1)

Sol.  $a_1 + a_2 = 4$

$$r^2 a_1 + r^2 a_2 = 16$$

$$\Rightarrow r^2 = 4 \Rightarrow r = -2 \quad \text{as } a_1 < 0$$

$$\text{and } a_1 + a_2 = 4$$

$$a_1 + a_1(-2) = 4 \Rightarrow a_1 = -4$$

$$4\lambda = (-4) \left( \frac{(-2)^9 - 1}{-2 - 1} \right) = (-4) \times \frac{513}{3}$$

$$\Rightarrow \lambda = -171$$

## 3. NTA Ans. (3)

Sol. Let the A.P is

$$a - 2d, a - d, a, a + d, a + 2d$$

$$\therefore \text{sum} = 25 \Rightarrow a = 5$$

$$\text{Product} = 2520$$

$$(25 - 4d^2)(25 - d^2) = 504$$

$$4d^4 - 125d^2 + 121 = 0$$

$$\Rightarrow d^2 = 1, \frac{121}{4}$$

$$\Rightarrow d = \pm 1, \pm \frac{11}{2}$$

$d = \pm 1$  is rejected because none of the term

$$\text{can be } \frac{-1}{2}.$$

$$\Rightarrow d = \pm \frac{11}{2}$$

$$\Rightarrow \text{AP will be } -6, -\frac{1}{2}, 5, \frac{21}{2}, 16$$

Largest term is 16.

## 4. NTA Ans. (3)

Sol.  $1 + 49 + 49^2 + \dots + 49^{12}$

$$= (49)^{126} - 1 = (49^{63} + 1) \frac{(49^{63} - 1)}{(48)}$$

So greatest value of  $k = 63$

## 5. NTA Ans. (2)

$$\text{Sol. } T_{10} = \frac{1}{20} = a + 9d \quad \dots(i)$$

$$T_{20} = \frac{1}{10} = a + 19d \quad \dots(ii)$$

$$a = \frac{1}{200} = d$$

$$\text{Hence, } S_{200} = \frac{200}{2} \left[ \frac{2}{200} + \frac{199}{200} \right] = \frac{201}{2}$$

(2) Option

## 6. NTA Ans. (504)

$$\text{Sol. } \frac{1}{4} \left( \sum_{n=1}^7 2n^3 + \sum_{n=1}^7 3n^2 + \sum_{n=1}^7 n \right)$$

$$= \frac{1}{4} \left( 2 \left( \frac{7 \times 8}{2} \right)^2 + 3 \left( \frac{7 \times 8 \times 15}{6} \right) + \frac{7 \times 8}{2} \right)$$

$$= 504$$

Ans. 504.00

## 7. NTA Ans. (1540.00)

$$\text{Sol. } \sum_{k=1}^{20} \frac{k(k+1)}{2} = \frac{1}{2} \sum_{k=1}^{20} \frac{k(k+1)(k+2) - (k-1)k(k+1)}{3}$$

$$= \frac{1}{6} \times 20 \times 21 \times 22 = 1540.00$$

## 8. NTA Ans. (3)

$$\text{Sol. } x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n} \theta = 1 - \tan^2 \theta + \tan^4 \theta + \dots$$

$$\Rightarrow x = \cos^2 \theta$$

$$y = \sum_{n=0}^{\infty} \cos^{2n} \theta \Rightarrow y = 1 + \cos^2 \theta + \cos^4 \theta + \dots$$

$$\Rightarrow y = \frac{1}{\sin^2 \theta} \Rightarrow y = \frac{1}{1-x}$$

$$\Rightarrow y(1 - x) = 1$$

9. NTA Ans. (4)

Sol.  $\sum_{n=1}^{100} a_{2n+1} = 200 \Rightarrow a_3 + a_5 + a_7 + \dots + a_{201} = 200$

$$\Rightarrow ar^2 \frac{(r^{200} - 1)}{(r^2 - 1)} = 200$$

$$\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + \dots + a_{200} = 100$$

