MATRICES

1. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is :

(1) 3 (2) 1/3
(3) 1/81 (4) 1/9

2. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to:

(1) A^3 (2) A
(3) A^2 (4) I_3

3. If $A = \begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$ and $I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to

(1) 4I – A (2) A – 6I
(3) 6I – A (4) A – 4I

4. The number of all 3×3 matrices A, with enteries from the set {–1,0,1} such that the sum of the diagonal elements of AA^T is 3, is

5. If the matrices $A = \begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, B = adjA and C = 3A, then $\frac{|\text{adj}B|}{|C|}$ is equal to :

(1) 72 (2) 2
(3) 8 (4) 16

6. Let A be a 2×2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements:

(P) If $A \neq I_2$, then $|A| = -1$
(Q) If $|A| = 1$, then $tr(A) = 2$,
where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then:

(1) (P) is true and (Q) is false
(2) Both (P) and (Q) are false
(3) Both (P) and (Q) are true
(4) (P) is false and (Q) is true

7. Let a, b, $c \in R$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix $A = \begin{pmatrix}a & b & c\\ b & c & a\\ c & a & b\end{pmatrix}$ satisfies $A^TA = I$, then a value of abc can be:

(1) $\frac{2}{3}$ (2) $-\frac{1}{3}$
(3) 3 (4) $\frac{1}{3}$

8. Let $A = \{X = (x, y, z)^T : PX = 0$ and $x^2 + y^2 + z^2 = 1\}$ where $P = \begin{bmatrix}1 & 2 & 1\\ -2 & 3 & -4\\ 1 & 9 & -1\end{bmatrix}$, then the set A :

(1) is a singleton
(2) contains exactly two elements
(3) contains more than two elements
(4) is an empty set

9. Let $A = \begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to ________ .

10. Let A be a 3×3 matrix such that adj $A = \begin{bmatrix}2 & -1 & 1\\ -1 & 0 & 2\\ 1 & -2 & -1\end{bmatrix}$ and B = adj (adj A).

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

(1) $\left(9, \frac{1}{9}\right)$ (2) $\left(9, \frac{1}{81}\right)$
(3) $\left(3, \frac{1}{81}\right)$ (4) (3, 81)

11. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$$x = \begin{bmatrix}1\\1\\1\end{bmatrix}, x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix}, x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}, b_1 = \begin{bmatrix}1\\0\\0\end{bmatrix},$$

$b_2 = \begin{bmatrix}0\\2\\0\end{bmatrix}$ and $b_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to :-

(1) $\frac{1}{2}$ (2) 4

(3) $\frac{3}{2}$ (4) 2

12. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$.

If B = A + A^4, then det(B) :

(1) is one (2) lies in (1, 2)

(3) is zero (4) lies in (2, 3)

SOLUTION

1. NTA Ans. (4)

Sol. $b_{ij} = (3)^{(i+j-2)} a_{ij}$

$$B = \begin{bmatrix} a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{bmatrix}$$

$$\Rightarrow |B| = 3 \times 3^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{vmatrix}$$

$= 3^6|A|$

$$\Rightarrow |A| = \frac{81}{27 \times 27} = \frac{1}{9}$$

2. NTA Ans. (1)

Sol. $x^2 + x + 1 = 0$

$\alpha = \omega$

$\alpha^2 = \omega^2$

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow A^4 = A^2 . A^2 = I_3$

$A^{31} = A^{28}. A^3 = A^{3.}$

3. NTA Ans. (2)

Sol. $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$; $|A| = 8 - 18 = -10$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{pmatrix} 4 & -2 \\ -9 & 2 \end{pmatrix}}{-10}$$

