PARABOLA

1. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to :

(1) 128 (2) –64
(3) –128 (4) –32

2. Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to __________.

3. The locus of a point which divides the line segment joining the point (0,–1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2 is-

(1) $9x^2 - 3y = 2$ (2) $9x^2 - 12y = 8$
(3) $x^2 - 3y = 2$ (4) $4x^2 - 3y = 2$

4. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2},-2\right)$, then the equation of the tangent to it at B is :

(1) $2x + y - 24 = 0$ (2) $x - 2y + 8 = 0$
(3) $2x - y - 24 = 0$ (4) $x + 2y + 8 = 0$

5. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$
(3) $192\sqrt{3}$ (4) $128\sqrt{3}$

6. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then :

(1) $MQ = \frac{1}{3}$ (2) PN = 3
(3) $MQ = \frac{1}{4}$ (4) PN = 4

7. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is :

(1) 8 (2) $4\sqrt{5}$
(3) 12 (4) $8\sqrt{5}$

8. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is :

(1) $\frac{4}{3\sqrt{3}}$ (2) $\frac{1}{3\sqrt{3}}$
(3) $\frac{4}{3}$ (4) $\frac{2}{3\sqrt{3}}$

9. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4}$

10. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) $x + 3 = 0$ (2) $x + 2y = 0$
(3) $2x + 1 = 0$ (4) $x + 2 = 0$

11. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is :

(1) $\left(\frac{3}{10},\frac{16}{5}\right)$ (2) $\left(\frac{-16}{5},\frac{53}{10}\right)$
(3) $\left(\frac{6}{5},\frac{53}{10}\right)$ (4) $\left(\frac{-53}{10},\frac{16}{5}\right)$

SOLUTION

1. NTA Ans. (3)

Sol. $y = mx + 4$ is tangent to $y^2 = 4x$

$\Rightarrow m = \frac{1}{4}$

$y = \frac{1}{4}x + 4$ is tangent to $x^2 = 2by$

$\Rightarrow x^2 - \frac{b}{2}x - 8b = 0$

$\Rightarrow D = 0$

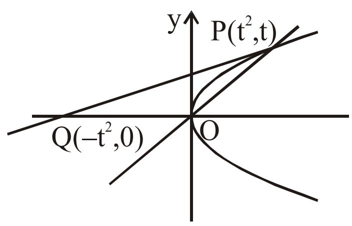
$b^2 + 128b = 0$

$\Rightarrow b = -128, 0$

$b \neq 0 \Rightarrow b = -128$

2. NTA Ans. (0.50)

Sol. $\Delta OPQ = 4$



$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$

$t = 2$ ($\because$ $t > 0$)

$\therefore\ m = \frac{1}{2}$

Ans. 0.50

3. NTA Ans. (2)

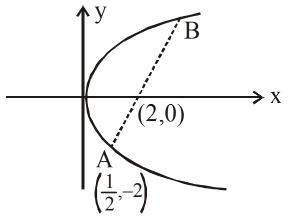
Sol. $\underset{A(0,-1)}{\bullet}\overset{1}{\text{———}}\underset{P(h,k)}{\bullet}\overset{2}{\text{———}}\underset{Q(2t,t^2)}{\bullet}$

$\Rightarrow 3h = 2t$ and $3k = t^2 - 2$

$\Rightarrow 3y = \left(\frac{3x}{2}\right)^2 - 2 \Rightarrow 12y = 9x^2 - 8$

4. NTA Ans. (2)

Sol. $y^2 = 8x$



$4t_1 = -2 \Rightarrow t_1 = -\frac{1}{2}$,

$t_1.t_2 = -1$

$t_2 = -\frac{1}{t_1}$

$\Rightarrow t_2 = 2$

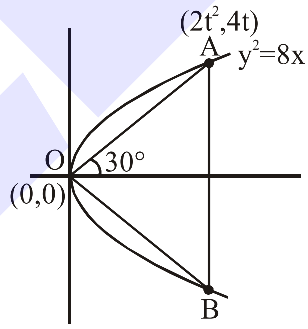
So coordinate of B is (8, 8)

$\therefore$ Equation of tangent at B is

$8y = 4(x + 8) \Rightarrow 2y = x + 8$

5. Official Ans. by NTA (3)

Sol.