$$\Rightarrow \frac{ar(r^{200} - 1)}{(r^2 - 1)} = 100$$

On dividing  $r = 2$

on adding  $a_2 + a_3 + a_4 + a_5 + \dots + a_{200} + a_{201} = 300$

$$\Rightarrow r(a_1 + a_2 + a_3 + \dots + a_{200}) = 300$$

$$\Rightarrow \sum_{n=1}^{200} a_n = 150$$

10. NTA Ans. (14)

Sol. Common term are : 23, 51, 79, .....  $T_n$

$$T_n \leq 407 \Rightarrow 23 + (n - 1)28 \leq 407$$

$$\Rightarrow n \leq 14.71$$

$$n = 14$$

11. NTA Ans. (1)

Sol.  $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \dots \infty$

$$= 2^{\frac{1}{4}} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \cdot \dots \infty$$

$$= 2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \infty}$$

$$= 2^{\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \infty} = (2)^{\left(\frac{1/4}{1-1/2}\right)} = 2^{1/2}$$

12. Official Ans. by NTA (1)

Sol.  $|x| < 1, |y| < 1, x \neq y$

$$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \dots$$

By multiplying and dividing  $x - y$  :

$$\frac{(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + \dots}{x - y}$$

$$= \frac{(x^2 + x^3 + x^4 + \dots) - (y^2 + y^3 + y^4 + \dots)}{x - y}$$

$$= \frac{\frac{x^2}{1-x} - \frac{y^2}{1-y}}{x - y}$$

$$= \frac{(x^2 - y^2) - xy(x - y)}{(1-x)(1-y)(x - y)}$$

$$= \frac{x + y - xy}{(1-x)(1-y)}$$

13. Official Ans. by NTA (4)

Sol. Let three terms of G.P. are  $\frac{a}{r}, a, ar$

product = 27

$$\Rightarrow a^3 = 27 \Rightarrow a = 3$$

$$S = \frac{3}{r} + 3r + 3$$

For  $r > 0$

$$\frac{\frac{3}{r} + 3r}{2} \geq \sqrt{3^2} \quad (\text{By AM} \geq \text{GM})$$

$$\Rightarrow \frac{3}{r} + 3r \geq 6 \quad \dots(1)$$

$$\text{For } r < 0 \quad \frac{3}{r} + 3r \leq -6 \quad \dots(2)$$

From (1) & (2)

$$S \in (-\infty - 3] \cup [9, \infty)$$

**14. Official Ans. by NTA (2)****Sol.**  $a_1 + a_2 + a_3 + \dots + a_{11} = 0$ 

$$\Rightarrow (a_1 + a_{11}) \times \frac{11}{2} = 0$$

$$\Rightarrow a_1 + a_{11} = 0$$

$$\Rightarrow a_1 + a_1 + 10d = 0$$

where  $d$  is common difference

$$\Rightarrow \boxed{a_1 = -5d}$$

$$a_1 + a_3 + a_5 + \dots + a_{23}$$

$$= (a_1 + a_{23}) \times \frac{12}{2} = (a_1 + a_1 + 22d) \times 6$$

$$= \left( 2a_1 + 22 \left( \frac{-a_1}{5} \right) \right) \times 6$$

$$= -\frac{72}{5} a_1 \Rightarrow K = \frac{-72}{5}$$

**15. Official Ans. by NTA (3)****Sol.**  $S = [x + ka + 0] + [x^2 + ka + 2a] + [x^3 + ka + 4a] + [x^4 + ka + 6a] + \dots + 9 \text{ terms}$ 

$$\Rightarrow S = (x + x^2 + x^3 + x^4 + \dots + 9 \text{ terms}) + (ka + ka + ka + \dots + 9 \text{ terms}) + (0 + 2a + 4a + 6a + \dots + 9 \text{ terms})$$

$$\Rightarrow S = x \left[ \frac{x^9 - 1}{x - 1} \right] + 9ka + 72a$$

$$\Rightarrow S = \frac{(x^{10} - x) + (9k + 72)a(x - 1)}{(x - 1)}$$

Compare with given sum, then we get,  $(9k + 72) = 45$ 

$$\Rightarrow \boxed{k = -3}$$

**16. Official Ans. by NTA (4)****Sol.** Sum of 1st 25 terms = sum of its next 15 terms

$$\Rightarrow (T_1 + \dots + T_{25}) = (T_{26} + \dots + T_{40})$$

$$\Rightarrow (T_1 + \dots + T_{40}) = 2(T_1 + \dots + T_{25})$$

$$\Rightarrow \frac{40}{2} [2 \times 3 + (39d)] = 2 \times \frac{25}{2} [2 \times 2 + 24d]$$