$$10A^{-1} = \begin{pmatrix} -4 & 2 \\ 9 & -2 \end{pmatrix} = A - 6I$$

(2) Option

4. NTA Ans. (672.00)

Sol. trace $(AA^T) = \Sigma a_{ij}^2 = 3$

Hence, number of such matrices

$= {}^9C_3 \times 2^3 = 672.00$

5. NTA Ans. (3)

Sol. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

$\Rightarrow \qquad |A| = 6$

$$\frac{|\text{adj}B|}{|c|} = \frac{|\text{adj}(\text{adj}A)|}{|9A|} = \frac{|A|^4}{3^3\,|A|} = \frac{|A|^3}{3^3}$$

$$= \frac{(6)^3}{(3)^3} = 8$$

6. Official Ans. by NTA (4)

Sol. $|A| \neq 0$

For (P) : $A \neq I_2$

So, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$|A|$ can be -1 or 1

So (P) is false.

For (Q); $|A| = 1$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow$ tr(A) = 2

$\Rightarrow$ Q is true

7. Official Ans. by NTA (4)

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$

and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = 1$

$\Rightarrow a + b + c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \pm 1 (1 - 0) = \pm 1$

$\Rightarrow 3abc = 2 \pm 1 = 3, 1$

$\Rightarrow abc = 1, \frac{1}{3}$

8. Official Ans. by NTA (2)

Sol. Given $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}$, Here |P| = 0 & also

given PX = 0

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{matrix} x+2y+z=0 \\ -2x+3y-4z=0 \\ x+9y-z=0 \end{matrix}\right\} \because D=0, \text{so system have}$$

infinite many solutions,

By solving these equation

we get $x = \frac{-11\lambda}{2}; y = \lambda; z = \frac{7\lambda}{2}$

Also given, $x^2 + y^2 + z^2 = 1$

$$\Rightarrow \left(\frac{-11\lambda}{2}\right)^2 + (\lambda)^2 + \left(\frac{7\lambda}{2}\right)^2 = 1$$

$$\Rightarrow \lambda = \pm\frac{1}{\sqrt{\frac{121}{4}+1+\frac{49}{4}}}$$

so, there are 2 values of λ.

$\therefore$ so, there are 2 solution set of (x,y,z).

9. Official Ans. by NTA (10)

Sol. $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2+x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$$

$a_{11} = (x^2 + 1)^2 + x^2 = 109$

$\Rightarrow x = \pm 3$

$a_{22} = x^2 + 1 = 10$

10. Official Ans. by NTA (3)

Sol. $C = \text{adj } A = \begin{vmatrix} +2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix}$

$|C| = |\text{adj } A| = +2(0 + 4) + 1.(1 - 2) + 1.(2-0) = +8 - 1 + 2$

$|\text{adj } A| = |A|^2 = 9 = 9$

$\lambda = |A| = \pm 3$

$|\lambda| = 3$

$B = \text{adj } C$

$|B| = |\text{adj } C| = |C|^2 = 81$

$|(B^{-1})^T| = |B|^{-1} = \frac{1}{81}$

$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$

11. Official Ans. by NTA (4)

Sol. $Ax_1 = b_1$

$Ax_2 = b_2$

$Ax_3 = b_3$

$$\Rightarrow |A|\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$\Rightarrow |A| = \frac{4}{2} = 2$

12. Official Ans. by NTA (2)

Sol. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$B = A + A^4$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$B = \begin{bmatrix} (\cos\theta + \cos 4\theta) & (\sin\theta + \sin 4\theta) \\ -(\sin\theta + \sin 4\theta) & (\cos\theta + \cos 4\theta) \end{bmatrix}$

$|B| = (\cos\theta + \cos 4\theta)^2 + (\sin\theta + \sin 4\theta)^2$

$|B| = 2 + 2\cos 3\theta$, when $\theta = \frac{\pi}{5}$

$|B| = 2 + 2\cos\frac{3\pi}{5} = 2(1 - \sin 18)$

$|B| = 2\left(1 - \frac{\sqrt{5}-1}{4}\right) = 2\left(\frac{5-\sqrt{5}}{4}\right) = \frac{5-\sqrt{5}}{2}$