PROBABILITY

1. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

(1) $\frac{17}{2}$ (2) 4

(3) $\frac{17}{8}$ (4) $\frac{17}{4}$

2. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5 otherwise X takes the value –1. Then the expected value of X, is :

(1) $\frac{3}{16}$ (2) $-\frac{3}{16}$

(3) $\frac{1}{8}$ (4) $-\frac{1}{8}$

3. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is

(1) 0.02 (2) 0.01

(3) 0.20 (4) 0.10

4. Let A and B be two independent events such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{6}$. Then, which of the following is TRUE ?

(1) $P(A/B)=\frac{2}{3}$ (2) $P(A/(A\cup B))=\frac{1}{4}$

(3) $P(A/B')=\frac{1}{3}$ (4) $P(A'/B')=\frac{1}{3}$

5. A random variable X has the following probability distribution :

X :	1	2	3	4	5
P(X) :	K^2	2K	K	2K	$5K^2$

Then P(X > 2) is equal to :

(1) $\frac{7}{12}$ (2) $\frac{23}{36}$

(3) $\frac{1}{36}$ (4) $\frac{1}{6}$

6. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

(1) $\frac{945}{2^{11}}$ (2) $\frac{965}{2^{11}}$

(3) $\frac{945}{2^{10}}$ (4) $\frac{965}{2^{10}}$

7. In a box, there are 20 cards, out of which 10 are lebelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is :

(1) $\frac{11}{16}$ (2) $\frac{13}{16}$

(3) $\frac{9}{16}$ (4) $\frac{15}{16}$

8. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :

(1) $\frac{8}{17}$ (2) $\frac{2}{3}$

(3) $\frac{4}{17}$ (4) $\frac{2}{5}$

9. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P(E_2^C \cap E_3^C / E_1)$ is equal to :

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$

(3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

10. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :

(1) $\frac{1}{8}$ (2) $\frac{1}{9}$

(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

11. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(1) $\frac{121}{10^4}$ (2) $\frac{150}{10^4}$ (3) $\frac{135}{10^4}$ (4) $\frac{134}{10^4}$

12. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________.

13. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six The game stops as soon as either of the players wins. The probability of A winning the game is :

(1) $\frac{31}{61}$ (2) $\frac{5}{6}$

(3) $\frac{5}{31}$ (4) $\frac{30}{61}$

14. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ____.

15. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _______.

16. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) 2!3!4! (2) $(3!)^3.(4!)$

(3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

17. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

(1) $\frac{15}{101}$ (2) $\frac{5}{101}$

(3) $\frac{5}{33}$ (4) $\frac{10}{99}$

18. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval:

(1) [0.36, 0.40] (2) [0.35, 0.36]

(3) [0.25, 0.35] (4) [0.20, 0.25]

SOLUTION

1. NTA Ans. (3)

Sol. Probability that at most 2 machines are out of service

$$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_2\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$$

$$= \left(\frac{3}{4}\right)^4 \times \frac{17}{8} \Rightarrow k = \frac{17}{8}$$

2. NTA Ans. (3)

Sol.

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

Expected value = $\sum XP(k)$

$$-\frac{1}{32} - \frac{12}{32} - \frac{11}{32} + \frac{15}{32} + \frac{8}{32} + \frac{5}{32}$$

$$= \frac{28-24}{32} = \frac{4}{32} = \frac{1}{8}$$

3. NTA Ans. (4)

Sol. $P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$P(A) + P(B) - P(A \cap B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{10}$

(4) Option

4. NTA Ans. (3)

Sol. (1) $P(A/B) = P(A) = \frac{1}{3}$

(2) $P(A/(A \cup B)) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)}$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6} - \frac{1}{18}} = \frac{3}{4}$$

(3) $P(A/B') = P(A) = \frac{1}{3}$

(4) $P(A'/B') = P(A') = \frac{2}{3}$

5. NTA Ans. (2)

Sol. $\sum P(X) = 1 \Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow 6K^2 + 5K - 1 = 0 \Rightarrow (6K - 1)(K + 1) = 0$

$\Rightarrow K = -1$ (rejected) $\Rightarrow K = \frac{1}{6}$

$P(X > 2) = K + 2K + 5K^2 = \frac{23}{36}$

6. NTA Ans. (3)

ALLEN Ans. (BONUS)

Note: Interpretating the given question, we find an answer that does not match with any of the given options. So, it should be bonus, but NTA retained the answer as option(3).

Sol. 10 different balls in 4 different boxes.

10

2,3,0,5   2,3,1,4   2,3,2,3

$$\frac{1}{4^{10}}\left(4! \times \frac{10!}{2! \times 3! \times 0! \times 5!} + 4! \times \frac{10!}{2! \times 3! \times 1! \times 4!} + 4! \times \frac{10!}{(2!)^2 \times 2! \times (3!)^2 \times 2!}\right)$$

$$= \frac{17 \times 945}{2^{15}}$$

7. NTA Ans. (1)

Sol. A : Event when card A is drawn

B : Event when card B is drawn.

$P(A) = P(B) = \frac{1}{2}$

Required probability = P(AA or (AB)A or (BA)A or (ABB)A or (BAB)A or (BBA)A)

$$= \frac{1}{2} \times \frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 2 + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 3$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{3}{16} = \frac{11}{16}$$

8. Official Ans. by NTA (1)

Sol. Let B_1 be the event where Box–I is selected.

& $B_2 \rightarrow$ where box-II selected

$$P(B_1) = P(B_2) = \frac{1}{2}$$

Let E be the event where selected card is non prime.