$$\Rightarrow d = \frac{1}{6}$$

**17. Official Ans. by NTA (3)**

$$\text{Sol. } S = \frac{100}{5} + \frac{98}{5} + \frac{96}{5} + \frac{94}{5} + \dots + n$$

$$S_n = \frac{n}{2} \left( 2 \times \frac{100}{5} + (n-1) \left( -\frac{2}{5} \right) \right) = 188$$

$$n(100 - n + 1) = 488 \times 5$$

$$n^2 - 101n + 488 \times 5 = 0$$

$$n = 61, 40$$

$$T_n = a + (n-1)d = \frac{100}{5} - \frac{2}{5} \times 60$$

$$= 20 - 24 = -4$$

**18. Official Ans. by NTA (39)****Sol.**  $3, A_1, A_2, \dots, A_m, 243$ 

$$d = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$$

Now  $3, G_1, G_2, G_3, 243$ 

$$r = \left( \frac{243}{3} \right)^{\frac{1}{3+1}} = 3$$

$$\therefore A_4 = G_2$$

$$\Rightarrow a + 4d = ar^2$$

$$3 + 4 \left( \frac{240}{m + 1} \right) = 3(3)^2$$

$$m = 39$$

**19. Official Ans. by NTA (2)****Sol.**  $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + \dots + (1 - 20^2 \cdot 19)$ 

$$= \alpha - 220 \beta$$

$$= 11 - (2^2 \cdot 1 + 4^2 \cdot 3 + \dots + 20^2 \cdot 19)$$

$$= 11 - 2^2 \cdot \sum_{r=1}^{10} r^2 (2r-1) = 11 - 4 \left( \frac{110^2}{2} - 35 \times 11 \right)$$

$$= 11 - 220(103)$$

$$\Rightarrow \alpha = 11, \beta = 103$$

**20. Official Ans. by NTA (3)**

**Sol.**  $a_n = a_1 + (n - 1)d$   
 $\Rightarrow 300 = 1 + (n - 1)d$   
 $\Rightarrow (n - 1)d = 299 = 13 \times 23$   
 since,  $n \in [15, 50]$   
 $\therefore n = 24$  and  $d = 13$   
 $a_{n-4} = a_{20} = 1 + 19 \times 13 = 248$   
 $\Rightarrow a_{n-4} = 248$   
 $S_{n-4} = \frac{20}{2} \{1 + 248\} = 2490$

**21. Official Ans. by NTA (1)**

**Sol.** Usnign AM  $\geq$  GM  
 $\Rightarrow \frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} \cdot 2^{\cos x}}$   
 $\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1 + \left(\frac{\sin x + \cos x}{2}\right)}$   
 $\Rightarrow \min(2^{\sin x} + 2^{\cos x}) = 2^{1 - \frac{1}{\sqrt{2}}}$

**22. Official Ans. by NTA (1)**

**Sol.** Given that  
 $3^4 - \sin 2\alpha + 3^{2 \sin 2\alpha} - 1 = 28$   
 Let  $3^{2 \sin 2\alpha} = t$   
 $\frac{81}{t} + \frac{t}{3} = 28$   
 $t = 81, 3$   
 $3^{2 \sin 2\alpha} = 3^1, 3^4$   
 $2 \sin 2\alpha = 1, 4$   
 $\sin 2\alpha = \frac{1}{2}, 2$  (rejected)  
 First term  $a = 3^{2 \sin 2\alpha} - 1$   
 $a = 1$   
 Second term = 14  
 $\therefore$  common difference  $d = 13$   
 $T_6 = a + 5d$   
 $T_6 = 1 + 5 \times 13$   
 $T_6 = 66$

