INVERSE TRIGONOMETRY FUNCTION

1. The domain of the function $f(x)=\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty,-a]\cup[a,\infty)$. Then a is equal to :

(1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$

(3) $\frac{\sqrt{17}}{2}+1$ (4) $\frac{\sqrt{17}}{2}$

2. $2\pi-\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right)$ is equal to:

(1) $\frac{7\pi}{4}$ (2) $\frac{5\pi}{4}$

(3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

3. If S is the sum of the first 10 terms of the series

$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\tan^{-1}\left(\frac{1}{21}\right)+\ldots,$

then tan(S) is equal to :

(1) $\frac{5}{11}$ (2) $-\frac{6}{5}$

(3) $\frac{10}{11}$ (4) $\frac{5}{6}$

ALLEN

SOLUTION

1. Official Ans. by NTA (1)

Sol. $f(x)=\sin\left(\frac{|x|+5}{x^2+1}\right)$

For domain :

$-1\le\frac{|x|+5}{x^2+1}\le 1$

Since |x| + 5 & x^2 + 1 is always positive

So $\frac{|x|+5}{x^2+1}\ge 0\ \forall x\in R$

So for domain :

$\frac{|x|+5}{x^2+1}\le 1$

$\Rightarrow |x| + 5 \le x^2 + 1$

$\Rightarrow 0 \le x^2 - |x| - 4$

$\Rightarrow\ 0\le\left(|x|-\frac{1+\sqrt{17}}{2}\right)\left(|x|-\frac{1-\sqrt{17}}{2}\right)$

$\Rightarrow\ |x|\ge\frac{1+\sqrt{17}}{2}$ or $|x|\le\frac{1-\sqrt{17}}{2}$ (Rejected)

$\Rightarrow\ x\in\left(-\infty,-\frac{1+\sqrt{17}}{2}\right]\cup\left[\frac{1+\sqrt{17}}{2},\infty\right)$

So, $a=\frac{1+\sqrt{17}}{2}$

2. Official Ans. by NTA (3)

Sol. $2\pi-\left(\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)\right)$

$=2\pi-\left(\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{16}{63}\right)\right)$

$=2\pi-\left(\tan^{-1}\left(\frac{63}{16}\right)+\tan^{-1}\left(\frac{16}{63}\right)\right)$

$=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$

3. Official Ans. by NTA (4)

Sol. $S=\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\ldots$

$S=\tan^{-1}\left(\frac{2-1}{1+1.2}\right)+\tan^{-1}\left(\frac{3-2}{1+2\times 3}\right)+\tan^{-1}\left(\frac{4-3}{1+3\times 4}\right)+\ldots.+\tan^{-1}\left(\frac{11-10}{1+10\times 11}\right)$

$S=(\tan^{-1}2-\tan^{-1}1)+(\tan^{-1}3-\tan^{-1}2)+(\tan^{-1}4-\tan^{-1}3)+\ldots+(\tan^{-1}(11)-\tan^{-1}(10))$

$S=\tan^{-1}11-\tan^{-1}1=\tan^{-1}\left(\frac{11-1}{1+11}\right)$

$\tan(S)=\frac{11-1}{1+11\times 1}=\frac{10}{12}=\frac{5}{6}$

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