For B_1 : Prime numbers :

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

For B_2 : Prime numbers :

{31, 37, 41, 43, 47}

$$P(E) = P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$$

$$= \frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}$$

Required probability :

$$P\left(\frac{B_1}{E}\right) = \frac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3}+\frac{3}{4}} = \frac{8}{17}$$

9. Official Ans. by NTA (1)

Sol. Given E_1, E_2, E_3 are pairwise indepedent events so $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

& $P(E_1 \cap E_2 \cap E_3) = 0$

Now $P\left(\frac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \frac{P\left[E_1 \cap (\bar{E}_2 \cap \bar{E}_3)\right]}{P(E_1)}$

$$= \frac{P(E_1) - [P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$$

$= 1 - P(E_2) - P(E_3)$

$= [1 - P(E_3)] - P(E_2)$

$= P(E_3^C) - P(E_2)$

10. Official Ans. by NTA (2)

Sol. A : Sum obtained is a multiple of 4.

A = {(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

B : Score of 4 has appeared at least once.

B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

Required probability = $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{1/36}{9/36} = \frac{1}{9}$$

11. Official Ans. by NTA (3)

Sol. First Case: Choose two non-zero digits ${}^9C_2$

Second Case : Number of 5-digit numbers containing both digits = $2^5 - 2$

Choose one non-zero & one zero as digit = ${}^9C_1$

Number of 5-digit numbers containg one non zero and one zero both = $(2^4 - 1)$

$\therefore$ Required prob.

$$= \frac{\left({}^9C_2 \times (2^5 - 2) + {}^9C_1 \times (2^4 - 1)\right)}{9 \times 10^4}$$

$$= \frac{36 \times (32 - 2) + 9 \times (16 - 1)}{9 \times 10^4}$$

$$= \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$$

12. Official Ans. by NTA (3)

Sol. We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$$\Rightarrow 1-\left(\frac{9}{10}\right)^n > \frac{1}{4}$$

$$\Rightarrow \frac{3}{4} > \left(\frac{9}{10}\right)^n \Rightarrow n \geq 3$$

13. Official Ans. by NTA (4)

Sol. $P(6) = \frac{1}{6}$, $P(7) = \frac{5}{36}$

P(A) = W + FFW + FFFFW + ...

$$= \frac{1}{6}+\frac{5}{6}\times\frac{31}{36}\times\frac{1}{6}+\left(\frac{5}{6}\right)^2\left(\frac{31}{36}\right)^2\frac{1}{6}+...$$

$$= \frac{\frac{1}{6}}{1-\frac{155}{216}} = \frac{36}{61}$$

14. Official Ans. by NTA (11)

Sol. 4 dice are independently thrown. Each die has probability to show 3 or 5 is

$$p = \frac{2}{6} = \frac{1}{3}$$

$$\therefore q = 1 - \frac{1}{3} = \frac{2}{3} \text{ (not showing 3 or 5)}$$

Experiment is performed with 4 dices independently.

∴ Their binomial distribution is

$(q + p)^4 = (q)^4 + {}^4C_1 q^3p + {}^4C_2 q^2p^2 + {}^4C_3 qp^3 + {}^4C_4p^4$

∴ In one throw of each dice probability of showing 3 or 5 at least twice is

$$= p^4 + {}^4C_3\, qp^3 + {}^4C_2 q^2p^2 = \frac{33}{81}$$

∴ Such experiment performed 27 times

∴ so expected out comes = np

$$= \frac{33}{81}\times 27$$

= 11

15. Official Ans. by NTA (11.00)

Sol. $P(H) = \frac{1}{2}$

$$P(\bar{H}) = \frac{1}{2}$$

Let total 'n' bomb are required to destroy the target

$$1 - {}^nC_n\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$$

$$1 - \frac{1}{2^n} - \frac{n}{2^n} \geq \frac{99}{100}$$

$$\frac{1}{100} \geq \frac{n+1}{2^n}$$

Now check for value of n

$\boxed{n = 11}$

16. Official Ans. by NTA (2)

Sol. Total numbers in three familes = 3 + 3 + 4 = 10

so total arrangement = 10!

Family 1	Family 2	Family 3
3	3	4

Favourable cases

$$= \underbrace{3!}_{\text{Arrangment of 3 Families}} \quad \underbrace{3!\times 3!\times 4!}_{\text{Interval Arrangment of families members}}$$

∴ Probability of same family memebers are together $= \frac{3!\,3!\,3!\,4!}{10!} = \frac{1}{700}$

so option(2) is correct.

17. Official Ans. by NTA (3)

Sol. Out of 11 consecutive natural numbers either 6 even and 5 odd numbers or 5 even and 6 odd numbers

when 3 numbers are selected at random then total cases = $^{11}C_3$

Since these 3 numbers are in A.P. Let no's are a,b,c

$2b \Rightarrow$ even number

$$a + c \Rightarrow \begin{pmatrix} \text{even} + \text{even} \\ \text{odd} \ + \text{odd} \end{pmatrix}$$

so favourable cases = $^6C_2 + {}^5C_2$

= 15 + 10 = 25

P(3 numbers are in A.P. $= \frac{25}{^{11}C_3} = \frac{25}{165} = \frac{5}{33}$)

18. Official Ans. by NTA (3)

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$\alpha = 1.5 - (0.2 + 0.3 + \beta) + 0.2$

$\alpha = 1.2 - \beta \in [0.85, 0.95]$

(where $\alpha \in [0.85, 0.95]$)

$\beta \in [0.25, 0.35]$