MATHEMATICAL REASONING

1. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

(1) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(2) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

(3) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(4) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

2. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) p

(2) q

(3) ~p

(4) ~q

3. Which of the following statements is a tautology?

(1) $\sim(p \vee \sim q) \rightarrow p \vee q$

(2) $\sim(p \wedge \sim q) \rightarrow p \vee q$

(3) $\sim(p \vee \sim q) \rightarrow p \wedge q$

(4) $p \vee (\sim q) \rightarrow p \wedge q$

4. Which one of the following is a tautology ?

(1) $P \wedge (P \vee Q)$ (2) $P \vee (P \wedge Q)$

(3) $Q \rightarrow (P \wedge (P \rightarrow Q))$ (4) $(P \wedge (P \rightarrow Q)) \rightarrow Q$

5. If $p \rightarrow (p \wedge \sim q)$ is false, then the truth values of p and q are respectively :

(1) F, T (2) T, T

(3) F, F (4) T, F

6. Negation of the statement :

$\sqrt{5}$ is an integer or 5 is irrational is :

(1) $\sqrt{5}$ is irrational or 5 is an integer.

(2) $\sqrt{5}$ is not an integer and 5 is not irrational.

(3) $\sqrt{5}$ is an integer and 5 is irrational.

(4) $\sqrt{5}$ is not an integer or 5 is not irrational.

7. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :

(1) If I will catch the train, then I reach the station in time.

(2) If I do not reach the station in time, then I will not catch the train.

(3) If I will not catch the train, then I do not reach the station in time.

(4) If I do not reach the station in time, then I will catch the train.

8. Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \rightarrow q$

(2) $(q \rightarrow p) \vee \sim(p \rightarrow q)$

(3) $(p \rightarrow q) \wedge (q \rightarrow p)$

(4) $(\sim q) \vee (p \wedge q) \rightarrow q$

9. The proposition $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to:

(1) $(\sim p) \vee q$

(2) q

(3) $(\sim p) \wedge q$

(4) $(\sim p) \vee (\sim q)$

10. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) T, F, T

(2) F, T, F

(3) T, T, F

(4) T, T, T

11. Given the following two statements :

(S_1) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

(S_2) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.

Then :

(1) only (S_1) is correct.

(2) both (S_1) and (S_2) are correct.

(3) both (S_1) and (S_2) are not correct.

(4) only (S_2) is correct.

12. Contrapositive of the statement:

'If a function f is differentiable at a, then it is also continuous at a', is :-

(1) If a function f is continuous at a, then it is not differentiable at a.

(2) If a function f is not continuous at a, then it is differentiable at a.

(3) If a function f is not continuous at a, then it is not differentiable at a.

(4) If a function f is continuous at a, then it is differentiable at a.

13. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

(1) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

(2) $(x \wedge \sim y) \vee (\sim x \wedge y)$

(3) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(4) $(x \wedge y) \wedge (\sim x \vee \sim y)$

14. The statement $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is :

(1) a contradiction

(2) equivalent to $(p \wedge q) \vee (\sim q)$

(3) a tautology

(4) equivalent to $(p \vee q) \wedge (\sim p)$

15. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

16. Consider the statement :

"For an integer n, if $n^3 - 1$ is even, then n is odd."

The contrapositive statement of this statement is :

(1) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(2) For an integer n, if n is even, then $n^3 - 1$ is odd.

(3) For an integer n, if n is odd, then $n^3 - 1$ is even.

(4) For an integer n, if n is even, then $n^3 - 1$ is even.

SOLUTION

1. NTA Ans. (2)

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

$(A \subseteq B) \wedge (B \subseteq D) \longrightarrow (A \subseteq C)$

Contrapositive is

$\sim(A \subseteq C) \longrightarrow \sim(A \subseteq B) \vee \sim(B \subseteq D)$

$A \not\subseteq C \to (A \not\subseteq B) \vee (B \not\subseteq D)$

2. NTA Ans. (3)

Sol. $(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$

$\equiv \sim p \vee (q \wedge \sim q)$

$\equiv \sim p \vee C \equiv \sim p$

3. NTA Ans. (1)

Sol. $\sim(p \vee \sim q) \to p \vee q$

$(\sim p \wedge q) \to p \vee q$

$\sim\{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\sim(\sim p \wedge f)$

