QUADRATIC EQUATION

1. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true ?

(1) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(2) $p_5 = 11$

(3) $p_3 = p_5 - p_4$

(4) $p_5 = p_2 \cdot p_3$

2. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

(1) is an empty set.

(2) contains at least four elements.

(3) contains exactly two elements.

(4) is a singleton.

3. The least positive value of 'a' for which the equation $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is

4. Let $a, b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :

(1) 26 (2) 25

(3) 28 (4) 24

5. If $A = \{x \in \mathbf{R} : |x| < 2\}$ and $B = \{x \in \mathbf{R} : |x - 2| \geq 3\}$; then :

(1) $A \cup B = \mathbf{R} - (2, 5)$ (2) $A \cap B = (-2, -1)$

(3) $B - A = \mathbf{R} - (-2, 5)$ (4) $A - B = [-1, 2)$

6. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is :

(1) 4 (2) 2

(3) 3 (4) 1

7. Let α and β be the roots of the equation $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, $n = 1,2,3....$, then :

(1) $5S_6 + 6S_5 = 2S_4$

(2) $5S_6 + 6S_5 + 2S_4 = 0$

(3) $6S_6 + 5S_5 + 2S_4 = 0$

(4) $6S_6 + 5S_5 = 2S_4$

8. Let f(x) be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in :

(1) $(-3, -1)$ (2) $(1, 3)$

(3) $(-1, 0)$ (4) $(0, 1)$

9. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to :

(1) $\frac{9}{4}(9 + p^2)$ (2) $\frac{9}{4}(9 - q^2)$

(3) $\frac{9}{4}(9 - p^2)$ (4) $\frac{9}{4}(9 + q^2)$

10. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(1) $(-3, -1)$ (2) $(1, 3]$

(3) $(0, 2)$ (4) $(2, 4]$

11. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is :

(1) 3 : 1 (2) 33 : 31

(3) 9 : 7 (4) 5 : 3

12. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to :

(1) 36 (2) 27
(3) 9 (4) 18

13. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is

(1) $\frac{25}{9}$ (2) $\frac{25}{81}$

(3) $\frac{5}{27}$ (4) $\frac{5}{9}$

14. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to:

(1) $\frac{27}{16}$ (2) $\frac{1}{24}$

(3) $\frac{27}{32}$ (4) $\frac{3}{8}$

15. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

(1) 1 (2) 3
(3) 4 (4) 2

16. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :

(1) $2\alpha^2$ (2) $2\alpha(\alpha + 1)$
(3) $-2\alpha(\alpha + 1)$ (4) $2\alpha(\alpha - 1)$

SOLUTION

1. NTA Ans (4)

Sol. $\alpha + \beta = 1, \alpha\beta = -1$

$P_k = \alpha^k + \beta^k$

$\alpha^2 - \alpha - 1 = 0$

$\Rightarrow \alpha^k - \alpha^{k-1} - \alpha^{k-2} = 0$

& $\beta^k - \beta^{k-1} - \beta^{k-2} = 0$

$\Rightarrow P_k = P_{k-1} + P_{k-2}$

$P_1 = \alpha + \beta = 1$

$P_2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 + 2 = 3$

$P_3 = 4$

$P_4 = 7$

$P_5 = 11$

2. NTA Ans.(4)

Sol. Let $3^x = t$; $t > 0$

$t(t - 1) + 2 = |t - 1| + |t - 2|$

$t^2 - t + 2 = |t - 1| + |t - 2|$

Case-I : $t < 1$

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 + 2 = 3 - t$

$t^2 + t - 1 = 0$

$$t = \frac{-1 \pm \sqrt{5}}{2}$$

$t = \frac{\sqrt{5} - 1}{2}$ is only acceptable

Case-II : $1 \le t < 2$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 1 = 0$

$D < 0$ no real solution

Case-III : $t \ge 2$

$t^2 - t + 2 = t - 1 + t - 2$

$t^2 - 3t \;\; 5 = 0 \Rightarrow D < 0$ no real solution

(4) Option

3. NTA Ans. (8.00)

Sol. $D \ge 0 \Rightarrow (a - 10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \ge 0$

$\Rightarrow a^2 - 4a - 32 \ge 0$

$\Rightarrow a \in (-\infty, 4] \cup [8, \infty)$

4. NTA Ans. (2)

Sol. $ax^2 - 2bx + 5 = 0 \begin{smallmatrix} \alpha \\ \alpha \end{smallmatrix}$

$\Rightarrow \alpha = \frac{b}{a};\ \alpha^2 = \frac{5}{a} \Rightarrow b^2 = 5a$

$x^2 - 2bx - 10 = 0 \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \Rightarrow \alpha^2 - 2b\alpha - 10 = 0$

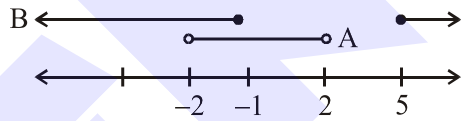
$\Rightarrow a = \frac{1}{4} \Rightarrow \alpha^2 = 20;\ \alpha\beta = -10 \Rightarrow \beta^2 = 5$

$\Rightarrow \alpha^2 + \beta^2 = 25$

5. NTA Ans. (3)

Sol. A : $x \in (-2, 2)$; B : $x \in (-\infty, -1] \cup [5, \infty)$

$\Rightarrow B - A = R - (-2, 5)$



6. NTA Ans. (4)

Sol. $e^{4x} + e^{3x} - 4e^x + e^x + 1 = 0$

Divide by e^{2x}

$$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

Let $e^x + \frac{1}{e^x} = t \Rightarrow (e^x - 1)^2 = 0 \Rightarrow x = 0.$

$\therefore$ Number of real roots = 1

7. Official Ans. by NTA (1)

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^n = 0$ …(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^n = 0$ …(2)