**23. Official Ans. by NTA (3)**

**Sol.**  $a = 2^{10}; r = \frac{3}{2}; n = 11$  (G.P.)  
 $S' = (2^{10}) \frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1} = 2^{11} \left(\frac{3^{11}}{2^{11}} - 1\right)$   
 $S' = 3^{11} - 2^{11} = S - 2^{11}$  (Given)  
 $\therefore S = 3^{11}$

**24. Official Ans. by NTA (4)**

**Sol.**  $460 = \log_7 x \cdot (2 + 3 + 4 + \dots + 20 + 21)$   
 $\Rightarrow 460 = \log_7 x \cdot \left(\frac{21 \times 22}{2} - 1\right)$   
 $\Rightarrow 460 = 230 \cdot \log_7 x$   
 $\Rightarrow \log_7 x = 2 \Rightarrow x = 49$

**25. Official Ans. by NTA (2)**

**Sol.** Let first term =  $a > 0$   
 Common ratio =  $r > 0$   
 $ar + ar^2 + ar^3 = 3$  ....(i)  
 $ar^5 + ar^6 + ar^7 = 243$  ....(ii)  
 $r^4(ar + ar^2 + ar^3) = 243$   
 $r^4(3) = 243 \Rightarrow r = 3$  as  $r > 0$   
 from (1)  
 $3a + 9a + 27a = 3$   
 $a = \frac{1}{13}$

$S_{50} = \frac{a(r^{50} - 1)}{(r - 1)} = \frac{1}{26} (3^{50} - 1)$

**26. Official Ans. by NTA (2)**

**Sol.**  $f(x + y) = f(x) \cdot f(y)$   
 $\sum_{x=1}^{\infty} f(x) = 2$  where  $x, y \in \mathbb{N}$   
 $f(1) + f(2) + f(3) + \dots = 2$  ... (1) (Given)  
 Now for  $f(2)$  put  $x = y = 1$   
 $f(2) = f(1 + 1) = f(1) \cdot f(1) = (f(1))^2$   
 $f(3) = f(2 + 1) = f(2) \cdot f(1) = (f(1))^3$   
 Now put these values in equation (1)  
 $f(1) + (f(1))^2 + [(f(1))^2 + \dots] = 2$   
 $\frac{f(1)}{1 - f(1)} = 2$

$$\Rightarrow f(1) = \frac{2}{3}$$

$$\text{Now } f(2) = \left(\frac{2}{3}\right)^2$$

$$f(4) = \left(\frac{2}{3}\right)^4$$

$$\text{then the value of } \frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}$$

**27. Official Ans. by NTA (3)**

**Sol.**  $(a^2 + b^2 + c^2)p^2 + 2(ab + bc + cd)p + b^2 + c^2 + d^2 = 0$

$$\Rightarrow (a^2p^2 + 2abp + b^2) + (b^2p^2 + 2bcp + c^2) + (c^2p^2 + 2cdp + d^2) = 0$$

$$\Rightarrow (ab + b)^2 + (bp + c)^2 + (cp + d)^2 = 0$$

This is possible only when

$$ap + b = 0 \text{ and } bp + c = 0 \text{ and } cp + d = 0$$

$$p = -\frac{b}{a} = -\frac{c}{b} = -\frac{d}{c}$$

$$\text{or } \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$\therefore a, b, c, d$  are in G.P.

**28. Official Ans. by NTA (2)**

**Sol.**  $a_1, a_2, \dots, a_n \rightarrow (CD = d)$

$$b_1, b_2, \dots, b_m \rightarrow (CD = d + 2)$$

$$a_{40} = a + 39d = -159$$

$$\dots(1)$$

$$a_{100} = a + 99d = -399$$

$$\dots(2)$$

$$\text{Subtract : } 60d = -240 \Rightarrow d = -4$$

using equation (1)

$$a + 39(-4) = -159$$

$$a = 156 - 159 = -3$$

$$a_{70} = a + 69d = -3 + 69(-4) = -279 = b_{100}$$

$$b_{100} = -279$$

$$b_1 + 99(d + 2) = -279$$

$$b_1 - 198 = -279 \Rightarrow b_1 = -81$$