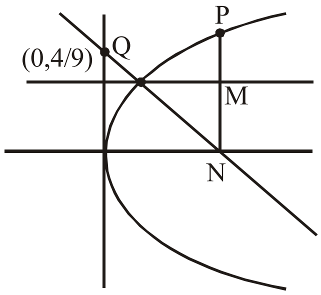
$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} \Rightarrow t = 2\sqrt{3}$

$AB = 8t = 16\sqrt{3}$

$\text{Area} = 256.3\cdot\frac{\sqrt{3}}{4} = 192\sqrt{3}$

6. Official Ans. by NTA (3)

Sol. Let P = $(3t^2, 6t)$; N = $(3t^2, 0)$



$M = (3t^2, 3t)$

Equation of MQ : $y = 3t$

$\therefore\ Q = \left(\frac{3}{4}t^2, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}(x - 3t^2)$$

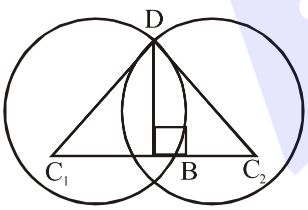
y-intercept of NQ = $4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$

$\therefore\ MQ = \frac{9}{4}t^2 = \frac{1}{4}$

PN = 6t = 2

7. Official Ans. by NTA (1)

Sol. Length of latus rectum = 4



DB = 2

$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$

$C_1C_2 = 8$

8. Official Ans. by NTA (1)

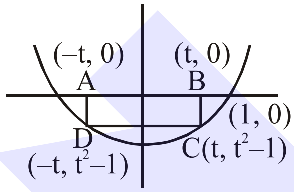
Sol. Area (A) = $2t \cdot (1 - t^2)$

$(0 < t < 1)$

$A = 2t - 2t^3$

$\frac{dA}{dt} = 2 - 6t^2$

$t = \frac{1}{\sqrt{3}}$



$\Rightarrow A_{max} = \frac{2}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = \frac{4}{3\sqrt{3}}$

9. Official Ans. by NTA (3)

Sol. $y = mx + \frac{1}{m}$ (tangent at $y^2 = 4x$)

$y = mx - m^2$ (tangent at $x^2 = 4y$)

$\frac{1}{m} = -m^2$ (for common tangent)

$m^3 = -1$

$\boxed{m = -1}$

$y = -x - 1$

$x + y + 1 = 0$

This line touches circle

$\therefore$ apply p = r

$c = \left|\frac{0+0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

10. Official Ans. by NTA (1)

Sol. $y^2 = 4(x + 1)$

equation of tangent $y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m}$

$y^2 = 8(x + 2)$

equation of tangent $y = m'(x+2) + \frac{2}{m'}$

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

since lines intersect at right angles

$\therefore\ mm' = -1$

Now $y = mx + m + \frac{1}{m}$ ...(1)

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

$y = -\frac{1}{m}x + 2\left(-\frac{1}{m} - m\right)$

$y = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$ ...(2)

From equation (1) and (2)

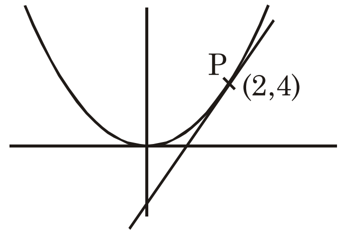
$mx + m + \frac{1}{m} = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$

$\left(m + \frac{1}{m}\right)x + 3\left(m + \frac{1}{m}\right) = 0$

$\therefore\ x + 3 = 0$

11. Official Ans. by NTA (2)

Sol. $y = x^2$



$\left.\frac{dy}{dx}\right|_P = 4$

$(y - 4) = 4(x - 2)$

$4x - y - 4 = 0$

Circle : $(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

passes through (0, 1)

$4 + 9 + \lambda(-5) = 0 \quad \Rightarrow \lambda = \frac{13}{5}$

Circle : $x^2 + y^2 + x(4\lambda - 4) + y(-\lambda - 8) + (20 - 4\lambda) = 0$

Centre : $\left(2 - 2\lambda, \frac{\lambda + 8}{2}\right) \equiv \left(\frac{-16}{5}, \frac{53}{10}\right)$