(1) Option

4. NTA Ans. (4)

Sol. (1) $P \wedge (P \vee Q) \equiv P$

(2) $P \vee (P \wedge Q) \equiv P$

(3) $Q \to (P \wedge (P \to Q))$

$\equiv Q \to (P \wedge (\sim P \vee Q)) \equiv Q \to (P \wedge Q)$

$\equiv (\sim Q) \vee (P \wedge Q) \equiv (P \vee (\sim Q))$

(4) $(P \wedge (P \to Q)) \to Q$

$\equiv (P \wedge (\sim P \vee Q)) \to Q \equiv (P \wedge Q) \to Q$

$\equiv ((\sim P) \vee (\sim Q)) \vee Q \equiv (\sim P) \vee t \equiv t$

5. NTA Ans. (2)

Sol. $p \to (p \wedge \sim q)$ is F $\Rightarrow$ p is T & $p \wedge \sim q$ is F $\Rightarrow$ q is T

$\therefore$ p is T, q is T

6. NTA Ans. (2)

Sol. $p = \sqrt{5}$ is an integer.

q : 5 is irrational

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$= \sqrt{5}$ is not an integer and 5 is not irrational.

7. Official Ans. by NTA (3)

Sol. Let p denotes statement

p : I reach the station in time.

q : I will catch the train.

Contrapositive of $p \to q$

is $\sim q \to \sim p$

$\sim q \to \sim p$: I will not catch the train, then I do not reach the station in time.

8. Official Ans. by NTA (1)

Sol. Option (1) is

$\sim p \wedge (p \vee q) \to q$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q) \to q$

$\equiv C \vee (\sim p \wedge q) \to q$

$\equiv (\sim p \wedge q) \to q$

$\equiv \sim(\sim p \wedge q) \vee q$

$\equiv (p \vee \sim q) \vee q$

$\equiv (p \vee q) \vee (\sim q \vee q)$

$\equiv (p \vee q) \vee t$

so $\sim p \wedge (p \vee q) \to q$ is a tautology

9. Official Ans. by NTA (1)

Sol. $p \to \sim (p \wedge \sim q)$

$= \sim p \vee \sim (p \wedge \sim q)$

$= \sim p \vee \sim p \vee q$

$= \sim (p \wedge q) \vee q$

$= \sim p \vee q$

10. Official Ans. by NTA (3)

Sol. $(p \wedge q) \rightarrow (\sim q \vee r)$ = false

when $(p \wedge q)$ = T

and $(\sim q \vee r)$ = F

So $(p \wedge q)$ = T is possible when p = q = true

$\therefore$ ~q = False (q = true)

So $(\sim q \vee r)$ = False is possible if r is false

$\therefore$ p = T, q = T, r = F

11. Official Ans. by NTA (3)

Sol. Let TV(r) denotes truth value of a statement r.

Now, if TV(p) = TV(q) = T

$\Rightarrow TV(S_1) = F$

Also, if TV(p) = T & TV(q) = F

$\Rightarrow TV(S_2) = T$

12. Official Ans. by NTA (3)

Sol. p = function is differantiable at a

q = function is continuous at a

contrapositive of statement $p \rightarrow q$ is

$\sim q \rightarrow \sim p$

13. Official Ans. by NTA (3)

Sol. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$x \leftrightarrow \sim y \equiv (x \rightarrow \sim y) \wedge (\sim y \rightarrow x)$

$\because (p \rightarrow q \equiv \sim p \vee q)$

$x \leftrightarrow \sim y \equiv (\sim x \vee \sim y) \wedge (y \vee x)$

$\sim (x \leftrightarrow \sim y) \equiv (x \wedge y) \vee (\sim x \wedge \sim y)$

14. Official Ans. by NTA (3)

Sol.

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$p \vee q$	$p \rightarrow p \vee q$	$(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

15. Official Ans. by NTA (3)

Sol. Negation of $\phi \vee (\sim p \wedge q)$

$p \vee (\sim p \wedge q) = (p \vee \sim p) \wedge (p \vee q)$

$= (T) \wedge (p \vee q)$

$= (p \vee q)$

now negation of $(p \vee q)$ is

$\sim (p \vee q) = \sim p \wedge \sim q$

16. Official Ans. by NTA (2)

Sol. Contrapositive of $(p \rightarrow q)$ is $\sim q \rightarrow \sim p$

For an integer n, if n is even then $(n^3 - 1)$ is odd