By adding (1) & (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

For n = 4

$\boxed{5S_6 + 6S_5 = 2S_4}$

8. Official Ans. by NTA (3)

Sol. $f(x) = a(x - 3)(x - \alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1 + \alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \neq 0 \Rightarrow 5\alpha = -2$

$\alpha = -\frac{2}{5} = -0.4$

$\alpha \in (-1, 0)$

9. Official Ans. by NTA (3)

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0$ & $\beta^2 + p\beta + 2 = 0$

$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p \quad \alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

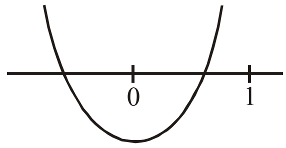
$$= \frac{(-p\alpha - 3)(-p\beta - 3)(\alpha\beta + 1)^2}{(\alpha\beta)^2}$$

$$= \frac{9}{4}(p\alpha\beta + 3p(\alpha + \beta) + 9)$$

$$= \frac{9}{4}(9 - p^2) = \frac{9}{4}(9 - 4q^2)$$

10. Official Ans. by NTA (2)

Sol. If exactly one root in (0, 1) then



$\Rightarrow f(0).f(1) < 0$

$\Rightarrow 2(\lambda^2 - 4\lambda + 3) < 0$

$\Rightarrow 1 < \lambda < 3$

Now for $\lambda = 1$, $2x^2 - 4x + 2 = 0$

$(x - 1)^2 = 0$, $x = 1, 1$

So both roots doesn't lie between (0, 1)

$\therefore \lambda \neq 1$

Again for $\lambda = 3$

$10x^2 - 12x + 2 = 0$

$\Rightarrow x = 1, \frac{1}{5}$

so if one root is 1 then second root lie between (0, 1)

so $\lambda = 3$ is correct.

$\therefore \lambda \in (1, 3]$.

11. Official Ans. by NTA (3)

Sol. $x^2 - 3x + p = 0 \begin{array}{l} \alpha \\ \beta \end{array}$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3$ ....(1)

$x^2 - 6x + q = 0 \begin{array}{l} \gamma \\ \delta \end{array}$

$\alpha r^2 + \alpha r^3 = 6$ ...(2)

(2) ÷ (1)

$r^2 = 2$

So, $\frac{2q + p}{2q - p} = \frac{2r^5 + r}{2r^5 - r} = \frac{2r^4 + 1}{2r^4 - 1} = \frac{9}{7}$

12. Official Ans. by NTA (4)

Sol. $\alpha + \beta = 1$, $\alpha\beta = 2\lambda$

$\alpha + \beta = \frac{10}{3}$, $\alpha\gamma = \frac{27\lambda}{3} = 9\lambda$

$\gamma - \beta = \frac{7}{3}$,

$$\frac{\gamma}{\beta} = \frac{9}{2} \Rightarrow \gamma = \frac{9}{2}\beta = \frac{9}{2}\times\frac{2}{3} \Rightarrow \gamma = 3$$

$$\frac{9}{2}\beta - \beta = \frac{7}{3}$$

$$\frac{9}{2}\beta = \frac{7}{3} \Rightarrow \beta = \frac{2}{3}$$

$$\alpha = 1 - \frac{2}{3} = \frac{1}{3}$$

$$2\lambda = \frac{2}{9} \Rightarrow \lambda = \frac{1}{9}$$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}\times 3}{\frac{1}{9}} = 18$$

13. Official Ans. by NTA (2)

Sol. $9x^2 - 18|x| + 5 = 0$

$9|x|^2 - 15|x| - 3|x| + 5 = 0$ $(\because x^2 = |x|^2)$

$3|x|(3|x| - 5) - (3|x| - 5) = 0$

$$|x| = \frac{1}{3}, \frac{5}{3}$$

$$x = \pm\frac{1}{3}, \pm\frac{5}{3}$$

Product of roots $= \frac{25}{81}$

14. Official Ans. by NTA (1)

Sol. $7x^2 - 3x - 2 = 0$

$$\alpha + \beta = \frac{3}{7} \quad \alpha\beta = \frac{-2}{7}$$

$$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{\alpha+\beta-\alpha\beta(\alpha+\beta)}{1-\alpha^2-\beta^2+\alpha^2\beta^2}$$

$$= \frac{\frac{3}{7}+\frac{2}{7}\left(\frac{3}{7}\right)}{1-(\alpha+\beta)^2+2\alpha\beta+\alpha^2\beta^2} = \frac{27}{16}$$

15. Official Ans. by NTA (4)

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64, \alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8} = \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$= \frac{\alpha+\beta}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}} = 2$$

16. Official Ans. by NTA (3)

Sol. α and β are the roots of the equation $4x^2 + 2x - 1 = 0$

$$4\alpha^2 + 2\alpha = 1 \Rightarrow \frac{1}{2} = 2\alpha^2 + \alpha \quad ...(1)$$

$$\beta = \frac{-1}{2} - \alpha$$

using equation (1)

$\beta = -(2\alpha^2 + \alpha) - \alpha$

$\beta = -2\alpha^2 - 2\alpha$

$\beta = -2\alpha(\alpha + 1)$